

Analog Electronic Circuits
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Lecture - 74
Phase Margin Examples

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So, with this background let us basically take some examples. So, let us take a second order system. Remember the uncompensated system in a second order case had some $A_o^2 f / (1 + s/\omega_o)^2$ and what was the closed loop quality factor in that case?

Student: A_o .

$A_o \sqrt{f/2}$ which was too high anyway, correct. So, I mean if a dominant pole compensation helps a third order system it must also help a second order system and another practical matter is that I remember as we were talking, I mean as we were discussing the other day. See it's always very easy to slow a system down.

So, rather than add a new pole which is much lower than these two existing poles one might as well take that one of those poles and make it deliberately lower. I mean in circuits that are very easy because all poles are consequences of capacitance at a node and where this capacitance is coming from which node this pole is coming from. So, if you want to make one of those poles lower in frequency you identify a node, take a big fat capacitor and put it on that node. So, in other words rather than add a new pole and make the system a third order one you can make one of the poles dominant with respect to the other.

So, in other words the dominant pole compensation system is $(1 + s/\omega_d)/(1 + s/\omega_o)$. So, I mean let us do first of all what comment we can make. I mean how low should we choose ω_d or? So, let us let us say that we choose a what is the unity gain frequency if ω_d is much smaller than ω_o $A_o^2 f \omega_d$ is the unity gain frequency and let us assume that ω_d is chosen such that the unity gain frequency is $0.1 \omega_o$, ok.

So, by the way. So, let us first check out the fact that our assumption that ω_o does not result in any appreciable magnitude response at the unity gain frequency. So, what is the magnitude response of $1/s (1 + s/\omega_o)$ at one tenth the frequency? It's $1/\sqrt{1 + 0.01}$.

Student: (Refer Time: 04:08).

0.01. So, its $\sqrt{1.01}$ with $1/1.05$ is 0.95. So, it basically only changes the meaning. So, point I mean sorry 1.05 I mean 1.05. So, 0.995 is the magnitude at one tenth of ω_o . So, for all practical purposes therefore, the unity gain frequency is quite accurately equal to $A_o^2 f \omega_o$. If this ratio is not 0.1, we may not be so lucky.

So, let us say ω_u was only half of ω_o then what is it now it becomes now $1/\sqrt{1.25} = 1.12$ and 1 by that is 0.87 something like that. So, basically you can see this is a 10 percent reduction in the magnitude response. So, the unity gain frequency is actually slightly small. So, the ω_u is $0.1 \omega_o$.

So, what is the angle of the loop gain at ω_u ? I mean to make $\omega_u = 0.1 \omega_o$ what is ω_d ? $0.1 \omega_o/A_o^2 f$, ok. So, what is the loop gain at ω_u ? What is the angle of the loop gain at ω_u ? -90° that is coming from the dominant pole $-\tan^{-1}(0.1)$. $\tan^{-1}(0.1)$ is roughly a small number. So, that is basically roughly 0.1 radians. So, this is so, this is approximately 0.1 radians is how many degrees? 1 radians 56° come on fellows. So, 0.1 radian 5.5° something less has roughly 6° . So, this basically means that the phase 90° is about the 96° phase line, correct.

So, what is the phase margin? 84° , ok. And the unity gains bandwidth as we say if we said this. So, if so, we chose ω_d to be $0.1 \omega_o/A_o^2 f$. Now, let us say we say ok well this phase margin is too high correct and that is why I am getting killed in my bandwidth.

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$(1 + \frac{s}{\omega_o})^2$
 Dom. pole compensated system: $L(s) = \frac{A_o^2 f}{(1 + \frac{s}{\omega_d})(1 + \frac{s}{\omega_o})}$
 $\omega_u = A_o^2 f \omega_d$ Assume $\omega_u = 0.1 \omega_o$ $\Rightarrow \omega_d = \frac{0.1 \omega_o}{A_o^2 f}$
 $\angle L_f @ \omega_u = -\frac{\pi}{2} - \tan^{-1}(0.1) \approx -96^\circ$
 $\phi_{PM} = 84^\circ$
 Choose $\omega_d = \frac{0.2 \omega_o}{A_o^2 f} \Rightarrow \omega_u = 0.2 \omega_o$
 $\angle L_f @ \omega_u = -\frac{\pi}{2} - \tan^{-1}(0.2) \approx -102^\circ$
 $\phi_{PM} = 78^\circ$

So, let me become brave and choose ω_d as saying I do not know $0.2 \omega_o/A_o^2 f$, correct. So, what comment can you make about ω_u now? $0.2 \omega_o$. So, what has happened to my closed loop bandwidth? It Has gone up by a factor of 2. So, the angle of the loop gain at ω_u therefore, is nothing but $-\pi/2 - \tan^{-1}(0.2)$ which is roughly -102° . What is the phase margin?

Student: 78° .

So, I mean as you can see going from 84° phase margin to 78 degree phase margin has increased your bandwidth by a factor of 2.

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Example: $L_f(s) = \frac{10000}{(1 + \frac{s}{\omega_0})^2} \Rightarrow \frac{10000}{(1 + \frac{s}{\omega_u})(1 + \frac{s}{\omega_0})}$

What dominant pole frequency must one choose for $\phi_{PM} = 60^\circ$?

+ Assume $\omega_u < \omega_0 \Rightarrow \omega_u = 10000 \omega_0$

$\phi_{PM} = 60^\circ \Rightarrow \angle L_f @ \omega_u = -120^\circ$ $2 \tan^{-1}\left(\frac{\omega_u}{\omega_0}\right) = 30^\circ$

So, now let me turn this whole thing upside down. So, what dominant pole frequency must I choose for a phase margin of say I do not know I mean say 60° they are reasonable.

So, basically, I mean in a practical situation the DC loop gain that you have where the locations of the poles are the design problem is for me to figure out now what dominant pole frequency I must choose. So, that I have some desired level of stability.

So, again with dominant pole compensation the loop gain becomes $1 + s/\omega_d$ as we discussed you can make one of the poles dominant, ok. So, first cut as let us assume that the unity gain frequency is $\omega_u < \omega_0$ and this may basically mean that ω_u is 10,000 ω_d . What is the angle of the loop gain I mean what is the phase margin? 60° which basically means that the angle of the loop gain at ω_u must be - 120° is that clear. So, out of these 120° how much must be coming from the dominant pole. - 90 is coming from the dominant pole. So, basically this extra 30° must be coming from this $2 \tan^{-1}(\omega_u/\omega_0)$ must be equal to 30°, ok.

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$\phi_{PM} = 78^\circ$

Example: $Lf(s) = \frac{10000}{(1 + \frac{s}{\omega_0})^2} \Rightarrow \frac{10000}{(1 + \frac{s}{\omega_d})(1 + \frac{s}{\omega_u})}$

What dominant pole frequency must one choose for $\phi_{PM} = 60^\circ$?

+ Assume $\omega_u < \omega_0 \Rightarrow \omega_u = 10000 \omega_d$

$\phi_{PM} = 60^\circ \Rightarrow \angle Lf Q \omega_u = -120^\circ$ $\tan^{-1}\left(\frac{\omega_u}{\omega_0}\right) = 15^\circ$

$\Rightarrow \frac{\omega_u}{\omega_0} = 0.26$

$\Rightarrow 10000 \omega_d = 0.26 \omega_0$

$\omega_d = 2.6 \times 10^{-5} \omega_0$

So, this is basically $\tan^{-1}(\omega_u/\omega_0)$ is so, which means that ω_u/ω_0 is $\tan 15$ which is about I would say something like 0.26. So, what does this mean ω_u which is $10,000 \omega_d$ is $0.26 \omega_0$ and therefore, ω_d is nothing but I do not know $2.6 \times 10^{-5} \omega_0$. Does it make sense?

Now, let us go back and check if this ω_u less than I mean if this our expression for ω_u is consistent with our assumption. So, if we assume that the second pole does not affect the magnitude response then our ω_u is $10,000 \omega_d$, but our ω_u/ω_0 is basically 0.26. So, what is the magnitude response of this $1/(1 + \text{of this extra factor})$? What is the magnitude response of that at 0.26? ω_u/ω_0 is 0.26.

So, basically what is the magnitude response $1/(1 + 0.26^2)$. It is 1.06. So, that is roughly 1.25^2 points whatever. So, basically.. So, that is basically 1.06. $1/1.06 = 0.95$ roughly. So, 0.95 is like we are about 5 percent off. So, what does this mean?

Well like a higher order term is I mean if we are squeamish about it then we should basically account for this thing. So, what this means is that this ω_u our estimate is slightly higher than the magnitude response at this frequency is actually smaller than 1 because of the extra attenuation because of the extra of those two extra poles.

So, you just move this by we had an error by 5%. So, this has to be moved a little by 5%. I mean if the other thing to do is basically do the unit calculation, I mean find the frequency at which this whole thing becomes 1.

So, the bottom line is as you can see and this is how you would design in practice hm. So, you have some DC gain that you need to accommodate you have want some degree of stability, the number of poles, where the poles are in this example of course, to make life easy for me on the blackboard, I just chose identical three identical poles, but I mean nothing prevents one pole from being at ω_0 the other one being at I do not know $1.5 \omega_0$ the other one being at $0.9 \omega_0$ something like that in any case this is the bottom line.

And the closed loop bandwidth is going to be the unity gain frequency which is $0.26 \omega_0$. Now, if I all of a sudden if I wanted a phase margin not a 60° , but I wanted a phase margin of I would know let us say 80° . What would change in the calculation? Angle of the loop gain must be 100° , out of which 90° is coming from the dominant pole. So, 10° must be coming from 2 poles. So, $\tan^{-1} \omega_u$ by ω_0 must be 5° . 5° is rough. I mean $\tan 5^\circ$ is roughly 0.1. So, 5° is what I mean is 1/11th actually roughly 0.9 something like that. And so, ω_u/ω_0 is basically 0.1 rather than 0.25. So, what comment can we make about the closed loop bandwidth therefore, it is reduced by a factor of 2.5. So, you can see therefore, that there is a penalty to be paid to have a large phase margin. Does it make sense?

So, with this we are this is all that I had to say about stabilizing the system using dominant pole compensation. Why is this dormant pole compensation so popular? Well, I mean it is easy to do. There is nothing if the system is oscillating, what do you do? Take a big capacitor and put it somewhere. And if the oscillation does not go away it means that I mean that is not enough you keep putting more and more and more until the system stops oscillating. Anyway, this is all the theory of dominant pole compensation that is needed at this level for an elementary course like this. So, now we are now armed with this knowledge we will jump on to our two stage op-amp and then stabilize it.