Analog Electronic Circuits
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Lecture - 73
Dominant Pole Compensation Summary

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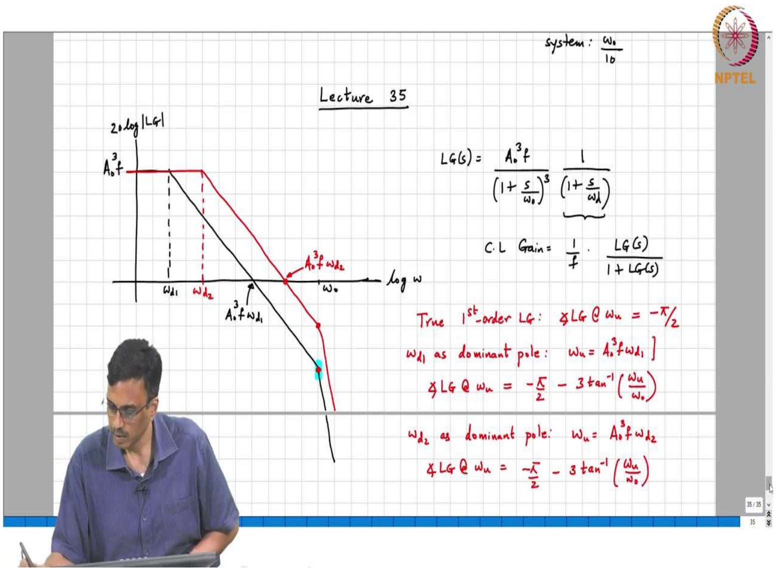

In the last class, we were looking at stabilizing the third order system. And we all $(1 + s/\omega_d)$ and we saw that well the whole idea in placing this $(1 + s/\omega_d)$ having this extra pole is to make the frequency response of the loop gain resemble that of a first order system. And if we draw the log magnitude of the loop gain versus log ω, we basically have say for this choice of this ω_{d1} and this goes 20 db per decade up to here. And then starts to go at 80 db a decade. Now, this is so, evidently, I mean over the useful frequency range of interest.

And what did we say the useful frequency range of interest was? I mean at all frequencies where the loop gain is greater than 1 or around abouts 1 as long as at those frequencies in that frequency range as long as the frequency response of the loop gain function resembles that of a first order system.

We should expect that the stability properties also pretty much resemble that of a first order system. And of course, at this frequency at ω_o and there the frequency response deviates from that of the first order system. But at that point the magnitude response is so small that the closed loop gain remember is nothing but is nothing but 1/f (loop gain/(1 + loop gain)).

So, if the angle of the loop gain for instance becomes 180^o, but the magnitude is very very small then that (1 + loop gain) can never become 0. And therefore, the system we do not really worry about becoming unstable because the magnitude response is so small already.

Ok so, that brings us to the next question well if what are we paid for I mean what is that we are paying. So, thanks to dominant pole compensation therefore, we have the luxury of having a large DC loop gain which is A_o^3 f while at the same time inheriting the stability of a first order system. But there is no free line. So, what are we paid in this whole bargain?

We have seen that the bandwidth of the system has gone down. What is the bandwidth now?

Student: A_o^3 f ω_{d1}.

Right whereas, earlier the loop gain had a very high magnitude all the way up to ω_o. But I mean that did not really help because well the system was not stable in the first place. I mean it is like basically having 300 crores in my bank account. But my bank account is frozen by the enforcement directorate and therefore, I cannot draw any money. It does not help to have any money like that.

So, this is here at least you have less bandwidth, but at least the whole system works. Now, the next obvious question comes is what is so special about ω_{d1}. What prevents me for instance from choosing I do not know say ω_{d2} where for this choice of ω_{d2} I have this Bode plot correct.

And what is the difference we see with ω_{d1} versus ω_{d2}? Well, the frequency at which the response deviates from true first order behavior. At that frequency the gain with when you use ω_{d1} is the loop gain at which the frequency, I mean the frequency response deviates from true first order behavior is much lower when you choose ω_{d1} versus ω_{d2}. And consequently, I mean it is fair to say that choosing ω_{d1} results in a better approximation to a first order system having said that; however, here also it seems like well when the deviation occurs the magnitude response is much smaller than unity anyway. So, it does not look like this is going to give us any problem either, correct.

So, in other words we need to find some way of quantifying the closeness or lack thereof from the response of a true first order system. So, one immediate obvious advantage of choosing ω_{d2} is that the unity gain frequency is now definitely much larger than what we had with ω_{do}. So, the question is how do we quantify the resemblance to a true first order system and that as we discussed in the previous class is by measuring the phase of the loop gain. So, for a true 1st order loop gain function. What is the angle of the loop gain at the unity gain frequency? Let ω_u be denote the unity gain frequency. So, if you had a true 1st order system with say ω_{d1} what would be the angle of the loop gain at the unity gain frequency?

It will be approximately - 90 °. Why do I say approximate? Because it is $\tan^{-1}$(some large number which) is 90°. So, the angle of the loop gain is negative $\pi/2$ radians. Now, with our ω_{d1} as the dominant pole when I say dominant pole it; obviously means that there are other poles. What is the angle of the loop gain at ω_u? First of all, what is ω_u?

Student: A_o^3 f ω_{d1}.

Now, I would like to add a small warning here. See this ω_u being A_o^3 f ω_{d1} is only true if under what circumstance is it exactly equal to A_o^3 f ω_{d1}? The higher order poles will also cause a magnitude I mean what is the definition of ω_u? The frequency at which the magnitude becomes 1. So, the assumption here is that the higher order poles of the three poles that ω_o in our example are not causing an appreciable change in the magnitude. Response has compared to that of a true first order system. So, that will only be true if the unity gain frequency is sufficiently low compared to 2 ω_o, is it clear? So, that is the assumption that you have to bear in mind when you make this.

If the unity gain frequency gets very close to ω_o then you will find that this is not quite true there is going to be some error in any. So, this is just something that you need to do. So, the angle of the loop gain at ω_u is therefore, what is it? Well, there will be four contributions 1 due to ω_d and and 3 due to ω_o.

You know each of the poles, all of them which are sitting at ω_o. So, this is basically $- \pi/2 - 3 \tan^{-1}(\omega_u/\omega_o)$. And sanity check as ω_o tends to infinity, we should get $- \pi/2$ and. So, therefore, and if we chose ω_{d2} as the dominant pole frequency what comment can you make? ω_u is now we just have to replace ω_{d2} instead of ω_{d1} in all our expressions. So, the angle of the loop gain at ω_u is nothing but $- \pi/2 – 3\tan^{-1}(\omega_u/\omega_o)$.

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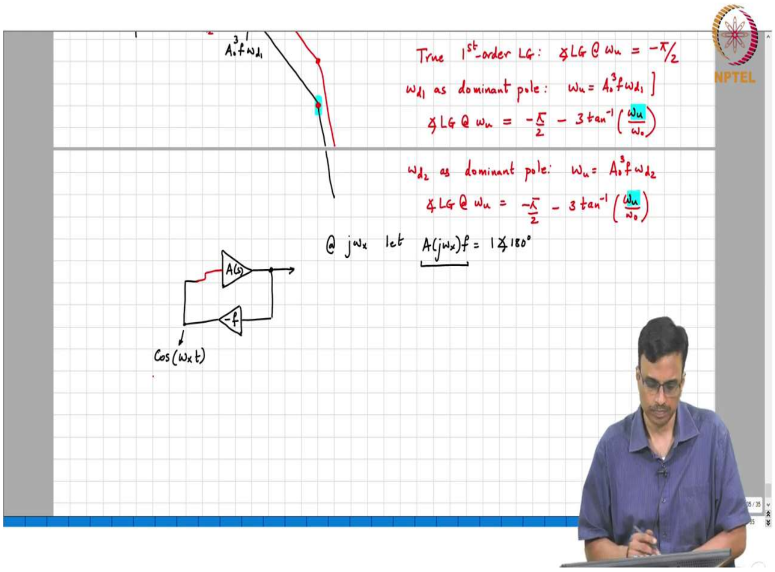

And note that this ω_u and this ω_u are different. So, which of them is the choice of ω_1 or ω_2 will have a larger phase lag ω_{d2}. And so, therefore, the deviation of the phase from 90° is an indicator is an indicator of the resemblance of the response to a true first order system. Does it make sense? And the phase measurement is a lot easier than I mean is a lot easier to do than magnitude because when the deviation occurs the magnitude may be so small that ok. Another way of looking at it from a more theoretical angle is that a pole affects the phase a lot a lot earlier than the magnitude because at low frequency they remember that the magnitude is an even function of frequency.

So, therefore, at low frequency the magnitude remains approximately flat whereas, the phase is a linear function of frequency and therefore, you will get something which is. So, now, that brings us to the next point which is the what you do not what you do not want is the angle of the loop gain at the unity gain frequency to be 180°.

If that is the case the system will be unstable and of course, it comes out of the math, but let me give you some intuition why that makes sense. So, let us say you have a system that is the forward amplifier. This is the feedback block now the question is so, let us assume that at some $j\omega_x$ let $A(j\omega_x)f$ be $1 \angle 180°$. I mean from the expressions of course it is apparent that what comment can we make about the closed loop gain at ω_x at the frequency $j\omega_x$.

Student: Infinity.

Infinity. So, there are two poles at $\pm j\omega_x$. So, let me mean give you time domain intuition about why this will lead to instability. So, for example, I am going to break open the loop. So, now there is no feedback loop to make this unstable. So, and then what I am going to do is I am going to have a switch here. So, I am going to move the negative sign into the f. So, that this is - f correct. And then, I am going to have a switch here which can go into either.

Student: A.

Either A or B and this basically. So, now let us say originally the system is in position A. Is there a feedback loop or not? There is no feedback loop. So, now let us say I put say $\cos(\omega_x t)$. What is so special about ω_x? At this frequency the loop gain is $1 \angle 180$. So, A f at this frequency is $1 \angle 180$. So, what is the signal that comes here?

Think carefully, people. Roy, what is the signal I will get there in the time domain? This is a time domain argument. I am putting $\cos(\omega_x t)$. I am telling you $A(j\omega_x)f$ is $1\angle 180$. So, what is finally, what is the signal at B man? What will you get? I got all possible answers. What is it?

A f is $1 \angle 180$ which is - 1. There is a negative sign. So, what do you get here? Cos $(\omega_x t)$. Is this clear to everybody? Ok so, the signal at A and the signal at B are exactly the same. So, now suddenly the position of the switch was at a. At some time I suddenly moved the switch to position B. What will happen? No change meaning what? What will be the signal at B?

It will remain Cos $(\omega_x t)$ because the signal at A and signal at B were the same anyway so, rather than coming from the generator you I mean you might as well feedback, I mean you take the output at - f. So, what is this telling you? Now, you have a system where you close the loop and the signal here is Cos $(\omega_x t)$. What does this refer to? What does this represent if you take a stable system and you leave it without any input, what will happen to all the signals everywhere? Eventually they will go to 0. What is happening here? The signal remains the same, and amplitude does not decay. So, this basically means that there are poles on the $j\omega$ axis.

If there were poles in the right half s plane, what do you think will happen? It will go on increasing exponentially until in practice something saturates in them. So, this is the meaning of system becomes unstable if the loop gain angle of the loop gain becomes $1 \angle 180^o$. Ok.

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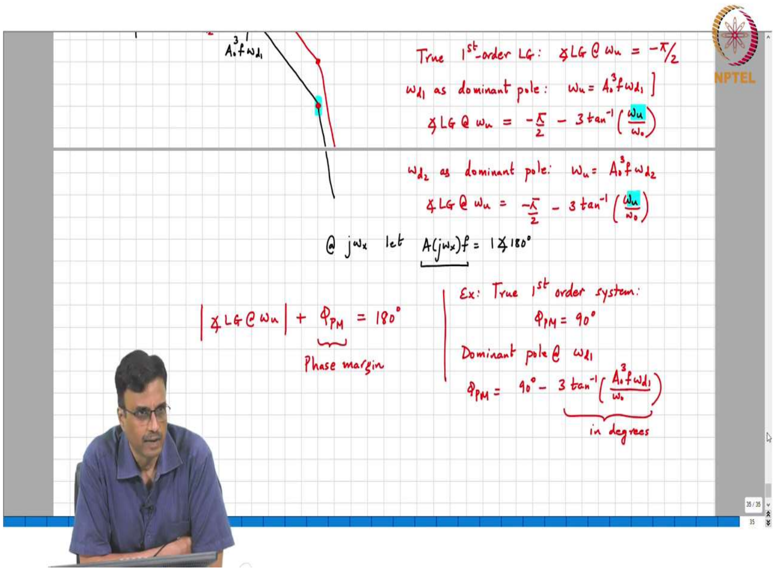

So, the system definitely becomes unstable with the angle of with a loop gain being 180º. The angle of the loop gain remains, becoming 180º when the magnitude is 1. So, in other words if you want to look at how far is this negative feedback loop from the edge of stability. What one can think of is how much phase so, the angle of the loop gain at ω_u.

So, to prevent all this lag lead and all that I will just take the magnitude of the angle at ω_u. How much extra phase must I add to the loop gain ? Ok so, this phi extra phase I am going to call that φ_{PM} for good reason which I will explain going forward. So, how much must I add to this and how much phase lag must I add to the loop to make it unstable is a measure of how far I am away from stability.

So, this is either π radians or 180º your favourite ok. So, this φ_{PM} therefore, represents the extra phase lag I must add to the loop to get the system to become unstable. So, this is so, because this is the margin you have before the system becomes unstable and because it is a phase this is what is called the phase margin. Does it make sense ? So, for example, a true 1st order system, what is the phase margin? Phase margin is well the lag of the loop gain function is 90º. So, you add extra 90º and therefore the phase margin is 90º. Now, the dominant pole at ω_{d1} has a phase margin is 90º - 3 $\tan^{-1}(\omega_u/\omega_o)$ Remember that this has to be expressed in degrees because we have 90º. Which is basically 3 $\tan^{-1}(A_o^3 f \omega_{d1}/\omega_o)$ and for ω_{d2} it is 90 - 3 $\tan^{-1}(A_o^3 f \omega_{d2}/\omega_o)$. So, the moral of the story is that the closer and closer a system is to true 1st order behavior the loop gain is the closer the system's loop gain is to true 1st order behavior. The phase margin will be closer to 90°.

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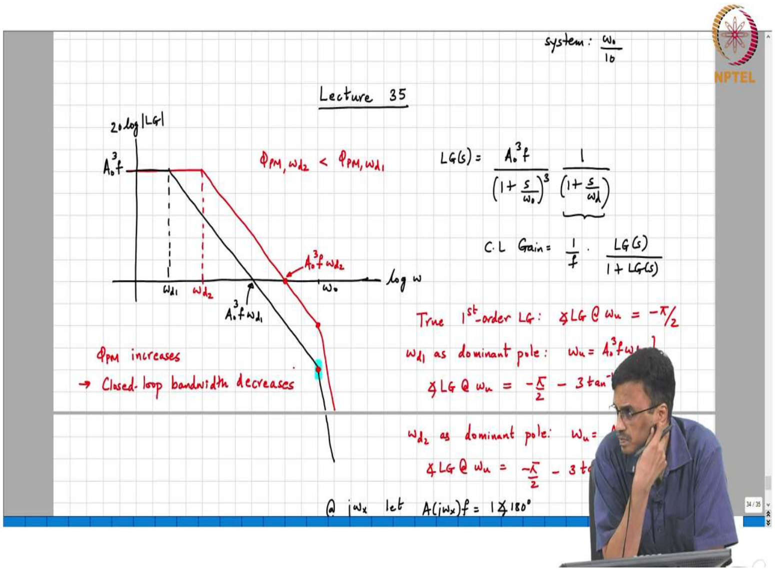

So, in this particular example, the phase margin where ω_{d2} is less than the phase margin for choosing ω_{d1}. So, phase margin is smaller, basically means that the margin you have for stability is smaller which means it's closer to be unstable. But what is it that you are gaining when you sacrifice phase margin? What are you gaining?

You are gaining back. So, closed loop bandwidth increases and phase margin decreases. Does it make sense to people? So, we have to summarize our discussion so far. To make a system stable using dominant pole compensation what do we do? Well, we choose the dominant pole such that the unity gain frequency is much smaller than all the other poles that can occur in our example we have used 3 poles at ω_o it could be 2 poles at ω_o it could be 4 poles at ω_o, it could be 1 pole at ω_o does not matter. And the phase margin is basically the angle I mean the extra phase you need to add to make the system unstable and that is that is given by the magnitude of the phase lag at the unity gain frequency.

And you look at the difference between that and 180° and by adding that extra phase you make the system unstable. So, that extra phase that you need to add is the phase margin as the phase margin reduces if the system is closer to being unstable.

And what comment can you make with regard to unity gain bandwidth of the closed loop system it increases. So, like everything else there is a trade-off I mean there is. So, the so, in other words what this discussion is telling us is that well there is nothing holy about making a system behave like a true 1st order system. Of course, if you do that you will have a large phase margin, but you will be paying in terms of bandwidth. On the other hand if you make the phase margin too little then your system will have a large bandwidth, but is on the verge of being unstable. So, like everything else in life there is a middle path. You do not want to choose a phase margin which is close to 90^{o} likewise you do not want to choose a phase margin which is 5^{o} ok.

So, somewhere in the middle, many people like to use conservative people like to use 30^{o} as the phase margin. People who like to live life on the edge may use 60^{o} of phase margin. Yeah, I mean it is the engineers, it is the engineers choice.