Modern Computer Vision

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Lecture-35

Now, if you if you and then the other kind of gradient that you do is kind of you know second order this one a differencing and that is called that is called a Laplacian. So, Laplacian is like you know the the the the symbol is a del square that is how that is how you indicate it and this is like you know in 2 D it is like dou square by dou x square + dou square by say dou y square it on whatever you are applying it. So, for example, if I say if I say del square f right. So, f is my image and I and if I and if I wanted to wanted to apply you know if I say that I am applying a Laplacian it is what it is what it means that means I am doing this over square and the the filter right that discrete equivalent of this actually looks like this − 4 1 or of course, I write I mean it depends 1 1 1 1 sorry is there a doubt somewhere. Now, one way one way to explain I mean there are there are you know different ways to ways to actually write explain this and one way is simply to use use a Taylor series kind of thing. So, what you can say is you know if I had f of x + h of course, you know this is a discrete case, but first you know let us take a continuous case then right then we have f of f of x + h * f ' of x + h square by 2 * f double ' of x and so on.

Then the other trick that you play is you know do this f of x − h and there you have like f of x − h * f ' of x + h square by 2 * f 2 ' of x and so on. And then we will we will neglect these higher order terms and then if you simply add these two right you will get like f of x + h + f of x − h it is equal to 2 f of x and then + h square f ' f double ' of x or in a sense the approximation for f double ' of x is f of x + h − 2 f of x + f of x − h by h square right and discrete approximation if you take a reach to be 1 right. So, then it looks like you know when you wanted a horizontal kernel right then what you are looking at is something like this right you are looking at 1 − 2 1 all else 0 and if you now repeat this for the for the for the for y right for the other this one dimension then you will get something like this like you will get 1 − 2 1 and then you will get 0's elsewhere and now you have to add the 2 if you add the 2 then you will end up with 1 1 1 1 on the on the 4 corners and then − 4 in the middle there are other approximations also right it is not like you know this is only one, but the most common approximation right is is this. And one of the uses of actually a Laplacian is that and it is also called a 0 crossing this one a detector 0 0 crossing a detector and this we will encounter this 0 crossing and all is something like that that we will be certainly interested in.

So, for example, I think of see normally your your edges are not like you know see seldom will you find an edge like that in an image which is jumps up you know suddenly from 0 to 255 what is normally common is something like that I mean you know you have a gradient you know and then and then it eventually becomes like 0 to 255. So, it is it is normally hard to pin down right where to where the where the where the edge location should be right. So, for example, if you had say f of x like this then what you can do is you know if you do and if you do and say f ' of x right then what happens then we have then right then we will have something like that right I mean we will have an increase and then finally, right it is good now it would kind of you know goes down to 0 always increasing therefore, it is always on the positive side. Then now if you try f 2 ' of x right what will happen if you tried f 2 ' of x on this signal on let us say f f ' if you take a derivative of that then it is like it is like right it is like you know initially climbing and then then right eventually hitting a 0 and then the signal is actually falling right f '. So, it goes right negative yeah.

So, yeah. So, right. So, you can. So, you can think of something like that right. So, at at 0 right it will cross the 0 will change its sign and then it kind of right goes on to the other side.

Now this is a 0 crossing right and this and this and this 0 crossing is actually is an you know is an important information provided it is also accompanied by kind of examining what is happening you know what what is the value of f ' of x because I mean if I just give you a give you a uniform image right I mean you will also have 0s everywhere. So, it does not mean that just because you have a 0 right for the for the Laplacian it does not mean that you know that is that is a very interesting point you will get 0s even when you have a homogeneous image. See one of the things that you must have observed is that whereas, an average always sums to 1 the weights of difference will always sum to 0. So, like 1 2 − 1 − 1 − 2 − 1 sum to 0 1 1 1 1 − 4 in the middle sums to 0 right that is all because it is supposed to they are all supposed to flag activity this when you have a difference right it is like you know you want to kind of you know find out activity where there is some activity and activity typically is in terms of a gradient right and that is why we call them as gradient operators. So, so there is no activity in an image right it is like a smooth wall and you go apply a Laplacian what will you get you will get 0s everywhere right, but that is not a 0 crossing right.

So, when you have a 0 crossing you can directly probably go and find out where the 0 crossing is or if you find that you know a Laplacian is ending at to be 0 you can go and find out with respect to what is going on and that way right you will know whether it is an important point or not. And that and you know typically a Laplacian is not actually used as a standalone because it is very sensitive to noise I mean the first derivative itself is sensitive to noise doing a second derivative makes it even more sensitive to noise and therefore, what one does is one uses what is called a log which is a Laplacian of the Gaussian. So, this is called Laplacian of the Gaussian and that and then given that you know this is all commutative operation just as in 1 d right I mean H 1 convolved with H 2 is the same as H 2 convolved with H 1 and so on right. So, so here also here also right I mean here also if you see right what you can say is you can say that you know I want to I want to apply del square right my Laplacian, but then I am not going to apply directly on the image because then it makes it very noisy. So, if my image is f then I will actually convolve with with actually Gaussian.

So, g is a Gaussian and after I convolve that is like you know smoothing it out a little bit that is like low pass filtering it and then and then I would apply a Laplacian because that way I keep the noise under control, but this is also the same as right I mean this is all straight forward right for you to show that this is also the same as f convolved with say right del square of g and this is the convolution right. So, all of that will will go through. So, so in a sense right this is an operator by itself just like del square is a Laplacian del square g where g is a Gaussian is a is a is a Laplacian of the Gaussian and this goes by a particular name what is that Laplacian it has a particular shape which you have seen in movies. Mexican hat. So, if you if you plot it right no it will look like a Mexican hat and and you know it is it is again it is again in the Fourier domain and all that you can what kind of a filter do you think it is if you have not attended my class before I mean otherwise you would know it, but somebody read from the rest if you have not heard me before what kind of what kind of a function do you think this must be in a in the the Fourier domain after all you can you can I mean you can you can examine it you know the Fourier domain what do you think it might be like anybody a ∂ I mean sorry a d by d t if you do what do you get I mean if I if I have f of t it has let us say f of j omega 1 d right if I do d by d t what happens j omega f of j omega right that is what you get.

So, d square by d t square second derivative – omega square. So, now, can you tell me what is what is what is that kind of a signal if it is a Gaussian – omega square and then g of omega is a is a

Gaussian right. So, in 1 d itself what will it be that is forget about 2 d and omega square e raise to – omega square by 2 * sigma square what is that what will it will it be what kind of filter will that be at omega equal to 0 at omega very high 0. So, what kind of filter is that a band pass. So, it is a band pass filter right I think we will stop here I will sort of exceeded that.