Analog Electronic Circuits
Prof. Shanthi Pavan
Department of Electrical Engineering
Indian Institute of Technology, Madras

Lecture - 71
Phase Margin

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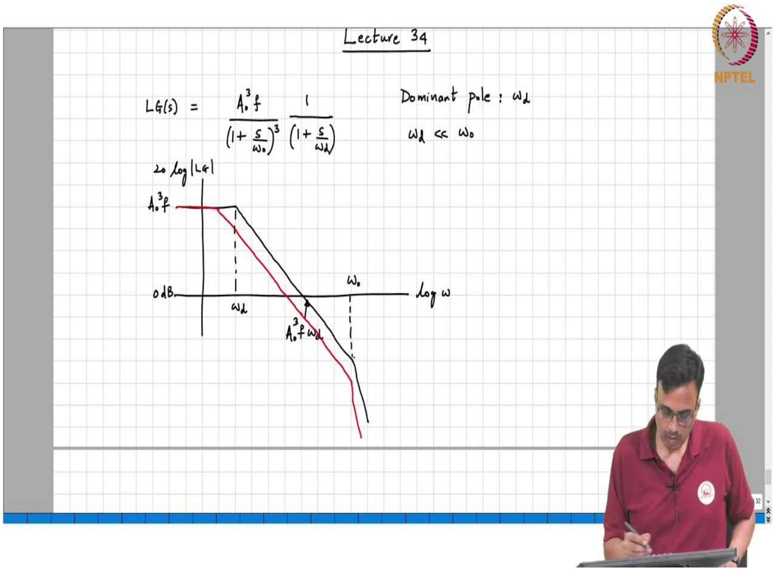

Good morning and welcome to Analog Electronic Circuits. This is lecture 34. So, in the last class, we were looking at trying to stabilize a high order system and for example, we had taken a system where the loop gain function had third order and recognized that the system is unstable for $A_o^3 f \geq 8$.

Then, we said well if we try to make it look like a first order system by creating an extra pole, where this ω_d is called the dominant pole, and the dominant pole ω_d is much much smaller than ω_o, alright. And so, the idea is that over the important; so, this is a log plot and this is 0 dB, right.

So, over the important frequency range or over the frequencies of interest, the frequency response of the loop gain function resembles that of a first order system, right? And consequently, the stability is that I mean you should expect it to behave like a first order system as far as stability is concerned. And the unity gain frequency, so for all practical purposes this should behave like a system with the unity gain frequency of $A_o^3 f\ \omega_d$, ok.

So, now, so yesterday we also said that there are several choices of ω_d which you know will make the system look like a first order system. Here I mean the one in black is one choice. Another choice could perhaps be something like this where you know you choose the dominant pole frequency. I think this is messed up, ok. So, this is $A_o^3 f\, \omega_{d1}$.

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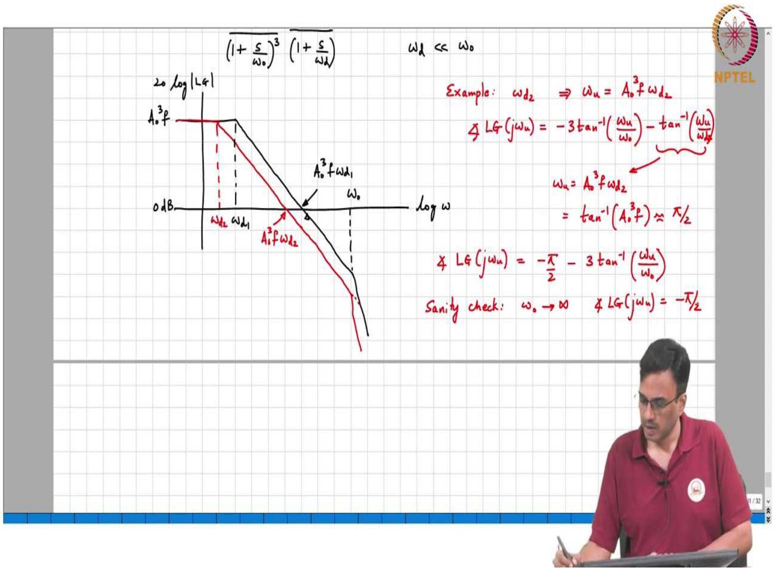

And this unity gain frequency is $A_o^3 f\, \omega_{d2}$, alright. And we yesterday also discussed that you know choosing ω_{d2} as the dominant pole frequency makes the frequency response a better approximation to that of a true first order system. Because at the frequency at which the frequency response deviates from that of a true first order system, the gain of the magnitude of the loop gain is much smaller than what it would be for ω_{d1}. Is this clear? Ok. And consequently we before needed you know a way of quantifying, I mean it is alright to say well you know you can plot the frequency response of a true first order system, and then you know check out at what gain this the two of them deviate, right.

But then, you know at the point of deviation, you can see that the magnitude is very small. So, a better way of quantifying what you call the bitterness of you know how well the dominant pole compensated system resembles a true first order system is to measure the phase of the loop gain function at the, I mean to measure the phase you need some frequency correct, right.

And so, the phase is basically a good place to measure, it is the unity gain frequency, right. And in this case, it is either $A_o^3 f\, \omega_{d2}$ or $A_o^3 f\, \omega_{d1}$ as the case may be. So, what is the so, for ω_{d2}

the unity gain frequency ω_u is $A_o^3f\ \omega_{d2}$. So, the angle of the loop gain function at ω_u, so angle of the loop gain function at j ω_u is nothing but - 3 $\tan^{-1}(\omega_u/\omega_o)$, alright, ok. And is it all?

$-\tan^{-1}(\omega_u/\omega_d)$, in this case ω_{d2}. And what is $\tan^{-1}(\omega_u/\omega_{d2})$? Remember, ω_u is nothing but $a_o^3f\ \omega_{d2}$. So, $\tan^{-1}(\omega_u/\omega_{d2})$ is nothing but $\tan^{-1}(A_o^3f)$. And what comment can we make about A_o^3f?

Student: Very very large.

Very large. So, $\tan^{-1}$ q, or $\tan^{-1}(a_o^3f)$ is basically going to be 90°. So, the angle of the loop gain therefore, $-\pi/2 - 3\tan^{-1}(\omega_u/\omega_o)$. So, and sanity check as ω_o tends to infinity, right. Basically, what comment can you make? What is the magnitude? What is the angle of the loop gain at the unity gain frequency? $-\pi/2$.

So, the angle of the loop gain at j ω_u will be $-\pi/2$, alright. And that is what we expect for a true first order system. Is this clear?

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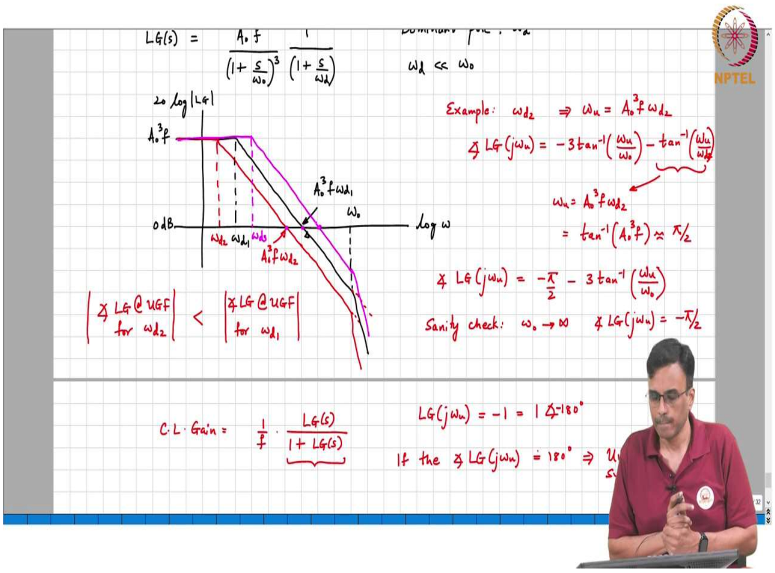

So, ok looking at this picture, what comment can you make about the angle of the loop gain function at the unity gain frequency when we choose ω_{d2}. So, in other words loop gain at the angle of the loop gain at the unity gain frequency for ω_{d2}, angle of the loop gain at the unity gain frequency for ω_{d1}, right. So you know not to get confused about what you call phase lag and phase lead and negative and positive and so on.

So, this will give you simply the phase lag, right at the unity gain frequency for both choices of dominant pole frequency. Which one will have a larger phase lag? Which choice of dominant pole frequency will have a larger phase lag?

So, loop gain at the unity gain frequency for the angle of the loop gain at the unity gain frequency for ω_{d2} versus ω_{d1} which one do you think has got a higher phase lag at the unity gain frequency, which choice of ω_d will result in a higher phase lag, right at the unity gain frequency?

Student: ω_{d1}.

Why?

Student: ω_{d1} is greater than ω_{d2}.

ω_{d1} is greater than ω_{d2} and therefore, the angle of the loop gain is I mean the phase shift added due to those poles ω_o, right at the unity gain frequency is you know it is much higher than what you would get at ω what you get for ω_{d2}, right. So, basically, the angle of the loop gain here is smaller than in absolute value smaller than the angle of the loop gain at ω_{d1}, alright.

So, next thing is the next thing that I would like to talk about therefore, so, in other words this angle of the loop gain at the unity gain frequency is basically an indicator of you know the location and the effect of all these high order poles on the stability of the system, correct.

So, if all the poles were infinitely far away from the unity gain frequency, the angle would simply be negative 90º, correct. So, any increase of the phase, any increase of the phase lag beyond negative 90º, basically is reflective of how far the poles are from the unity gain frequency.

And you know what we have to be very very about? Remember, that the closed loop gain is nothing but 1/f (loop gain(s)/(1 + loop gain(s)). And if this loop gain(s) becomes, so if loop gain($j\omega_u$), the unity gain frequency let us say this becomes - 1 which is basically $1 \angle 180^o$, right I mean phase lag, so $1\angle - 180^o$.

Then, what happens? This 1 + loop gain simply becomes 0. So, the closed loop gain becomes infinite which basically means as the system is unstable, alright. So, this so, what so, if the angle of the loop gains at the unity gain frequency, right is 180º then the system is unstable, yeah.

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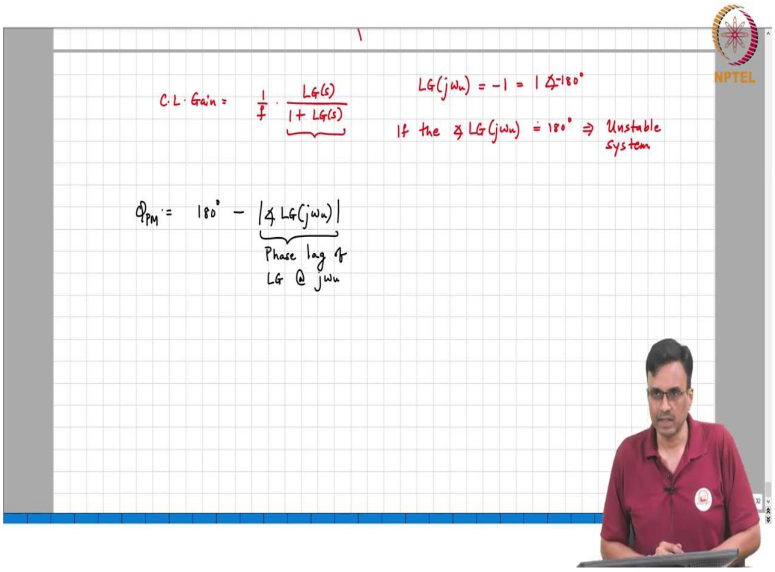

So, basically that brings us to the next topic. So, basically, the question is, you know, is there, I mean clearly ω using a lower dominant pole frequency makes the loop gain function a better approximation to a true first order system. But we are paying a big penalty for choosing the ω_{d2}, right. And what is that penalty by paying? You can see that the unity gain bandwidth of the loop gain function which is also the 3 dB bandwidth of the closed loop system, right. Remember, that if you have a first order system, the unity gain bandwidth of the unity gain bandwidth of the loop gain function is also the 3 dB bandwidth of the closed loop system.

If choosing ω_{d2} therefore, basically reduces the unity gain bandwidth as you can see. And consequently, the 3 dB bandwidth of the closed loop system is also reduced. So, that is the price you are paying to make a better approximation to a true first order system. So, but then the obvious question is, you know as long as my system is stable, right with this A_o^3f DCloop gain. Why do I care about making it approximate a good first order system? Correct, ok. So, for instance what is wrong with me in this example choosing ω_{d1} as opposed to ω_{d2}. Even with ω_{d1} you can see that the unity gain, I mean the frequency at which the response deviates from that of a true first order system, is already considerably below 1. So, then the next obvious question is, why am I losing out on bandwidth, right. What if I choose an even higher dominant pole frequency, do something like that. So, you can see that here I get an even larger one, this is ω_{d3}. So, I now get an even larger unity gain frequency.

But what comment can we make about the angle of the loop gain, you know as we keep progressing from ω_{d2} to ω_{d1} to ω_{d3}? The phase lag of the loop gain function at the unity gain frequency is going on increasing, reflective of the fact that the higher order poles are getting closer and closer to the unity gain frequency, right. So, so and how much phase can we tolerate before the loop gain becomes unstable? How much phase lag can we afford in the loop before the loop becomes unstable?

Student: 180 degrees.

We can tolerate 180^o, right. So, this quantity the angle of the loop gain at $j\omega_u$, right must be how much? So, the margin we have for the phase before the loop becomes unstable is the difference between it is 180^o - the phase lag of the loop gain at $j\omega_u$, correct.

We can tolerate a phase lag of 180^o, correct or π radians. And the loop gain function at the unity gain frequency has some phase lag, right. So, the margin we have, right for excess phase before the closed loop system becomes unstable is, 180^o - the phase lag that we have at the loop.