Analog Electronic Circuits
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Lecture - 68
Stability of Third-Order Negative Feedback Systems

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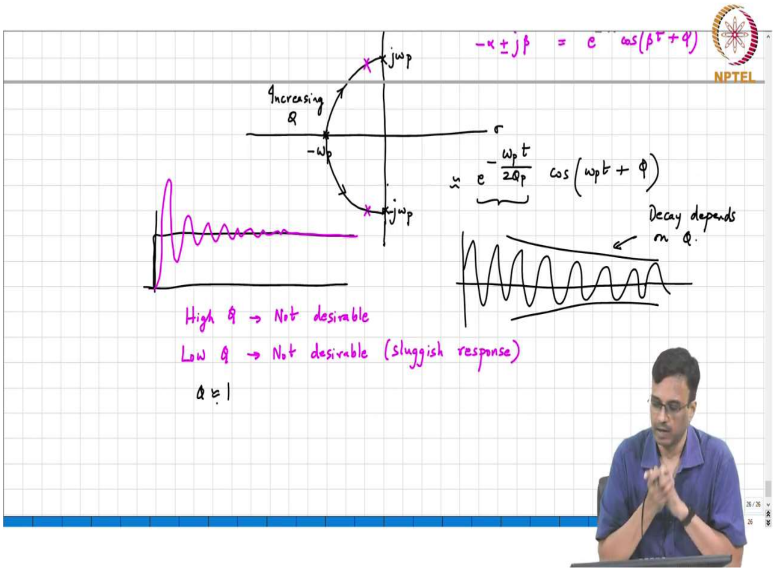

Theoretically 2 roots right, but the dominant root basically is going to be -1.

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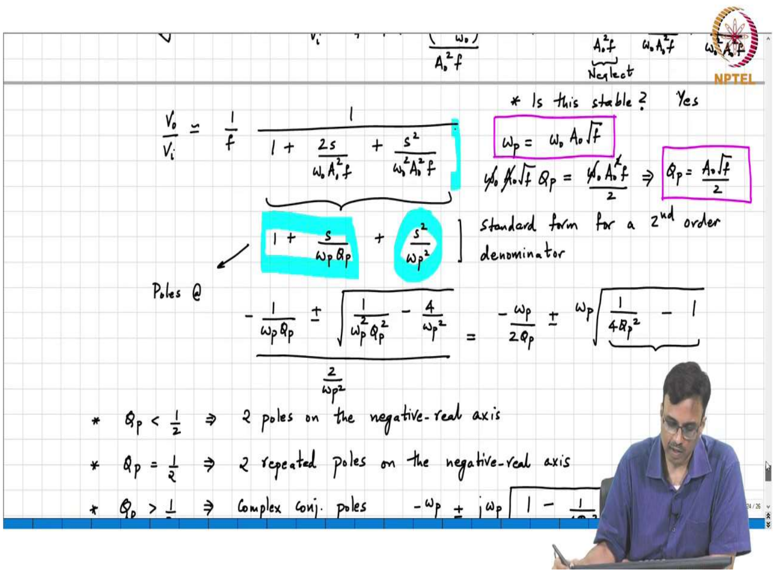

So, in the limit if you try to make the quality factor Q too low what is happening? It looks like a first-order system with a bandwidth ω_p Q_p correct. I mean the ω_p is inconsistent with what you said earlier. You said that it is going to become very slow, correct ok. So, why is it slow because the effective bandwidth is $\omega_p Q_p$ and because Q_p is very low the effective bandwidth is?

Student: Low.

Very very small right. So, the response of the system will be very very sluggish right. Again so, therefore, not desirable, you get a sluggish response ok. So, therefore, you know what is a; I mean high Q is not desirable, low Q is not desirable. So, what is desirable then? So, basically like everything else in life you know it is the middle path which makes sense right.

You do not want to have too much of anything or too little of anything ok. You do not want too much rain, you do not want too much sun, you do not want too much money, you do not want too little money, right I mean. So, joy lies always being in the middle, right. And therefore, you know here therefore, what is your; what is your you know what you think as engineers, what is the reasonable Q? So, Q of the order of 1 is a reasonable Q. So, as you said 1.5 heavens are not going to fall if you choose 1.5, heavens are not going to fall if you choose 0.8, right. But the Q you want for a closed loop system for the poles is roughly of the order of 1, right. Where there is one whereas, maybe is just critically damned or may slightly some small overshoot ok. So, this is what the Q we must be we must have alright.

Now, with this background let us go back to our actual system and figure out what Q we have, what Q do we have? The Q we got for our closed loop system is $A_o \sqrt{f}/2$ alright. So, this is $A_o \sqrt{f}/2$ is this a large number or a small number?

Student: Bigger number.

It is a large number. Remember that $A_o^2 f$ we want it to be very very large. Let us say I do not know maybe we want 10,000 right, then $A_o \sqrt{f}$ by 2 will be $\sqrt{10000}/2$ which is 50 alright. So, that basically means that you go and hit the amplifier with the step and then it will ring 50 before it finally settles. So, definitely I mean as you can see here this, I mean by cascading two identical stages in and with the hope of being able to get a high loop gain. I mean if you definitely got the high loop gain, but unfortunately, we seem to be in a situation where the closed system is technically stable but the response is got two or the closed loop poles have a very high Q and therefore, making the amplifier.

Student: Slow.

You know not slow I mean unusable because it keeps?

Student: Ringing.

It keeps ringing all the time, right. There is a lot of overshoot, undershoot and this kind of ok. So, it is like me asking you know what is 1 + 2 and then you know some fellow saying 10 and then the other fellow saying 0 and then the other fellow saying you know 0.5 and then the other guy saying 8.5 and after 400 iterations we converge to the answer which is 3. You understand? Definitely not you know, not a fun thing, alright.

It would be ok if you know the first fellow said you know 2.5 and then the next fellow said 3 and that is basically reasonable because we settle to the final answer quickly. Alright.

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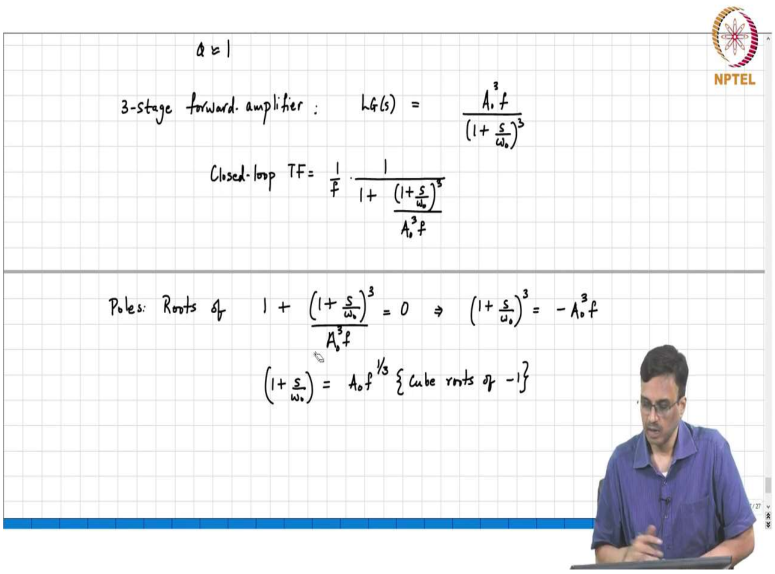

So, now, maybe we say ok well perhaps you know two being even is not our lucky number. So, maybe we try with 3 stages and see what happens. So, what do we do? Well, again same old same old. So, we will go through the steps quickly. So, what is the loop gain function? It is $A^3 f/\left(1 + \frac{s}{\omega_o}\right)^3$. So, the closed loop transfer function is

$$= \frac{1}{f} \frac{1}{1 + \frac{\left(1 + \frac{s}{\omega_o}\right)^3}{A^3 f}}$$

And rather than write this out like you know as a polynomial in the first two cases we wrote the whole thing out as a polynomial and looked at its roots.

But I do not know if you guys will, but you know I have given a third-order polynomial. I do not know of any quick formula to find the roots ok. So, rather than do that you know by exploiting the fact that the three poles are the same ok. If we mean by hook or crook we need to find where the poles of the denominator are right whatever trick works.

So, closed loop poles are simply the roots of $1 + \frac{\left(1 + \frac{s}{\omega_o}\right)^3}{A^3 f} = 0$ which basically means that this is $\left(1 + \frac{s}{\omega_o}\right)^3 = -A^3 f$. So, $\left(1 + \frac{s}{\omega_o}\right)$ therefore, is nothing but $A_o f^{1/3}$

Remember we have it as a third-order polynomial, it must have three roots and the three roots you know are $A_o f^{1/3}$.

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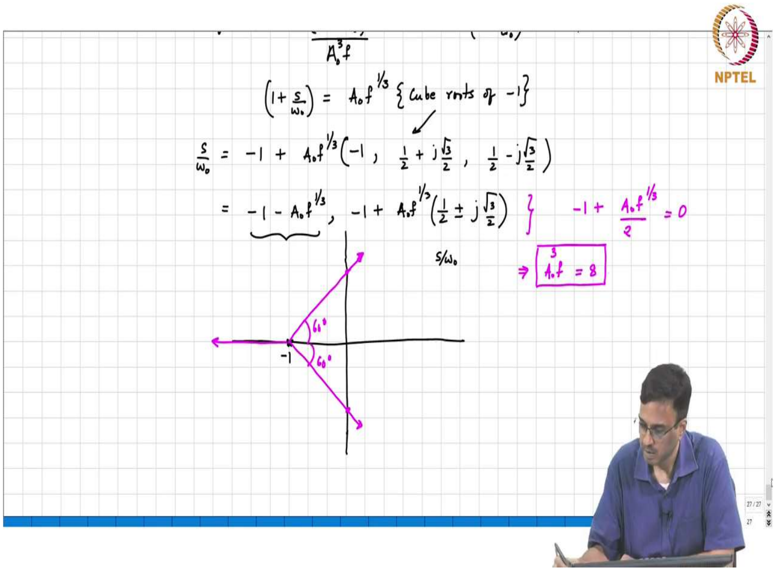

So, s/ω_o therefore, must be equal to $- 1 + A_o f^{1/3}$. What are the cube roots of - 1. The three roots are?

Student: $-1 + A_o f^{1/3}$ (½ ± j $\sqrt{3/2}$)

So, the poles are therefore, at $-1 + A_o f^{1/3}$ (½ + j $\sqrt{3/2}$) and $-1 + A_o f^{1/3}$ (½ - j $\sqrt{3/2}$). So, if you plot the locus of these roots as A_o is varied for A_o is 0. Where are the roots?

Student: Negative.

Where on the negative real axis? So, s/ω_o will basically be here. So, I am plotting s/ω_o. So that I do not have to keep carrying ω_o everywhere. So, this is basically going to be the three poles at - 1. As A_o as the magnitude of the loop gain keeps increasing which is what we want to do. What do we want that A_o^3 f to be?

Student: Large.

We want to be very large right. So, as A_o f to the one-third keeps increasing right what comment can we make about the locus of the roots? What happened to this guy? Yeah. One this branch is the root locus stuff that you are familiar with from control right. So, that goes to the left. What happens to the other two? They basically go at 60° and - 60° right, ok. And you know what is the thing that you notice from this picture, as you keep increasing A_o^3f eventually Two poles are going to go into the right half s plane. If the second-order system was bad enough where the Q is so high that it was unusable.

These poles are downright crazy right because they are in the right half s plane, alright. So, now, the question is, you know, perhaps we are still ok if that A_o^3f needed to push the poles into the right half s plane is so large that you know it does not bother us right. So, if you need the 10 million A_o^3f for the poles to go into the right half s plane, we say oh well that is 10 million sufficiently large numbers and maybe I work with a closed loop A_o^3f of 100,000 and I am still good right.

So, we may need to find out therefore, what is the maximum A_o^3f that will still result in poles in the left half s plane, ok, alright. So, what comment can you make about that? So, how will we find that? What is the limit of A_o^3f where the roots have 0 real parts? How will you figure it out $(-1 + A_o f^{1/3})/2$ and this must be equal to 0. So, what is A_o^3f?

Student: 8.

Alright. So, what is this and what is this whole story telling us? What is this telling us? That if the DC loop gain namely A_o^3f is greater than 8 then the system is?

Student: Unstable.

Unstable right and at this point you say well what is the whole point in going to this third-order system? There is a whole idea behind going to this third-order system was to get you know a large DC loop gain so that the properties of your negative feedback system will be largely independent of the forward amplifier right. But we see that we have a serious problem here, correct. And this the serious problem is that with a third-order system the closed loop poles are; the closed loop poles are in the right half s plane even when the DC loop gain is as small as 8 and then the obvious question is you know why did I do all this in the first place?

I might have used a first-order system right since its loop gain was low in the first place, but at least it was not unstable alright, ok. So, this first-order system is basically like you know you are in a; you are in a company and you know you have this employee who is very nice and easy to work with right, very stable right, but you know is not very smart ok alright because gain is not very high right.

The third-order system is like a very smart employee, but you know the person keeps fighting with everybody right and is impossible to work with because it was so unstable, correct, ok. So, I mean so, you are either one side you have the devil on the other side you have the deep sea right. So, what you actually want is I mean you want you know you want a smart person who is also good to work with right, ok. And you know does not get into fights with his teammates all the time. So, that is basically the intention alright.

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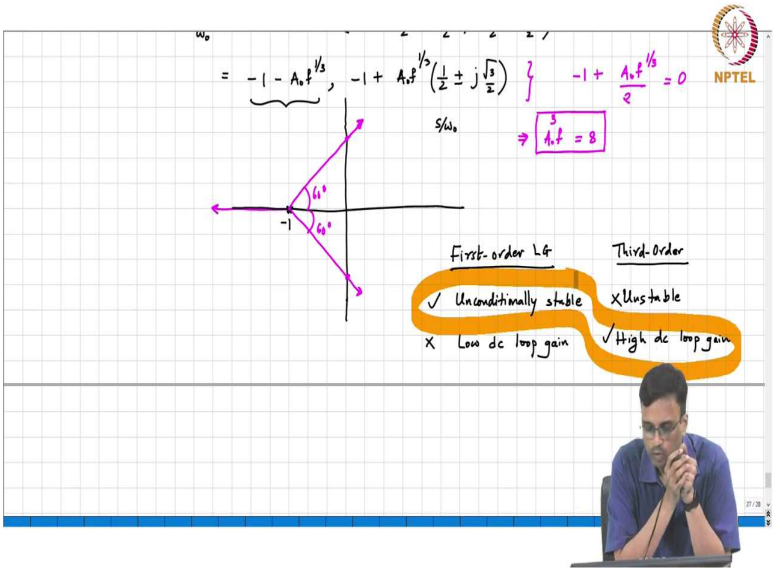

So, what would ideally like to do? Therefore, it figures out a way where so, let us say a first-order loop gains what are the good things about it; unconditionally stable, ok. What are the bad things? Low DC loop gain. Third-order or high order in general I mean ok, well we did this for third-order you know now perhaps you say oh maybe third-order is unlucky again right maybe we should do it for fourth-order. What do you think will happen? If you look at the trend, what do you expect to see?

Student: Four poles.

There you know that now you will have three poles you totally will have four poles right; two will go into the left half plane two will go into the right half plane. So, and you know higher order this thing basically you get things become even worse. So, third or higher order loop gain basically the nice thing is high DC loop gain and this is, but this is you know unstable. I am writing unstable because you know if you use high DC loop gain it is unstable. If you want to make it stable then the loop gain is low in which case the legitimate question to ask is why did I want to do all this in the first place, alright. So, this is the bad aspect of this and this is the good aspect of this. And like in most engineering situations you know what you want to do?

You want to combine these two so that you get, I mean you basically want to find a solution where you hope to get these two guys married so that you know the result will be something with the good points of both parents right. But of course, we do things wrong. You know you get the bad things from both parents and then that is a disaster, you understand. So, that is basically the whole idea behind what we are going to do, right.

The basic idea is the following. If you somehow take a third-order system and make it look like a first-order system right then you know you basically will inherit the stability properties of first-order systems.

And therefore, we will hopefully inherit some of the nice stability properties of the first-order system.