

Analog Electronic Circuits
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Lecture - 78

2 Stage Operational Amplifier and Miller Compensation Canceling the R.H.P Zero

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Lecture 37

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$$\frac{V_o(s)}{V_i(s)} = \frac{A_{dc} \left(1 - \frac{s}{\omega_z}\right)}{\left(1 + \frac{s}{\omega_d}\right) \left(1 + \frac{s}{\omega_o}\right)}$$

$$\omega_z = \frac{g_{m2}}{C_c}$$

$$\omega_d = \frac{1}{r_1 C_c g_{m2} r_2}$$

$$\omega_u = \omega_z = A_{dc} \omega_d = \frac{g_{m1}}{C_c}$$

So, in the last class we were winding up looking at the frequency response of the 2 Stage Operational Amplifier and this is the small signal equivalent. So, this is the first stage $g_m v_i$ goes into resistance r_1 which denotes the total output resistance. For the first stage then there is a parasitic capacitance C_1 , this is the dominant pole compensating capacitor, this is the transconductance of the second stage, I call that v_x this is $g_{m2} v_x$.

This is r_2 , C_2 and there is a voltage control voltage source. Which we said has got such a large bandwidth that for all practical purposes it can be considered a unity gain bar. Now, after compensation we saw a couple of things. So, this $V_o(s)/V_i(s)$ is the dc gain $A_{dc} (1 - s/\omega_z)/(1 + s/\omega_d)(1 + s/\omega_o)$, this is the ω_o is the second pole.

ω_z is g_{m2}/C_c , ω_d is $1/r_1 C_c g_{m2} r_2$ approximately of course, and the unity gain frequency which is ω_u is basically $A_{dc} \omega_d$ which is the surprisingly simple expression g_{m1}/C_c , alright. Yesterday we saw the intuition for this result.

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$$V(s) = \frac{1}{\left(1 + \frac{s}{\omega_d}\right) \left(1 + \frac{s}{\omega_c}\right)}$$

$$\omega_u = \frac{1}{r_f C_c \beta_{ol} r_o}$$

$$UGF \equiv \omega_u = A_{dc} \omega_d = \frac{S_u}{C_c}$$

Miller compensation \equiv Pole-splitting compensation

$$LHP \text{ zero} = h(t) + k h'(t)$$

$$h(t - t_d) \approx h(t) - t_d h'(t) + \dots$$

$$H(s) = s t_d H(s)$$

And we woke up yesterday. We were basically looking at the poles positions of the uncompensated system, right. Remember that the two poles of the uncompensated system are close to each other, right? And so, let us call them P_1 and P_2 and after the Miller capacitor is added, one pole moves very close to the origin, the other one moves further away. So, this is basically you can see that the poles are split. So, Miller compensation is also often called pole splitting compensation.

And another, you know, minor point that I would like to make is that if somebody, if a company is selling you a general purpose op-amp right, where when you say a general purpose op-amp, it means that you have no real idea what the user is going to use the op-amp for, right. So, you just assume some broad set of boundary conditions in the sense that yeah you know you hope that the user will not put a feedback factor with a gain greater than 1, right.

And then basically make sure that your compensating capacitor C_c is chosen so that the amplifier is unity gain stable, correct. So, this way you know, you do not have to be a user as far as the user is concerned, you do not have to worry about, you know, will my closed loop be stable and so on, right?

As long as the feedback factor is less than 1, the system will be stable. That is one aspect. The other aspect that I would like to draw your attention to is this whole splitting business, right. So, I mean, have you seen something similar occur elsewhere? Ok, all of you, you know,

probably still remember that, you know, you have an atom and then there are electrons going around the atoms and therefore, there are only discrete levels of energy that these electrons can have, right.

So, you have discrete levels of energy, right. So, now when you bring two atoms close together, what happens? And they interact with each other, what happens to the energy levels possible now? Energy levels split, right. If you bring more and more atoms together, the split becomes a band, right, ok. And what is, you know, when you bring atoms close together, right. When you say they are interacting, what does it actually mean mathematically? There is some coupling between two systems, basically, correct.

And, you know, energy levels are related to the Eigenvalues of the Schrodinger equation, correct, ok. And so, you know, now can you see the analogy here? What are the poles? The poles are nothing but the roots of the characteristic equation, correct, ok. Analogous to the natural frequencies that you have with equal to h_0 type, energy calculations, right. And so, when C_c is 0, correct.

In other words, you have two linear systems which are not coupled to each other. They are not interacting with each other because the output of the first system is taken to the second system, you know, the second system does not care what is happening to the first one, it just takes whatever comes to it and then amplifies it, right? So, it has two poles P_1 and P_2 , correct. What is C_c doing? It is introducing coupling between these two systems and therefore, right? What, you know, it is only natural to see that. Just like how energy levels split, they are Eigenvalues of, you know, differential equation, likewise h_0 type.

The poles are basically Eigenvalues of the system, ok. And when two systems are coupled to each other, the Eigenvalues will split, right? And that is basically, you know, what you see here. If the coupling is weak, then the split will be small, right. If the coupling is strong, you basically will see a large split in the poles, ok, alright. And that, that is one aspect that I just wanted to draw your attention to. The next thing I would like to address is this fellow here. So, $(1 - s)/\omega_z$, you know, is a right half plane 0 and that is coming because of the feed forward due to this feed forward path due to C_c . And now, we have learned enough about stability to understand that this right half plane 0 is a, it is a problem for stability because it actually, even though it is a 0 because it is the right half plane, it causes phase lag, right.

And I mean, another way of looking at it is the following. So, let us say, there are a whole bunch of poles. So, let us say you have a system with say a single pole so some, you know, $A_{dc}/(1 + s/\omega_d)$, just for argument sake, right. So, if you put a step, a small step here, what kind of signal will you get here? Of course, you will get an amplified step due to A_{dc} , but what you will, there it will be because of the single pole, it will basically do that, correct, ok.

Now, if I add a right half plane 0, how will this look? In the time domain, what is in the time domain? Let us call this; let us call this $h(t)$, right. So, this has got two components, one which is, I will write this $(1 - s/\omega_z)$ as 1 and s/ω_z , and there is a subtraction here. Now, can somebody tell me what the waveform here is going to be?

Student: $h(t)$.

This is going to be $h(t)$.

What about this guy here?

Student: It is $1/\omega_z$.

So, how will this waveform look, how will this waveform look like? How will the derivative of this waveform, this is the $h(t)$? What will $1/\omega_z$, some constant derivative, look like? It will just do this, correct. With some constant which is basically some $1/\omega_z$, correct, ok. So, now, if I. So, this is some $1/\omega_z h'(t)$. So, if I subtract the two, what will happen?

So, first I will go negative then right. So, that is basically the waveform that you see here, ok. So, now, look at the blue curve and $h(t)$ and the input and output due to the 0, what comment can we make about the blue curve with respect to the black one? It is simply delayed. Does that make sense? Right. What if I had instead of, you know, subtracting if I had added? So, if I added a 0 in the R.H.P zero, what would the output look like? It basically goes, does this.

And L.H.P is zero, what will it, you expect to see? You get something like this and then you will get that and without any 0, ok, alright. So, I mean, so this basically, you can see there is a time domain intuitive way of seeing why? The right half plane V_s causes more delay in the system. A left half plane zero reduces, you know, another way of thinking about it is that L.H.P zero is $h(t) + k h'(t)$.

And remember, what is $h(t)$ if you want to advance waveform, what by a time t_d , what is the expression? $h(t)$ is a waveform. I want to advance it by t_d . The advanced version is $h(t + t_d)$, right. For small t_d , this is you know, if you expand it in Taylor series, what does this look like? $h(t) + t_d h'(t)$ and so on, right. So, what is this $h'(t)$, how do you get $h'(t)$? Yeah, this is basically, this is nothing but $H(s) + t_d s H(s)$. So, this is basically saying that you have a 0 and it is in the left half plane, right. So, this is equivalent to that 0 is attempting to advance the signal, right.

On the other hand, if you do $h(t) - t_d$, what do you see? This becomes negative. And so, this is the so if you have the, subtract the derivative, then it becomes delayed by $s t_d$ and I mean delayed by t_d and this is equivalent to a right half plane.

So, the bottom line is, you know, of course, it also all comes out to the math. But the important thing, the time domain, I mean, I always find time domain intuition, you know very useful and. So, it's always good to be able to look at something from multiple perspectives.

And so, basically this is telling you that, you know, the right half plane zero because the Miller capacitor is actually degrading the phase margin of the system. And therefore, you know, since it degrades phase margin, it, you know, we would like to be able to sit and fix this problem, correct. And that is what we are going to see next.

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The image shows two circuit diagrams and associated equations on a grid background. The top diagram is a Miller capacitor circuit. It features a dependent current source $\beta_{m1} v_i$ in parallel with a capacitor C_c . The output node is connected to a load capacitor C_L and a dependent current source $\beta_{m2} v_x$. The output voltage is $v_o - v_x$. The voltage across the Miller capacitor is v_x . The equations shown are:

$$H(s) = \frac{v_o - v_x}{v_i} = \frac{\beta_{m1} v_i}{s C_c} - \frac{\beta_{m2} v_x}{s C_c} = \frac{\beta_{m1}}{s C_c} \left(1 - \frac{s C_c}{\beta_{m2}} \right) v_i$$

The term $\frac{s C_c}{\beta_{m2}}$ is crossed out with a green 'X', indicating a right-half plane zero. The NPTEL logo is visible in the top right corner.

The bottom diagram shows a simplified equivalent circuit with a dependent current source $\beta_{m1} v_i$ in parallel with a capacitor C_c , connected to a load capacitor C_L and a dependent current source $\beta_{m2} v_x$. The output voltage is $v_o - v_x$.

So, again, let me remind you of the origin of the whole problem. So, this is $g_{m1} v_i$, I am going to throw out all the, you know non-essential stuff and focus on the core root cause of the problem. This is C_c , alright. So, ideally what do we want? I mean, let us assume that this right, to first order, basically we ideally like, we want this $g_{m1} v_i$ flows, this current flows through C_c causing a voltage drop, $g_{m1}/s C_c v_i$, ok.

But what is the problem? The actual output voltage is, you know, I am trying to understand the origin of the 0, correct. So, what is the output voltage v_o ?

Student: Gate voltage.

Now what is the gate voltage? What is the impedance looking in here? So, this is $g_{m2} v_x$. So, this voltage therefore is what is v_x therefore? The impedance is $1/g_{m2}$. So, what is the voltage v_x ?

Student: $-g_{m1}/g_{m2} v_i$.

So, what is the output voltage? That is basically the voltage across the capacitor across C_c - $g_{m1} v_i/g_{m2}$, correct. So, this is basically $g_{m1}/s C_c(1-s C_c/g_{m2}) v_x$. Does it make sense? I mean, basically this is saying that at high frequencies that C_c is short. So, whatever is there at v_x will show up at v_o , ok.

So, what is it to be of these terms you know which is the one. So, this is v_x . So, which is the term we want and which is the term we do not want? Which of those terms is responsible for the 0? v_x , correct. So, this is what we want this term we do not want, correct. So, you have the output which consists of two quantities, one which you want and one which you do not want.

So, if you want to get rid of something that you do not want, the two fundamental ways of doing it, what are they? You either divide by a large number or subtract, right. I mean multiplying by a small number is the same as dividing by a large number, ok. So, I mean. So, if you want to divide by a large number, what will you do?

You want to make g_{m2} infinitely, correct. So, one way of getting rid of the 0 is to say I am going to make g_{m2} infinity. Mathematically that sounds great, but in practice what problems will we have? Yeah, the transistor is going to become large, a lot of power, etcetera etcetera.

So, that is not a particularly smart way of doing it, right. The other way that we can try is to basically subtract v_x , ok. So, in principle therefore. So, if what we need to do is to subtract v_x is, I mean if somehow, we had a voltage source v_x , we could simply, you know, you know, add it in series with v_o and that will give you $v_o - v_x$. And therefore, we can get rid of the 0. Is this clear?

Now, you know all of you I am sure are aware of, you know, you can always take a voltage source and push it through a node, correct. So, this is equivalent to v_x . So, what is this voltage going to be? What is that voltage going to be? So, this is v_x . So, what is this voltage going to be? This is $v_o - v_x$, all I have done is pushed this voltage source through the node. Is this clear? Now, let us take a look at this voltage source. This is in series with a, what is the transistor? And what is this actually?

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This is nothing but a current source which is $g_{m2} v_x$. So, if you have a voltage source in series with the current source, what is the effect of the voltage source? What does the voltage source do? Nothing.

So, do we need that voltage source v_x in series with g_{m2} in the drain of the transistor? So, this v_x does not, I mean, it does not affect anything. So, it is not going to change. So, basically, because in series with a current source. So, we can remove it, ok.

Now, look at, let us concentrate on this voltage source. What is the current flowing through this voltage source? What is v_x , by the way? So, v_x is nothing but $-g_{m1}/g_{m2} v_i$. And the current flowing in that direction is $g_{m1} v_i$ in that direction. So, here is a voltage source.

A current $g_{m1} v_i$ is flowing through it. The voltage developed across that voltage source is $g_{m1} v_i / g_{m2}$. So, what can. So, what can you replace that voltage source by? A resistor of value? $1/g_{m2}$. Is this clear people?

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I mean, this gives you, I mean, this is a negative voltage, right. This is the same v_x in the opposite direction, right. So, the output voltage is basically $v_x + 1/s C - v_x$. So, the effect of v_x is canceled and therefore, you basically end up with $g_{m1}/s C_c$.

So, whenever, you know I mean, whenever you are trying to cancel two things, right. What must you be aware of? What can go wrong? Yeah, you know you are trying to subtract a with b, of course, you know what he points out is, you must make sure that you do not add, right. Well, I am assuming that we do not do something like that, ok. That is a pathological problem.

But even if you do it correctly, if you subtract, right. What you subtract will not be exactly, you know, what you want to subtract and consequently, there will be some, there will be some, you know, effect, residual effect. So, in other words, the 0 will not be perfectly

canceled, but it will become so far away that it becomes, I mean, that ω_z tends to infinity, you know, or even goes up by a factor of say 100.

Because there is 1 percent mismatch between, you know this g_{m2} and this $1/g_{m2}$, we are still much better off than we were without the 0 canceling. Let us stop here.