Analog Electronic Circuits
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Lecture - 69
Dominant - Pole Compensation part 1

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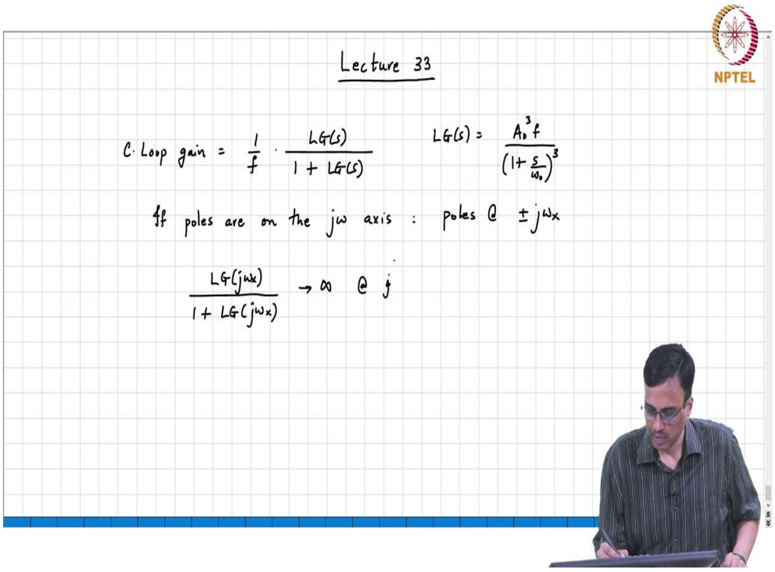

In the last class, we were basically looking at the stability of negative feedback systems and you know over the last couple of classes, we looked at we started off with a first order forward amplifier and we found that it is unconditionally stable. Unfortunately, the practical reality happens to be that the DC gain that you can get is very modest and consequently you will basically have you know that while it is great, it is not realizable in practice, right.

The high gain that we want is not realizable in practice with one stage. And, then what did we do? We said ok, well the easiest way of increasing gain seems to be to simply cascade stages. So, we cascade at two stages and we found that the cache of the closed loop system is technically stable.

But for all practical purposes, the transient response is got, I mean if we want to realize the ideal of having a large loop gain at DC, then the quality factor of the closed loop poles is going to be you know so high that the transient response you know when you know excited with the with the step for instance is going to ring for a very, very long time and in an amplifier that is actually quite undesirable, right.

Then we thought ok maybe you know it just so happens that the second order case we got unlucky and we said let us try and look at the, there we will put three stages in cascade and now you know all hell breaks loose. The poles are now you know if , A_o^3 f that is the DC loop gain is greater than even 8, right. We basically find that the closed loop system is unstable, ok.

Now, yesterday we actually solved the equations, right and we found the roots of the characteristic equation and you know plotted the locus of the poles as a function of the DC loop gain and you know we found when the system becomes unstable, right. As another way of doing it I just want to cover this briefly, right.

So, remember that the closed loop gain is 1/f (loop gain(s)/(1 + loop gain (s))) and if the poles are on the jω axis. So, I mean the aim therefore, is to find and in our case loop gain of s happens to be A_o^3 $f/(1 + s/\omega_o)^3$, ok.

And, yesterday we used root locus to find for what A_o^3 f the poles will just be on the jω axis. So, you know this is an alternate way of finding the same answer and the approach is the following. If the poles are on the jω axis, anyway we know that they must be complex conjugate, right. There must be at $\pm j\omega_x$, correct.

Now, what is the meaning of you know if you have a pole at a certain complex frequency for a transfer function? What will you get when you evaluate the transfer function at that pole frequency? So, basically the transfer function when evaluated at pole frequency goes to?

Student: Infinity.

So, now there are poles at ±jω , right. So, therefore, therefore, loop gain of jω/(1 + loop gain of jω)must go to infinity at $j\omega_x$, right? I mean so, basically must go to infinity, correct. So, if this must go to infinity what comment can you make about loop gain evaluated at $j\omega_x$?

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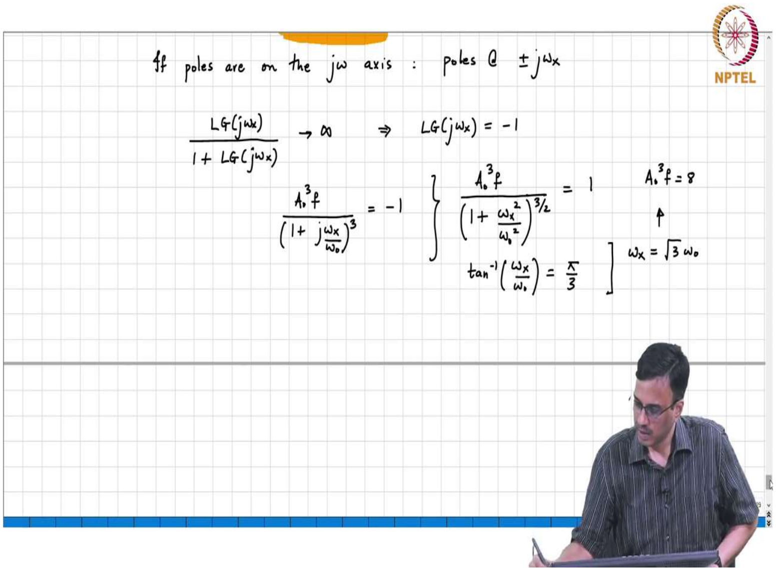

So, which basically means that loop gain of $j\omega$ = - 1, ok, alright. So, if so, now, if you look at the loop gain function it is A_o^3 f, it is a complex function, it is $(1 + j\omega/\omega_o)^3$ and this must be equal to - 1, correct. So, it is a complex number on the left hand side. So, yeah, how many equations do we get? Either real part, imaginary part or magnitude and phase. So, this basically leads to A_o^3 f/$(1 + \omega_x^2/\omega_o^2\ )^3$ halves must be equal to 1 and what about the angle? So, $\tan^{-1}(\omega_x/\omega_o)$ must be equal to $\pi/3$, alright. So, which basically means what ω_x is $\sqrt{3\omega_o}$ which basically what are the unknowns that we are trying to find?

Student: A_o^3 f.

A_o^3 f and ω_x, right. ω_x is $\sqrt{3\omega_o}$. So, what is A_o^3 f? So, which basically means that A_o^3 f is, 8, because ω_x^2/ω_o^2 is now 3, 3 + 1 = 4, $4^{3/2}$ is 8 and therefore, A_o^3 f is 8, which is you know not surprising because this is exactly, this is what we expected anyway which we got from root locus analysis, right.

So, you know, so, you know one way of basically checking, you know if a system is close to becoming unstable, right. So, if a system is close to becoming unstable what does it actually mean? I mean so, in other words, if the closed loop system has got poles on the $j\omega$ axis, or close to the $j\omega$ axis, what comment can you make about the closed loop transfer function? It will become very, very large at a certain frequency, right? And, if that happens, I mean looking at this expression, when do you think the gain can become very large at a certain frequency?

So, when the denominator is going close to?

Student: 0.

0 and that happens when the loop gain evaluated at some complex frequency basically at some jω, if that loop gain function is coming close to - 1, which mind you is basically magnitude is 1, the angle of the loop gain is, if loop gain of some jω is - 1, the magnitude of the loop gain is?

Student: 1.

What is the angle of the loop gain? The phase lag is basically 180 degrees, ok. If that happens then you know that 1 + loop gain basically goes to 0 and then therefore, the closed loop gain goes to infinity, which is telling you that there is a, there is a, there is a pole for the closed loop system at that frequency jω, alright. Does it make sense folks? Alright.

So, so, anyway so, the question is, you know, ok well all this analysis is fine, but the real question is, you know what we do to fix this problem, right. So, as we discussed yesterday, we want to have the height DC loop gain as well as you know stability, right, because this A_o^3 f being 8 is simply a non-starter, correct, ok.

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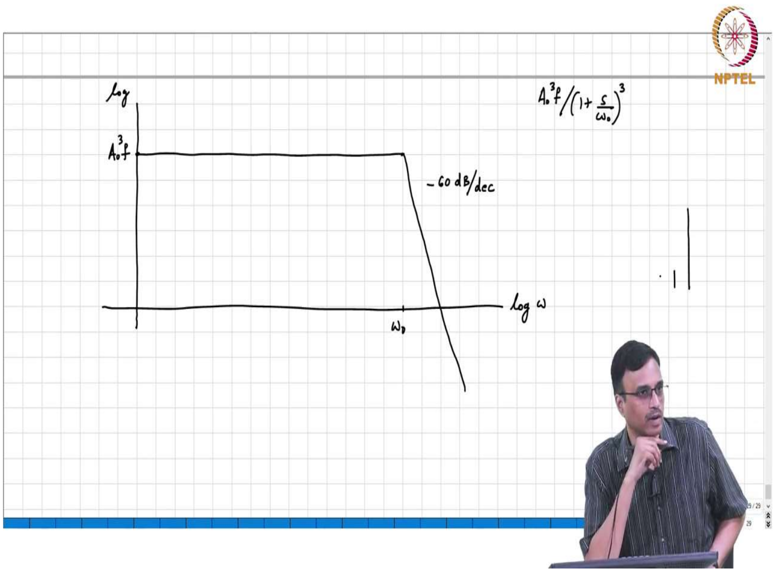

So, the basic idea is the following and you know again we take inspiration from the fact that a first order system has got none of these stability problems. So, the idea is well, you know if I have a third order loop gain function, the loop gain function, the magnitude of the loop gain function basically if I all plot this on a log plot, right.

So, you know when I mark this as A_o^3 f, what actually it means is that what I am plotting is 20 log(A_o^3 f), I do not want to keep writing this 20 log all the time and then make the whole plot very messy, ok. So, now if you plot the Bode plot of this, this guy, what does this look like? So, let us say this is ω_o, how will the Bode plot of this loop gain function $A_o^3 f/(1 + s/\omega_o)^3$, what does it look like?

Well, the bode plot is an asymptotic plot. So, it looks like you know and mind you this is log(ω), and after that how will it roll off? -60 dB/decade. So, this basically does something like this, ok. So, this is - 60 dB/decade. So, you want to make this third order system look like a first order system. So, any suggestions on what we can do? Ok, let us say, ok let me give an analogy, right, here is a stick, ok, alright. How do you make this stick smaller without touching it?

If you want to make this stick smaller without touching it, well you draw a longer line next to it, right, which basically is equivalent to saying that the small, large etcetera are all relative terms, right. And if you want to make the stick longer without having to touch it, what do you do?

You put a stick smaller next to it and then finally, this becomes long, right, ok. So, now, you know using this as inspiration, you know what do you think we can do to make this look like a first order system, ok?

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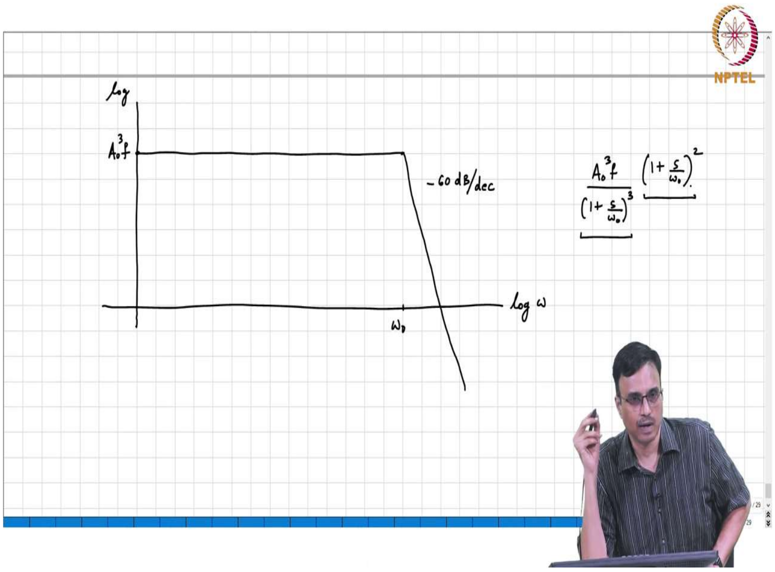

So, let us take a look at what all things are possible and then you know rule out some possibilities. So, this is the loop gain. So, what our Abhishek is saying here is hey, you know why do not I, this is easy. Why do not I just multiply this A_o^3f $(1 + s/\omega_o^2)/(1 + s/\omega_o)^3$? I will cancel off, right, on paper it looks perfect, right, and I get a first order system, you know move on, ok.

What is the problem with this with this approach? I mean so, you cascading this R forward amplifier with another block with this transfer function, correct. So, what is the gain of this transfer for this extra transfer function that you are introducing? What is the gain of that at infinity?

It is infinite, right. So, it is impossible to get a gain of infinity at infinite frequency, correct. So, this is and remember that you can never realize a transfer function 0 without adding an extra pole. How do you realize a 0? There must be some memory element. That memory element is going to add some poles, right and presumably it is, you know, physically impossible to get those poles to be at a higher frequency than ω_o because if I was able to make such high frequency poles, I may put them in my amplifier in the first place. You understand? Ok. So, mathematically though this seems, you know, the easiest thing to do, you just multiply by, you know, $(1 + s/\omega_o)^2$ and then we are done, right. So, that is not practical, ok, alright. So, what else can we do?

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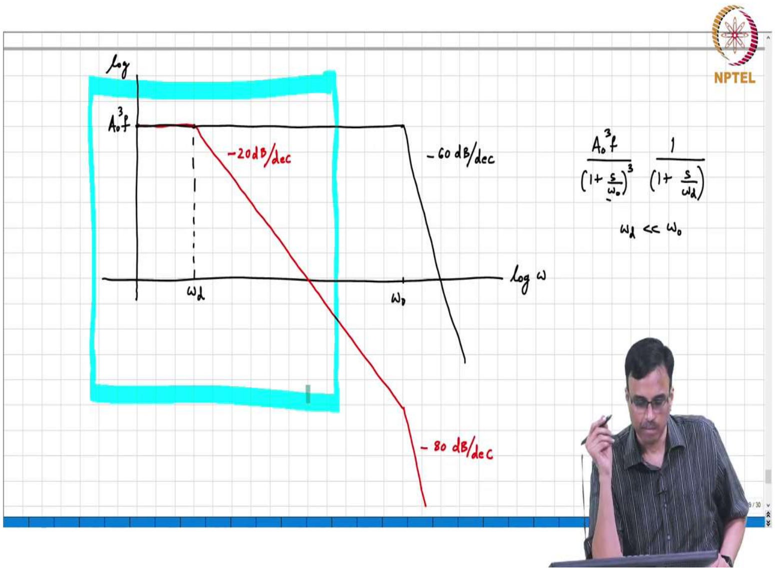

So, basically the idea is, well, you know, what if I deliberately multiply this over a function $1/(1 + s/\omega_d)$, where ω_d is much, much smaller than ω_o, right. First of all, is this practical? Can we do this? Well, you know, it is always difficult to make a slow person fast, but making a fast person slow is very easy, right ok. So, you can always pull people down, right? So, to slow a fast system which is very fast, you know, we are very good at doing it. So, it is very straightforward.

All that you take is, you know, find some node in the circuit, take a huge capacitor and put it on that node, the whole circuit will slow down, right, ok. So, what do you call, and if we choose ω_d to be very, very small compared to ω_o, how will the Bode plot of this resulting animal look like?

Student: Up to ω_o it is constant.

Up to ω_o it will basically be constant. After that what will happen? It will fall down at - 20 dB/decade, which is what it looks like for a first order system, right and at ω_o, it starts. Well, it starts to go down at - 80 dB/decade, ok. So, this is - 20 dB/decade, and this is - 80 dB/decade, and if I just showed you this part of the picture, imagine I covered up everything outside this blue rectangle right, and I showed you this picture and I asked you, what this red curve represents, what would you say?

Student: First order system.

Where you say this looks like a first order system. So, what do you call it? So, this basically is the basic idea behind trying to make, this is how you try to make, I mean, please note that the red curve corresponds to actually corresponds to a fourth order system, right. But, it looks like the important frequency range, right, we will come back to what it means to say important frequency range. Over frequencies of practical interest, it looks like a first order system, right.

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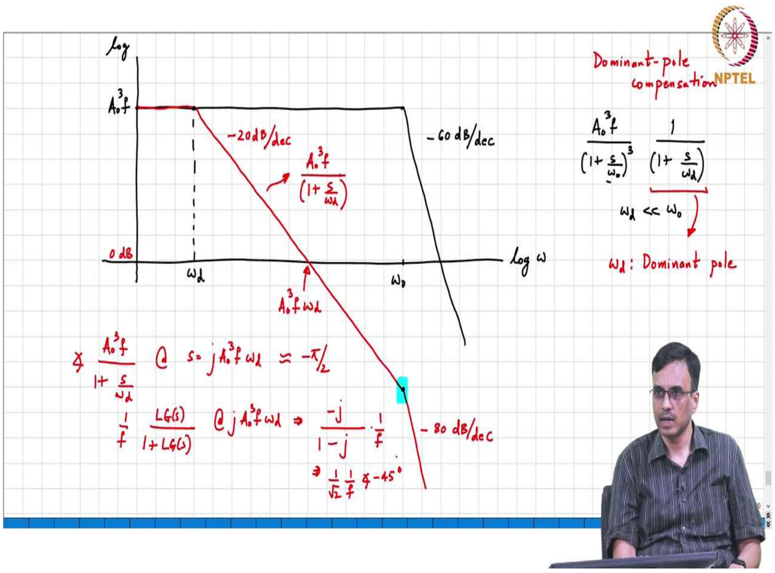

So, let me label this, this is, you know, 0 dB or 1, ok, alright. So, this pole, this ω_d is called the dominant pole and this basically, you know, dominant pole because it basically pretty much within quotes dominates the frequency response of the loop gain function, right.

So, for all practical purposes, therefore, this ω_d is the most significant pole. All the other, this ω_o now, you know, as far as we are concerned are poles which are so far away from ω_d, that their influence on the frequency response of the system, you know. They only kick in this particular example when the magnitude of the loop gain function has fallen way at the point at which the extra poles at ω_o, at the three extra poles at ω_o kick in, right we see that the magnitude of the loop gain is way smaller than 1, right.

So, going back to our, you know, our observation that the, we are going to be only in trouble when the loop gain function becomes close to -1, when the loop gain function becomes close to -1 and 0 is when the denominator goes to 0, right and, you know, where, and at the frequency at which the loop gain magnitude goes to 1, in this particular example where the dominant pole ω_d is chosen to be so small compared to ω_o right, that the frequency at which the higher poles kick in at that frequency, the magnitude response is already fallen to something which is very, very small compared to 1, right.

So, at this frequency, what comment at the unity gain frequency of the loop gain, which is the new unity gain frequency of the loop gain? So, basically the new unity gain frequency of the loop gain is, is $A_o^3f\ \omega_d$ approximately. Why because in this region, this curve can be approximated by this expression, $A_o^3\ f/(1 + A_o^3f\ )\ (1/(1 + s/\omega_d))$. So, it is a first order system with the DC gain of A_o^3 f and a pole at ω_d. Does it make sense, alright? And, we know very well that, at a frequency much greater than ω_d, you can neglect that 1 and the unity gain frequency of this system is A_o^3 f ω, d alright. So, at the unity gain frequency of this dominant pole compensated system. So we have stabilized the system by adding a dominant pole.

So, this is often called dominant pole compensation. And, so, at the unity gain frequency of the dominant pole compensated system, what common can we make about the angle of the loop gain? $A_o^3/(1 + s/\omega_d)$. What is the, you know, what is the unity gain frequency? A_o^3 f ω_d. So, what is the angle of the loop gain at that frequency? It is $-\tan^{-1}(A_o^3\ f)$ and is A_o^3 f, a small number or a large number?

Student: Large number.

Large numbers. So, what is $\tan^{-1}(A_o^3\ f)$? This is going to be approximately $-\pi/2$, which is what you will see if you had a first order system, correct? Ok. So, if you had a true first order system, you would see an angle of -, the angle at the, at the unity gain frequency for the loop gain function would be - 90°, ok.

And as you can see, if the angle of the loop gain is - 90°, what is that loop gain/(1 + loop gain) at A_o^3 f ω_d is what? What is the loop gain at A_o^3 f ω_d? Loop gain is a complex number. $1\angle{-90^o}$. So, basically it is $-\ j/(1\ -\ j)$, correct? So, what is the magnitude of this complex number? $1/\sqrt{2}\ \angle{-45^o}$. What is that telling you? So, the closed loop gain, remember is 1/f (loop gain/(1 + loop gain)). At the unity gain frequency of the loop gain function what is the closed loop gain? It is $1/\sqrt{2}\ (1/f)\ \angle{-45^o}$. And what does this mean? At what frequency will the magnitude of the gain of an amplifier go to $1/\sqrt{2}$. It is dB bandwidth, right. So, basically this is something that we knew already, ok and you basically will have, when we worked out the math, we saw that the unity gain frequency of the loop gain function for a first order system is the 3 dB bandwidth of the closed loop system, right. This is just us parroting the same thing all over again, right. There is nothing new here.