Modern Computer Vision

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Lecture-62

Now one thing right which we know is first of all right I mean based upon the camera see this one projection matrix that we have already learnt which we did from a single view and what we know is that I can write x in the left plane. Suppose now when I put a kind of you know this one a ~ or a wiggle on top it basically means that it is a homogeneous coordinate okay that means this is like xy1 okay. So x ~ right I know is a projection let us say if p is the projection matrix for the left camera whatever it you can write it as pl or whatever it if you wish then you can write it as p left and then you have x ~ and again this is a homogeneous representation of the 3D coordinate representation of the 3D coordinate. So it is like xyz1 and you know that pl is 3 X 4 and this is 3 X 1 this guy is 4 X 1 and this p it is a pl right itself we know some intrinsic matrix of the left camera that is like kl and then you will typically have some with respect to some world coordinate you would have had rt and then you would have had x ~. But now we are assuming that the world coordinate is centered at what you call centered at this first camera therefore this just simplifies to kl i0 x ~ okay this is what it is. This is the first coordinate right and suppose by some means right suppose I knew where was this okay anyway right let us see where the second guy will go second guy let me indicate it as x ~ prime or I could have written p prime right for that matter in k prime whatever okay for the for the right camera.

So this will have a p right matrix a projection of the right camera projection matrix corresponding to the right camera acting on the same guy x ~ then that we can write as k right and then some rt right because that is somewhere else the even center of that camera x ~ okay this we know right this we understand. Now what we want to know is this right epipolar geometry this comes from our single view thing okay this is still not relating the 2 only thing that relates the 2 is x ~ being common and the fact that there is an r and t right mutual sort of you know motion between the 2 cameras but otherwise right we still have not we still have not seen what is the what is the relation. Now if you kind of go back to this this thing right what you find for example what you find is if you do if you do a back projection right if you kind of if you go back on this ray right now what we know is I do not know where to stop right because I do not know where that exists but one thing is clear right if I try to if I try to do a projection of each of these points right I mean it has got to be one of these right it has got to be one of these points but if I try to if I try to try to act my p right right which is the projection matrix that I showed okay if I try to right if I try

to act it okay then what can be then what happens is right so you will have a line okay going through this I mean right I am you know I am I am just drawing it something like you know approximately right. So so right it will it will cut the image plane at you see some other location which actually means that right that is also that is also the one that gives you a sense for depth because eventually right when we are still not looking at how depth is calculated but then a disparity is how far is the shift between X and X ‘ right because you have a translation you expect that things that are closer will move on the image plane more that means the image of a point that is closer because of why is that so no no but then where does it if you want to talk into which equation governs that Fx by Z right Fx by Z is what actually governs that right so when you have Z is small the movement is large right.

Now when you when you have a situation like that so what this means is that a point that is farther off right it will it will it will actually introduce a shift between X and X ‘ that is higher sorry which is farther will introduce a shift that is smaller and the point that is closer will introduce a shift that is actually higher and that is what gives you a sense for depth okay. Now what what what I am showing here is that if I were to if I were to if I were to take any of these points on the ray but and if I try to say project it okay well I have a figure right which I will show later but then I just want you to understand what I am doing because when you when I will show a plotted figure right everything looks very easy and it is convenient okay. Now what happens is right this one okay now all these points will pass through what is called what is called an epipolar line that also that also is a line okay that is a line and and all these points right if you try to see okay where they are actually hitting the image plane they will all hit they will all lie on this line and this line right incidentally okay this line is not actually correct the way I have drawn how should I have drawn this line the way I have drawn it is incorrect why is it incorrect? I mean in the sense that not grossly wrong but somewhat wrong how should this be can somebody tell me? There is something there which should which should should also obey I cannot just draw it arbitrarily there is something else that should happen just just you look at the geometry and tell me exactly should cut the baseline because see the same point as I keep on coming right I will also hit the C no that is also a point on that ray right and if I if I apply if I actually P right on C ~ right it maps to this location here correct that means that this point should also be a part of that line no it cannot be somewhere else okay so well that is why I said that I have not drawn the diagram diagram correctly but then imagine that this line it could be inclined and all okay it does not have to be parallel to the image plane and all it could be some inclination okay so I hope that inclination is clear I mean I am not trying to draw lines parallel to the boundaries of the image plane and all okay it can be at any inclination so imagine that that right it is some inclined line like that okay and this inclined line is actually called the right epipolar line right epipolar line epipolar line and the place where this where the center of the left hits the right I mean after you do a projection hits

the image plane call that E ‘ that point okay that has a name it is called an epi pole it is called an epi pole it is called the right epi pole and similarly right you can also you can also imagine that that out here right there should be a left epi pole right so the same thing right so for example now the point is pole is well I mean I thought I will talk about it down the line but it is something like this what do you think it should be E ‘ is equal to what should it be you tell me know what can it be I mean it cannot be anything else P right acting on C but it is only way you can write it and E should be P left acting on C ‘ right okay that is that is why you are say payment just like we did X and X ~ no that was some 3D point of the world C is another 3D point of the world it map it to the right epipolar right image plane it will hit it at E ‘ and E ‘ is simply P right because it is it is it is from the 3D scene on to the right image plane therefore you have to use P right E is from the 3D world on to the left image plane therefore you have to write P you have to use P left and then but then that is C ‘ coming on to this and this is C coming on to the right plane right so that is the way you define the epi poles okay. Now yeah let me go to that figure I think that figure can kind of better explain something else that I want to say okay well the other thing right is also clear okay now before that right let us just go and look at this other slide that I wanted to sort of show you okay these are all I am going to share but then right I mean everything is there in these slides but huh just kind of look at this see there is something called the right epipolar plane right we talked about when I say epipolar line right which is this guy so now I think you know in this diagram it should probably be more clear now as to what I was trying to draw there it is a real nice picture right not the one that I showed. So here you can see that you can see that right you have this line and it passes through the epi pole and one other thing is that you know the epi pole need not be inside the inside the C image light on the image plane but then it does not mean that you know if you find the epipolar I mean epipolar lines well okay we will talk about it later okay right now right is being shown as though E ‘ is lying inside the image grid okay but that is not going to say necessary okay.

Now the question is this so the same way right what I could also do is perhaps is that so for example right I mean you know if this was my say x ~ whatever right I mean it is okay let me go back to my own diagram so right if this is my x this is my whatever what did I draw my x ‘ right so my x ‘ was here right. So similarly right if I had drawn a ray okay that is like this like the one right that is going through x ‘ and now I want to search for where is my x then again I would have to do the same thing in the sense that I would have to take a point right send it through this send it through this and so on and now this line should be somewhat like this right it should go through this epipolar right and that is the left epipolar line. So you could start from either image does not matter you can start from the left or the right but whichever way you start then you will say again this could be inclined okay I am drawing it almost it is also parallel to the image axis and all not true. So it can be some inclined line that passes that cuts this epipolar okay and that is the left

epipolar line epipolar line then let us go back to this picture now so now right this will become more clear. So now if you kind of if you kind of see think of the think of the think of the 3D point right we just oops we just somewhere oh okay I think it is an image so I cannot write on it oh maybe if I view so no I think I can write right so if this is your point right this now look at the plane right that gets formed between O, E, E ‘, O ‘ and X right that plane is called actually they see right epipolar plane okay that is the name of the plane that actually contains your 3D point and then the 2 centers and then the 2 you see right epi poles just to know that know that right you have something called an right epipolar plane.

Now suppose I asked you right if I had another point let us say some other point right let us say this is some X2 let us say this was X1 now I have another 3D point X2 in the world and I do the same thing right I repeat in the sense that I have right suppose I again do this thing I join this and similarly right I can join that with that center so it kind of hits it somewhere let us say here and maybe this guy hits it somewhere here. Now what should happen see again right you see I mean after all right the job is to find out all such points right the I mean depth of all those points the first thing is that right this we are doing in order to understand where to search for right this whole exercise of the epipolar geometry is basically directed to at knowing where should I look for right and of course you know we still have not shown how to arrive okay now this L ‘ is actually the epipolar line right we have not talked about mathematically how to arrive at that but yeah but we know that such a line exists and therefore the search should be carried out on that line along that line and similarly now if you have X2 which is another point and suppose I try to do this what kind of an epipolar line do you kind of expect now yeah there will be another epipolar line but then what will it what will be it will also pass through the you know epipolar right because every ray right every ray in the scene has to actually pass through this center and this center right which then means that when you travel on this ray eventually you will come to O and then and that point it should also pass through epipolar therefore it I mean if you think of if you think of any epipolar line right any number of epipolar lines right corresponding to different different 3D points you can actually you can actually imagine a bunch of epipolar planes right can you kind of can you imagine that I mean there is one epipolar line for X and then keeping the baseline fix you can actually rotate it right you can rotate the epipolar line to hit another point X1 and there is an there is an epipolar plane there and you can think of the epipolar line as intersection of this plane right with the image plane where it cuts the image plane right see for example this epipolar line right is where this epipolar plane cuts the image plane right whether left or right right that is where the epipolar line is so you can arrive at the solution in various different ways okay there is no unique way of telling how to arrive at the epipolar line I will follow one strategy but there could be people right that follow you know different strategy but now the point is the other thing to actually notice is that of course one thing is that the epipolar plane where it intersects the image plane is where the epipolar line is for that point right and you have

a family of such planes now right I mean on all of them which you can rotate about the about the baseline right when I can go like this the baseline remains the same because right everything is everything is around the baseline whatever you do everything comes back to the baseline but at the point that plane might go might go like that but then again the baseline is common I may have a point there again the baseline is common right that is because that is because of the way the way the underlying geometry is right and all of this is not assuming right so these are these are what are called what are called the converging cameras it is right as you can see that right these are courtesy converging camera converging means means that the mutual what you call the motion right should be sorry not parallel means the mutual motion should be such that r is identity and the translation vector is simply Tx I mean if at all you can allow for a Ty but you cannot allow for a Tz you cannot go back and forth or you cannot go like this right you cannot go what would be Tz right Tz will be like going forward and backward right that is called zooming right you cannot do that that is not that is not parallel cameras okay. So parallel cameras are exactly like this okay so you can only move on the image plane this baseline should coincide baseline motion should coincide with the x axis and that is what is parallel camera but our whatever we are kind of right you know some of these things right will take a will take slightly a tricky form if you make them into parallel cameras but actually parallel camera is what is simple to handle but this epi pole and all right which will sort of what do you think will happen to the epi poles? No I mean it is like this right no where do the epi poles go now anyway I mean we will see that later okay but right now right so for the converging camera it is easy to understand so we will actually go with that okay. So now the point is right so the idea is that right you have got a family of these planes right epi polar planes and that is called a pencil epi polar this one a pencil it is called a pencil okay these are all things right that are just nomenclatures right that we should be aware of because somebody says you know epi polar pencil right then we should not be wondering what that is right. So this epi polar geometry okay involves so the epi polar geometry involves a notion of epi poles okay which I have which I have already said what they are right and then it involves a involves a notion of epi polar lines it involves the notion of epi polar planes and it involves a notion of I think it is epi polar pencil.

Let me just check the last one I think it is called epi polar pencil anyway it does not matter I think that is what it is called oh yeah epi polar pencil okay. Now okay now the point is right now we want to kind of look at how to actually relate right relate the I mean just as a kind of homography right we related points okay aX the right 2 images but here right we have a complete full blown 3D kind of a situation but we still want to be able to say something that we want to know what would be the condition right between the between these 2 points right what they should satisfy algebraically right. So now the now the now the arrive is now what we want to look at is the algebraic or the algebraic sort of

representation of the epi polar geometry. Algebraic representation of epi polar I mean epi polar geometry geometrically it is clear right what it should be like.