

**Analog Electronic Circuits**  
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**Lecture - 76**  
**2-Stage Operational Amplifier and Miller Compensation (contd)**

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Lecture 36

NPTEL

$$\frac{V_o(s)}{V_i(s)} = \frac{g_{m1} r_1 g_{m2} r_2 (1 - \frac{s C_c}{g_{m2}})}{(1 + s/\omega_d) (1 + s/\omega_o)}$$

$$\omega_d = \frac{1}{r_1 g_{m2} r_2 C_c} \quad \omega_o = \frac{(C_c + C_1)}{g_{m2} C_c \cdot \left\{ C_2 + \frac{C_c C_2}{C_1 + C_c} \right\}}$$

In the last class we were looking at the frequency response of a 2-stage operational amplifier. This was the small signal equivalent and we said that  $V_o$  by  $V_i$  is nothing but the DC gain. What is the DC gain? It is  $g_{m1} r_1 g_{m2} r_2$  is the right half plane 0 which is  $s C_c / g_{m2}$ . And this is the 2 poles are widely separated and there is a dominant pole and there is a what did we call the second pole?

And what was the dominant pole frequency? The dominant pole frequency is 1 over the dominant pole frequency occurs due to the effective time constant in that node. And that is basically  $1 / r_1$ . Well, if you want to basically do the exact thing it's a long expression, but in any case, since  $C_c$  is a Miller-compensating capacitor the effective capacitance looking in at node  $v_x$  is for all practical purposes equal to  $g_m g_{m2} r_2 C_c$ .

There is a  $1 + g_{m2} r_2$  there, but since  $g_{m2} r_2$  is expected to be large this is the dominant pole frequency, The second pole, is at this node and at frequencies much beyond the dominant pole frequency, pretty much you basically no current flows through  $r_2$ , and no current flows to

$r_1$ . So, the circuit pretty much looks like  $g_{m2}$  with a capacitor divider and feedback the capacitive divider is that  $C_1$  and  $C_c$  in series, And so, essentially  $g_{m2}$  the effective  $g_{m2}$  will basically be you know smaller.

So, the resistance looking at that node is basically 1 over  $g_{m2}$ . What is it? What is the effective resistance looking in there? 1 over  $g_{m2}$ , yeah. So, basically there is what I mean if I apply a  $v_{test}$  here, what comment can you make about the voltage there assuming  $r_1$  is open?

$C_c / (C_1 + C_c) v_{test}$ . So, basically the resistive part of the current drawn will be  $1 / g_{m2} v_{test}$ , So, that is so this becomes  $1 + C_1 / C_c$ , ok. And the total capacitance looking. So, this is basically  $C_c + C_1 / C$  as  $C_c$  tends to infinity this must be tending to  $1 / g_{m2}$ .

If  $C_c$  tends to infinity, what does it mean? Yeah, at that high frequency basically  $C$  if  $C_c$  becomes very very large then it is pretty much like shorting  $v_x$  with  $v_{test}$ . And therefore, the current that will flow will be 1 over I mean it is  $g_{m2} v_{test}$  and consequently the resistance is going to be 1 over  $g_m$ , is this clear?

And the effective capacitance looking in basically consists of two components. One is  $C_2$  and the other one is the series combination of  $C_1$  and  $C_c$  that is  $C_1 C_c / (C_1 + C_c)$ , ok. And therefore, the second pole has a frequency.

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Lecture 36

$$\frac{V_o(s)}{V_i(s)} = \frac{g_{m1} r_1 g_{m2} r_2 \left(1 - \frac{s C_c}{g_{m2}}\right)}{(1 + s/\omega_d) (1 + s/\omega_o)}$$

$$\omega_1 = \frac{1}{r_1 g_{m2} r_2 C_c} \quad \omega_o = \frac{g_{m2} C_c}{(C_1 + C_c) \left\{ C_2 + \frac{C_1 C_c}{C_1 + C_c} \right\}}$$

$$= \frac{g_{m2} C_c}{C_1 C_2 + C_2 C_c + C_c C_1} = g_{m2}$$

This is  $g_m g_{m2} C_c / (C_1 + C_c) (C_2 + (C_1 C_c / C_1 + C_c))$ . And consequently, this basically is  $g_{m2} C_c / (C_1 C_2 + C_2 C_c + C_c C_1)$ .

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NPTEL

$$\omega_d = \frac{1}{r_1 g_{m2} r_2 C_c} \quad \omega_o = \frac{g_{m2}}{C_1 + C_2 + \frac{C_1 C_2}{C_c}}$$

For good phase margin:  $\omega_u \equiv A_{dc} \omega_d = \frac{g_{m1} r_1 g_{m2} r_2}{r_1 g_{m2} r_2 C_c} = \frac{g_{m1}}{C_c}$

$$\angle LG @ \omega_u = \angle \frac{A_{dc} (1 - s/\omega_o)}{(1 + s/\omega_d)(1 + s/\omega_o)} \Big|_{s=j\omega_u} = \frac{-\pi}{2} - \tan^{-1}\left(\frac{\omega_u}{\omega_o}\right) + \tan^{-1}\left(\frac{\omega_u}{\omega_d}\right)$$

So,  $\omega_d = 1 / r_1 g_{m2} r_2 C_c$ ,  $\omega_o$ , the second pole is nothing but  $g_{m2} / (C_1 + C_2 + C_1 C_2 / C_c)$ , ok. So, if you want to have a good phase margin, what comment can we make? How should we choose  $\omega_d$  versus  $\omega_o$ ?  $\omega_d$  is very less than  $\omega_o$  is given, but is that enough? To achieve good phase margin what should be smaller than  $\omega_o$ ? The unity gain frequency must be smaller than  $\omega_o$ , it is understood that  $\omega_d$  is smaller than  $\omega_o$ , The unity gain frequency  $\omega_u$  and what is the unity gain frequency?  $A_{dc} \omega_d$  I mean approximately, And this is what the  $A_{dc}$  in our case is ( $g_{m1} r_1 g_{m2} r_2 / r_1 g_{m2} r_2 C_c$ ). And therefore, what comment can we make? The  $r_1$  goes away,  $g_{m2}$  goes away,  $r_2$  goes away. So, almost unbelievably it seems like the unity gain frequency is largely independent of everything else. It only depends on  $g_m / C_c$ , ok.

So, we will take a look at why this makes intuitive sense you know going forward, but for a good phase margin  $\omega_u$  must be smaller than this much smaller than  $\omega_o$ . So, what, but what is the trade off between phase margin and unity gain bandwidth? In any case, ok. So, let us say  $\omega_u$  is much smaller than  $\omega_o$ , what is the angle of the loop gain at  $\omega_u$  is  $-\pi / 2$ . Why is this  $-\pi / 2$  coming?

That is because of the phase lag due to the dominant pole, and then you have basically the angle of  $A_{dc} (1 - s / \omega_o) / (1 + s / \omega_d)(1 + s / \omega_o)$ . So, evaluated at  $s = j\omega_u$ , correct. And ok, and therefore, this is nothing but  $-\pi / 2 - \tan^{-1}(\omega_u / \omega_o)$ . What about the numerator is  $-\tan^{-1}(\omega_u / \omega_d)$ . So, the question is a quick question, you know does the feed forward 0 hurt or help the phase margin?

Student: Hurts.

It hurts, because it is in the right half s plane, And what do you call it?

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$$\omega_u = \frac{g_{m1} g_{m2} / C_c}{C_1 + C_2 + \frac{C_1 C_2}{C_c}}$$

For good phase margin :  $\omega_u \equiv A_{dc} \omega_u = \frac{g_{m1} g_{m2} / C_c}{g_{m1} g_{m2} / C_c} = \frac{g_{m1}}{C_c}$   
 $\Rightarrow g_{m2} \uparrow$

$$\angle LG @ \omega_u = \angle \frac{A_{dc} (1 - \frac{s}{\omega_z})}{(1 + \frac{s}{\omega_{p1}})(1 + \frac{s}{\omega_{p2}})} \Big|_{s=j\omega_u} = \underbrace{-\frac{\pi}{2}} - \underbrace{\tan^{-1}\left(\frac{\omega_u}{\omega_{p1}}\right)} - \underbrace{\tan^{-1}\left(\frac{\omega_u}{\omega_{p2}}\right)}$$

R.H.P zero hurts phase margin

For a fo factor  $f$ ,  $LG(s) = \frac{A_{dc} f (1 - \frac{s}{\omega_z})}{(1 + \frac{s}{\omega_{p1}})(1 + \frac{s}{\omega_{p2}})}$

So, we will take a look at it, you know. So, basically R.H.P zero hurts phase margin. Because it increases the effective lag of the transfer function at the unity gain frequency. So, you have to get a large phase margin. What should you do? So, for good phase margin therefore, what is the bottom line what should be I mean what would you ideally like  $g_{m2}$  to be? How do we increase phase margin?

$\omega_z$  is  $g_{m2} / C_c$ . So, basically to increase phase margin therefore,  $g_{m2}$  must be increased, ok. So, what is the trade-off behind? I mean if you go on doing this what will happen  $g_{m2}$ . So, what happens to the unity gain frequency? How does it depend on  $g_{m2}$ ? Unity gains frequency; how does it depend on  $g_{m2}$ ? Independent, ok. So, if you go on increasing  $g_{m2}$  the unity gain frequency remains the same and the phase margin keeps improving and as  $g_{m2}$  tends to infinity you would expect the phase margin to be  $90^\circ$ , ok, So, but what is the price you are paying to get higher and higher  $g_{m2}$ ? So, you need to pump more current. I mean to be able to get a larger  $g_{m2}$ .

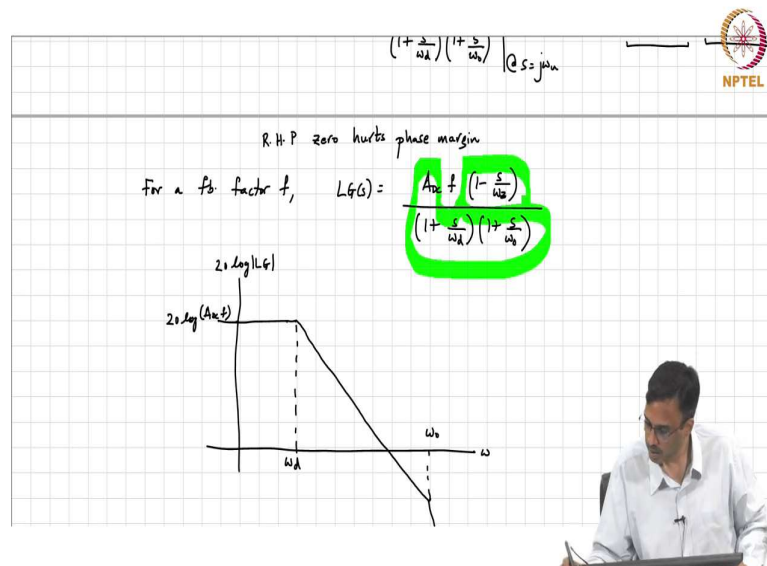
Another way of doing it is to increase the size as well as the current ok, but the problem with that is that well, if you increase the size of the transistor the parasitic capacitances also increase, So, basically you know there is some trade off there, but the bottom line is that you

know to increase phase margin you will need to; you will need to increase power dissipation, ok,

Now, a couple of things you know as you know there is a trade-off between phase margin and the unity gain and the unity gain bandwidth. So, what do you think and by the way the unity gain bandwidth is basically the unity gain bandwidth of the op amp itself is  $g_{m1} / C_c$ , correct. So, let us say the op amp is enclosed in a feedback loop with a feedback factor  $f$ .

So, what is the loop gain function? The loop gain function will be this  $A_{DC} f$  was  $(1 - s/\omega_z)/(1 + s/\omega_d)(1 + s/\omega_o)$ . So, what is the worst case feedback factor for stability analysis? When do we have for what values of  $f$  would we have to be careful about stability?

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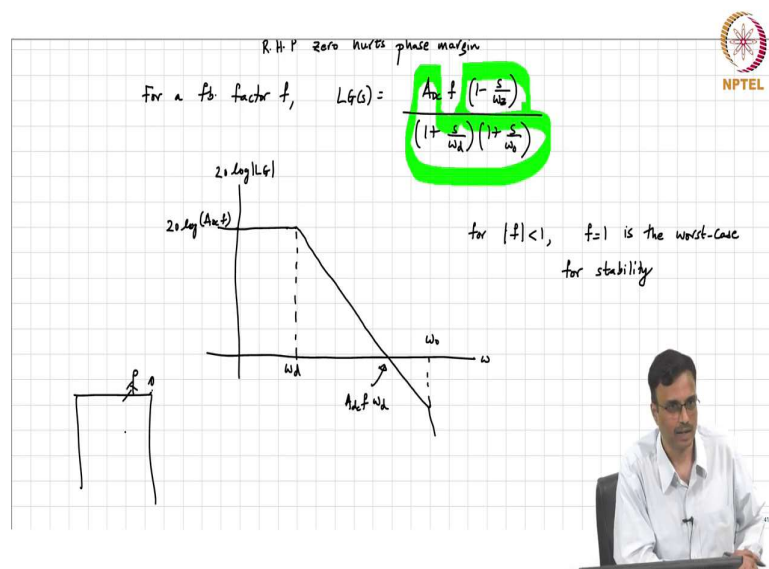
So, let us say all of this is exactly the same, none of that is changing, ok. Because the op amp is the same, we are varying the feedback factor around the op amp, correct. So, for which feedback factor would we have to be most concerned about the stability of the closed loop system?

And by the way what is the useful range of feedback factors? What would you know how you know how much would  $f$  vary in practice? If you are trying to build an amplifier voltage amplifier  $(1 + r_2/r_1)$  type system. It is  $1/f$ , correct. So, you want to build an amplifier. So,  $f$  must be less than 1, correct. So, the useful I mean if you are trying to build an amplifier the useful range of  $f$  is 0 to 1, if you want if you want to choose  $f = 4$  like he suggested you know

what will be the closed loop gain is  $1 / 4$ , if you want to get  $1 / 4$ , I think there is smarter ways of doing this than using an op amp, correct.

So, the feedback factors we have to worry about are basically  $f$  ranging from 0 to 1, So, in this range of  $f$  which is 0 to 1 where for what value of  $f$  would be very concerned about stability? So, if  $f$  is 0 is very clear that you know there is no feedback at all, correct. So, there is no nothing to worry about stability, ok. So, for higher values of  $f$ . So, if you have actually followed any of the discussion remember that the loop gain function if you plot it on a this is log magnitude of the loop gain, this is the frequency, then what is this value? That is  $20 \log A_{dc} f$  Then what is that breakpoint  $\omega_d$ . So, here it starts with a slope of 20 dB/decade, And this is then it ends with a slope of 40 dB / decade. So, this is  $\omega_o$ , ok,

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And so, what is the unity gain frequency here? It is  $A_{dc} f \omega_d$ , ok, So, now, can somebody take a look at this plot and tell me what is the worst case feedback factor as far stability is concerned?

Student: 1.

1, why? So, basically remember that for the feedback system to be stable the unity gain frequency must be as far away from  $\omega_o$  as possible, for good phase margin. So, as  $f$  keeps increasing, what happens? It is approaching that the unity gain frequency is becoming closer and closer to  $\omega_o$  and therefore the phase margin will decrease. So, in other words equivalently

the system is becoming closer to getting closer to becoming unstable, ok. If you know hypothetically, if you put a gain in the feedback path, the feedback factor lets us say you know I mean we listen to Abhishek and say we put a feedback factor of 3, what will happen? I mean the closed loop system could simply will simply become I mean it could simply become unstable, ok. So,  $f < 1$  for feedback factors,  $f = 1$  is the worst case for stability for a dominant pole compensation op amp, ok.

So, in many data sheets you know you will find the following description: the op amp unity gain stable, ok. So, when they say unity gains stability it means that it is stable even with a feedback factor of 1, ok. If an op amp is not stable for you know for unity gain feedback, that does not mean much, if you are not going to be using it in unity gain feedback mode, So, let us say you wanted to gain of 50, you only care that the phase margin of the closed loop system is sufficiently high for that particular gain, it does not help you if it does not help you if the system is unstable for I mean, it does not you know help you if the you know the system is stable for unity gain, nor does it hurt, ok. If it is unstable for unity gain because you are not going to be using it for unity gain anyway. Does it make sense to people?

That is one point, the other one is you know this trade-off between you know phase margin and unity gain frequency is pretty much like you know assume you are sitting on you know assume I do not know you walk up you know the new academic complex which is like basically very tall building,

And of course, if you are at that height, you know there are a lot of you know winds gusting. So, but if you stand on top of the you know if you go on to the terrace you know I do not know how many of you have gone there, but it is a great view, but then there is always the danger of the wind pushing you over the you know over the parapet, ok.

So, you can do you know one of two things, you can be very conservative and say I am going to stay 20 feet away from the parapet because who knows I mean you know the wind may decide to just take me along, correct, ok. So, what is the price you are paying for being 20 feet and you know behind the parapet wall?

Well, you know you are missing, you know, great view, ok. On the other hand, you may say you know this view is terrible, you know let me go hang around you know walk around walk on the parapet, ok, And you know of course, now you get a fantastic view, but you know if

there is you know if there is a gust of wind you know you might, you might just get pushed off the edge.

And so, therefore, you know if so therefore, if you have wide you know if you are you know if you say I will work with a very small phase margin, because I want wide bandwidth which is analogous to saying you know I will work walk on the parapet just because I can get a great wide view,

Then you know a small gust of wind will basically push you over the edge. In circuit terms basically if something changes, You thought these parasitics were something oh you missed a decimal point and like you know, And therefore, parasitics all turn out to be you know much larger, then you will end up with an interruption you understand. So, you know like standing on the top of a building you know it is like it is a; it is a trade of how far we are confident of you know how close you want to go to the wall,

So, if you know if you like to play things very very conservatively then you will shoot for a very large phase margin and lose bandwidth. If you know if you like to live life on the edge, then you know you will say ok, I will deal with you knowing I will have a  $15^\circ$  phase margin and then you know or now I get off bandwidth.

So, what do you call, but now I am lot more sensitive to everything, And this is a general principle in engineering, if you are pushing everything to the edge, to maximize performance many you know it is not merely you know the thrill of standing on the edge of a building,

You are desperately trying to kind of push the limits of technology, And so, basically you your margins are very thin, ok. And if margins are very thin it basically means that you know you are going to be in general sensitive to minor variations in everything, correct. And this is no exception. So, that is one thing.