Analog Electronic Circuits
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Lecture - 70
Dominant-Pole Compensation part 2

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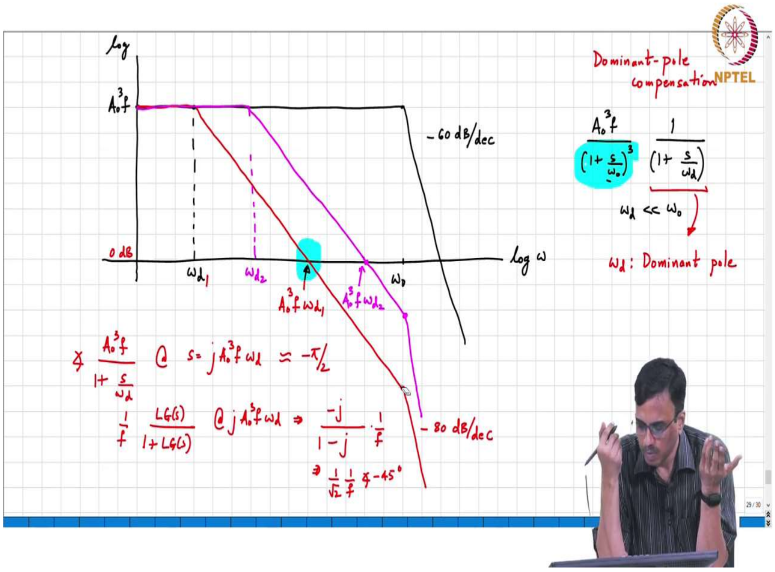

So, now I mean staring at this particular example right. So, because we have introduced this dominant pole ω_d which is much much smaller than ω_o right ok. The loop gain function approximates that of the response of the loop gain basically approximates that of a first order system and therefore, the closed and so, you basically get now the stability of a first order system right. Truly it's actually a fourth order system right, but what this is telling us is that since the higher the poles ω_o are so, far away right for all practical purposes this is looking like a first order system right and why do I say for all practical purposes.

So, basically you can see that when these higher poles kick in which add extra phase which can potentially make the angle of the loop gain equal to 180° right the magnitude has become so, small that 1 + loop gain function whatever you add right if the phase is 180°, but the magnitude is so, small that denominator can never become 0 does it make sense right.

So, for all practical purposes this looks like a first order loop gain function right which then, therefore, means that the 3 dB bandwidth of the closed loop system will be A_o^3 f what do you call the 3 dB bandwidth will be A_o^3 f ω_d. So, here we have a situation which we wanted, namely that we have the high DC loop gain corresponding to a third order system and the stability properties corresponding to a first order system. Now, the question is what have we lost in the bargain? I mean you know that there is no free lunch right. So, what have we lost? See in the original system remember that the loop gain function was high up to a frequency of almost ω_o. But now what is happening? You know at frequencies way below ω_o. We basically see that the loop gain has already fallen off to 1.

So, you know one could of course, complain that well in this whole process you know earlier we had this this loop gain magnitude which is very large for up to very high frequencies and now you know we end up in a situation where the magnitude response of the magnitude of the loop gain has fallen off to 1 right at frequencies way below ω_o. Remember that if you want this ω_o not to bother you at all right these poles at ω_o not to bother you the unity gain what comment can we make about the unity gain frequency versus the location of all these extra these poles at ω_o

Let me repeat the question. If we did not want these extra poles, I mean these poles at ω_o to bother us in any way as far as stability is concerned, what comment can we make about the relative position of the unity gain frequency which is A_o^3 f ω_d versus these poles at ω_o? So, basically you know to make this look like a very good first order system you must choose the unity gain frequency to be way smaller than the ω.

The location of the higher poles does it make sense ok? So, therefore, you know what we have paid in order to achieve this stability is that what you have paid in order to achieve stability is that well bandwidth has gone down right. Well, but this seems like a reasonable trade off because I mean you know you earlier had a system which was unusable because it was unstable right. Now, well you have a system which is usable and you know and you have the high DC loop gain that you want right.

So, this is the basic idea behind dominant pole compensation right. So, now the question is ok, well now what if for instance you know I chose this, let us say I chose this dominant pole frequency to be ω_{d1}. What if I had made another choice ω_{d2} like this ok. So, this is another choice of dominant pole frequency, this is another choice of dominant pole frequency right. Here also this is A_o^3 what is that? A_o^3 f ω_{d2}. Here also at the frequency that the higher order poles kick in alright. The magnitude of what comment we can make about the magnitude of the loop gain function is less than 1 right. So, you know for all practical purposes this also looks like a first order system. So, now the question and, but what is the advantage of choosing ω_{d2} over ω_{d1}?

Student: The unity gains bandwidth.

Well, the unity gain bandwidth is evidently increased by some factor. So, there must be a way of quantifying I mean which of these systems I mean which of these systems ω_{d1} which of these two choices of dominant pole frequency ω_{d1} ω_{d2} which of them you know make the loop gain a better approximation to a first order system ω_{d1} because you know it is the frequency at which you know the higher order poles kick in at that frequency the magnitude is much smaller. So, there must be some number or some metric to be able to qualify.

You know how close to the true first order system the dominant pole is making the system look like. Is that clear right? I mean you know as you can see in this example both ω_{d1} and ω_{d2} seem to be doing a reasonable job of making the system stable. But clearly, we see that ω_{d1} is; obviously, a much better I mean is doing a better job of making it look like a first order system when compared to ω_{d2}. So, now the question is how do you quantify it? How can we I mean yeah can we think of ways in which we can say visually it's apparent ok. The question is you know we are an engineering right we would like to attach a number to everything. So, what can we do?

The third order system, you compare it and then what? So, what will you do again? So, you will take a third order system. You will take this compensated system then you will draw the Bode plot of a true first order system with this with the same pole frequency. The true first order system will basically be something like that and then what will you do?

I mean you basically you will what you are saying is you will measure the magnitude of the loop gain function at which the Bode plot of the high order system deviates from the actual true first order system correct and if the magnitude is smaller. It is a better approximation to a true first order system. Does it make sense ok? Is that clear or not alright? So, unfortunately it turns out I mean if you have to do this in practice what practical difficulties do you think this will lead to?

I mean let us say you somehow make a first order system with you know with ω_{d1} ok. The problem with magnitude measurement is that you know at the frequency where the response of the first order system deviates from the higher order system the magnitude is very small and therefore, making any sort of measurement on this becomes fundamentally difficult.

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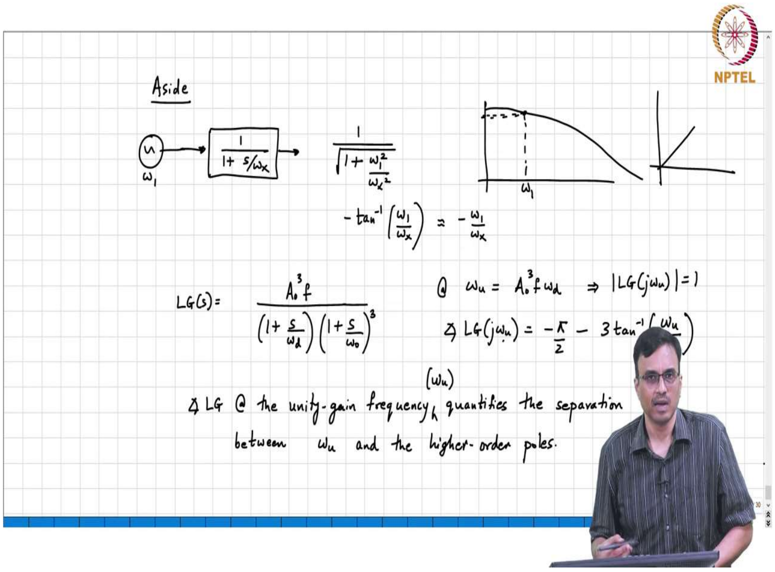

So, but fortunately it turns out and you should have seen this in your classes before, but let me pay attention to the following. So, let us say you have a; you have a single pole system right $1/(1 + s/\omega_x)$ and you would like to measure that you do not know what ω_x is; you only have a low frequency sinusoidal generator with a frequency ω_1.

When you are in a lab you are told that inside the box is basically a system with a DC gain of 1 and a pole frequency ω_x right 3 dB bandwidth ω_x. I have a low frequency sine wave ω_1 right where I know that ω_1 is much smaller than ω_x. How will we find ω_x? The many ways of doing this how will we find ω_x?

So, basically one way is to say well I will know plop in the sin wave look at the output what is the magnitude of the output?

Student: $1/\sqrt{(1+(\omega_1^2/\omega_x^2))}$.

That is the magnitude I will measure and from this I will be able to calculate ω_1/ω_x ok. So, now. So, this ω_1/ω_x is a small number right, which basically means that basically the magnitude response is looking like this at DC. What is the slope of that magnitude response?

Student: 0.

It is actually 0 right. So, if you measure the magnitude response at ω_1. The slope is very small. So, if you make a small error in measuring the amplitude what comment can you make about the error in measuring ω_x.

It will be very high. Does it make sense to people? On the other hand, what is the angle of the phase shift between the input and the output? $-\tan^{-1}(\omega_1/\omega_x)$. For small ω_1/ω_x what do you know the tan inverse approximately? This is approximately - ω_1/ω_x. So, this is more sensitive to ω_x, the magnitude falls off like this. What comment can you make about the phase lag? It is linearly correct. So, which would be which makes more sense to use to make a lot more sense to use phase rather than magnitude right. Because the effect of a pole therefore, is visible way earlier in the phase plot rather than in the magnitude plot ok alright.

I do not know how many of you have done this undergrad experiment where you build a filter. Let us say you build a band pass filter and you have to measure it center frequency right. What is the dumb way of doing it? If you go on changing frequency look where the magnitude peaks and then right? But the problem with that is that whenever the magnitude peaks its derivative is 0. So, you change the frequency it looks like all you know the magnitude looks largely flat correct. So, you are not able to make a good judgment. However, if you look at the phase it turns out that the phase will be very rapidly at the center frequency. So, if you just look at the phase shift between the input and output you will find it is extremely sensitive to center frequency which is basically what you want.

So, you will be able to accurately measure the center frequency right. So, the moral of both these stories is that the phase is a much more sensitive function of you know frequency than the magnitude is. So, the effect of these higher poles on the magnitude occurs only you know at ω_o right. But the phase basically occurs when the effect on the phase occurs at a much earlier frequency. So, for example, you know, a convenient way of measuring the phase I mean we want to. I mean basically we want these higher order poles to be far away from the unity gain frequency correctly. They you want them to occur way after this has become the magnitude of the loop gain function has become 1 right. So, a way of measuring how far these extra poles are from the unity gain frequency is to simply measure the phase shift caused by these higher order poles at the unity gain frequency not at ω_o.

I mean you know at ω_o they cause 45^o each that does not make any sense. So, basically, we want to look at to figure out how far these extra poles are from the unity gain frequency. We need to measure the phase of the loop gain function at the unity gain frequency. The phase will consist of two parts one is that due to ω_d itself. That dominant pole frequency will add some phase shift. How much phase shift does it add at unity gain? It basically adds -90^o. If there were no higher order poles at all.

Then this would be a true first order system and the angle of the loop gain at the unity gain frequency would be exactly -90^o assuming that $1/A_o^3$ f is very large. Now, if there were these extra guys at ω_o, what would be the phase shift at the unity gain frequency? So, basically the phase lag would be more than 90^o. So, if the loop gain function is A_o^3 f/(1 + s/ω_d) say (1 + s/ω_o)^3, then at the unity gain frequency which is I am going to call that A_o^3 f ω_d the angle of the loop gain what is the magnitude of the loop gain function at the unity gain frequency?

Student: 1.

By definition unity gain frequency means the frequency at which the loop gain has a magnitude 2. So, the magnitude of the loop gain is 1 by definition the angle of the loop gain function is what?

Student: Less.

Yeah, you know everything, just plug it in and tell me what it is man – π/2 -3 $\tan^{-1}(\omega_u/\omega_o)$. And so, clearly as you see a sanity check as ω_o tends to infinity, what will you get for the loop gain I mean for the angle of the loop gain at the unity gain frequency – π/2. The higher and higher the unity gain frequency becomes in relation to ω_o. You see that this phase lag becomes more and more negative. And you know at what point do we start becoming very worried?

We know that when the angle of the loop gain becomes -1 at the unity gain frequency the loop gain then becomes -1 and the closed loop gain becomes infinite which means that the system is unstable. Does it make sense? Is this clear? So, this basic summary is that the angle of the loop gains at the unity gain frequency. You know basically quantifies the separation between UGB and let me call this between ω_u and the higher order poles ok.

And we know that the danger mark is basically 180^o ok and so, you know the closer the loop gain angle of the loop gain at the unity gain frequency becomes to 180^o the more we need to be worried about instability right. So, you know a way to quantify that is to actually look at the difference between 180-degree phase lag and the actual phase. So, if it is a true first order system that difference would be how much?

Student: 90^{o}.

90^{o} because 180^{o} is the phase lag is the danger mark the first order system will give you a phase lag of 90^{o}. So, the difference is 90^{o}. Now, if you have extra higher order poles what comment can we make about that difference?

Student: Less than 90^{o}.

It will be less than 90^{o}. So, that is what is called the phase margin ok. So, we will continue in the next class.