Analog Electronic Circuits
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Lecture - 67
Stability of Second-Order Feedback Systems

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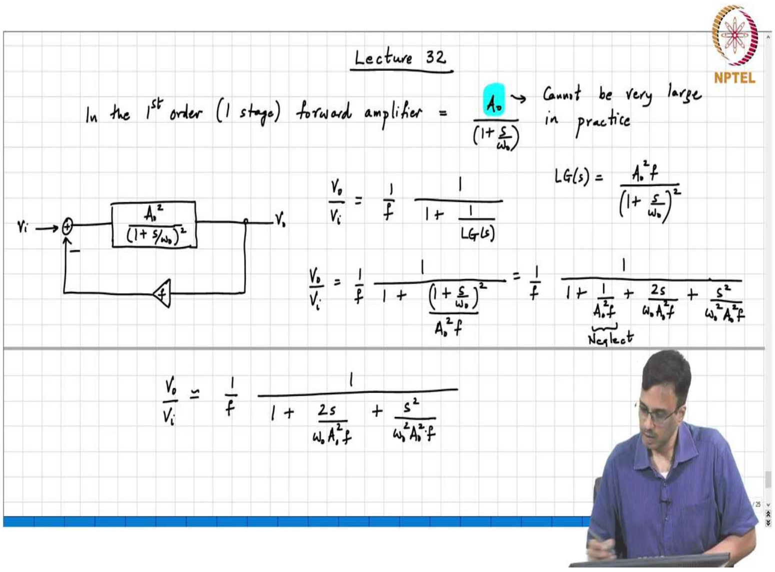

So, in the last class we basically looked at the stability of a 1st order closed loop system where the forward amplifier has got a single pole system. The nice thing about a single pole system is that the closed loop system is unconditionally stable right. And we also said that we also discovered that the unity gain bandwidth of the forward amplifier also happens to be the gain bandwidth product of the closed loop amplifier right.

So, if you want more closed loop DC gain, then you will end up having a smaller 3 dB band right and I mean this just happens to come out from the map right of a first order system. Now, many people take this out of context and say you know if you increase gain bandwidth it will reduce for any amplifier right and the gain bandwidth product is constant.

These are like these commonly you know circulating myths that you will encounter right, but that is not fully correct simply because you know it happens to hold for it happens to hold when the forward amplifier in a feedback loop is a single pole system and is you know you can easily come up with counter examples where this is not true right.

In general, you should expect that if you want more gain, you will get lesser bandwidth, but the fact that the gain bandwidth is a constant right only happens when you have a first order loop gain and the closed loop system you know the 4 behaves like you know 1st order system, but otherwise in general it is not known.

Now, what I mean is that everything is nice with the 1st order system except that there is a big practical problem and what is the practical problem? You cannot get that $A_o/(1 + s/\omega_p)$, which is what we used, s/ω_o. Is it what we use for the forward amplifiers gain?

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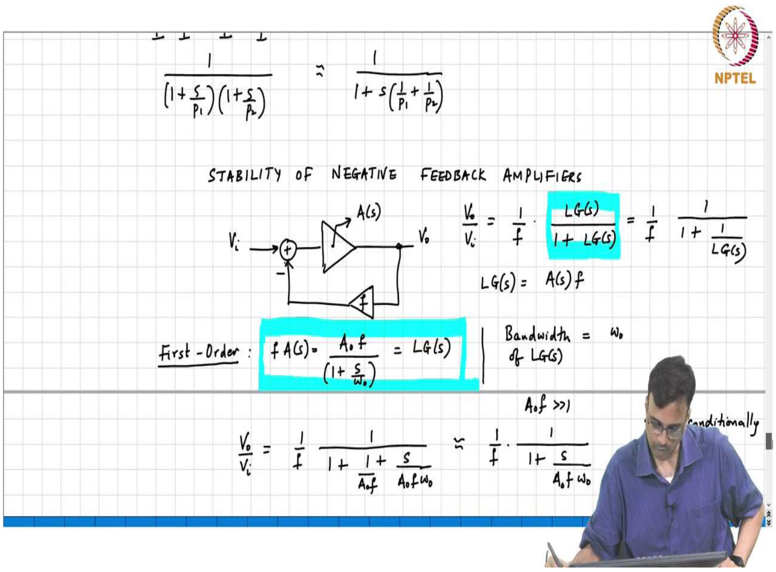

Yeah $1 + s/\omega_o$. So, in the 1st order case or basically 1 stage forward amplifier the forward amplifier's transfer function is $A_o/(1 + s/\omega_o)$ right. Unfortunately, in practice this A_o cannot be very large in practice and why do we need a very large A_o? Yeah, so basically the how well the negative feedback loop operates depends on how large the loop gain is. So, and the loop gain is basically $A_o f/(1+(s/\omega_o))$. So, we would like that A_o F to be as large as possible, but unfortunately if you use a single stage forward amplifier, then you know evidently there's not enough K right and you have seen the reasons for that when we talked about these transistor amplifier. So, straight forward way of getting more gain is to simply cascade stages.

So, basically, we say that is too bad. This is A_o if I cascade two stages then I have a forward amplifier which looks like this ok. And please note again you know at the risk of repeating myself, I will say that the need for a 2nd stage in the forward path is simply brought out by necessity right. Ideally if I could get you know very large A_o using a single stage then you know all this discussion is a moot point.

So, like everything else, the first thing we need to do whenever you see a feedback system is what did we do in the 1st order case? What are we trying to find? We want to find the input to output transfer function and you know what is the first thing we will check with respect to the transfer function? We want to check if the closed loop system is stable or not. So, as usual V_o/V_i in the Laplace domain is nothing but 1/f (1/1 + 1/LG(s)) which is the same as loop gain/(loop gain + 1) and what is the loop gain(s) here? Well, it is straightforward A_o^2 f/(1 + $s/\omega_o)^2$, correct. So, V_o/V_i therefore, is,

$$\frac{V_o}{V_i} = \frac{1}{f}\frac{1}{1+\frac{2s}{\omega_o^2 A_o^2 f}+\frac{s^2}{A_o^2 f \omega_o^2}}$$

Now, you know what can be. So, what term can be neglected in the denominator to make our lives a little less messy? Well, we say that well A_o^2 f the whole idea in cascading 2 stages was to have a large DC gain for the loop gain.

And therefore, A_o^2 f must therefore, be a very large number consequently we will be able to neglect that quantity thereby resulting in a transfer function which is for all practical purposes,

$$\frac{V_o}{V_i} = \frac{1}{f}\frac{1}{1+\frac{2s}{\omega_o A_o^2 f}+\frac{s^2}{A_o^2 f \omega_o^2}}$$

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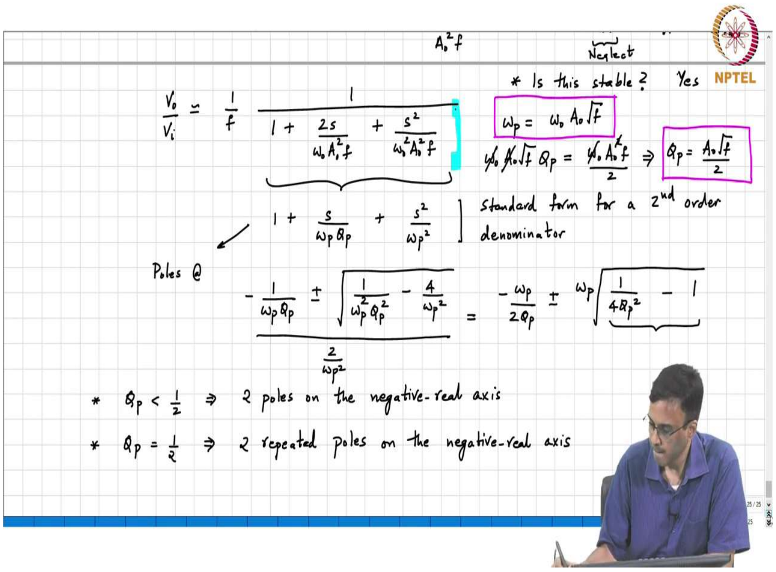

So, first things first you know the first question is this stable? Is this a closed loop transfer function right ? Is it stable? Well, it is a 2nd order polynomial in the denominator all the coefficients have the same sign, so all the roots are in the will be in the left half s-plane right. So, the answer to that question is an overwhelming yes. Now, the question is whether the left half plane is a big place. So, the question. So, a legitimate question to ask therefore, is where in the left half s plane are the poles located? Right. And to do that rather than work with you know all these multiple symbols I will use a standard form of a 2nd order denominator polynomial which I am sure you have seen in different contexts.

So, for example, I mean, so I am going to express that I am going to write this as $(1 + s/(\omega_P Q_P) + s^2/\omega_P^2)$ and this is what is called a standard form for a 2nd order denominator ok. I am sure you have seen the similar standard forms in control right where they write it as some you know ω_n^2 + 2 the damping factor you know s ω_n this is what we like to use in circuit work right and for good reason as we will see going forward.

So, if you want to map you know this polynomial that we have which is this guy here to the 2nd order denominator standard 2nd order denominator what comment can we make about ω_P and Q_P. What is ω_P? It is simply its very straightforward it's ω_o $A_{o\sqrt{f}}$ and ω_P Q_P which is ω_o A_o $_{\sqrt{f}}Q_P$ is nothing but ω_o a_o^2 f/2 which implies Q_P therefore, is nothing but.

Student: $A\sqrt{f}/2$. Does it make sense? Alright. So, let us now you know we mapped this 2nd order polynomial that we have for the denominator of our closed loop system to a standard

2nd order system right and the closed loop poles of the standard 2nd order system where are the poles located? Yeah, algebra man? So, basically - b = $\frac{-1}{\omega_P Q_P} \pm \sqrt{\frac{1}{\omega_P^2 Q_P^2} - \frac{4}{\omega_P^2}}$.

So, clearly you know as you can see there is something under the square root. So, we will have to consider cases where the discriminant you know is positive , 0 and negative. So, case 1 to Q_P less than 1/2 what comment can you make? What comment can you make about that quantity there? It is greater than . So, what comment can you make about the location of the poles?

Student: Greater.

So, basically for Q < ½, you have 2 poles on the negative real axis, ok alright. So, Q_P = ½ the denominator will be a perfect[2] and therefore, you have 2 repeated poles on the negative real axis ok.

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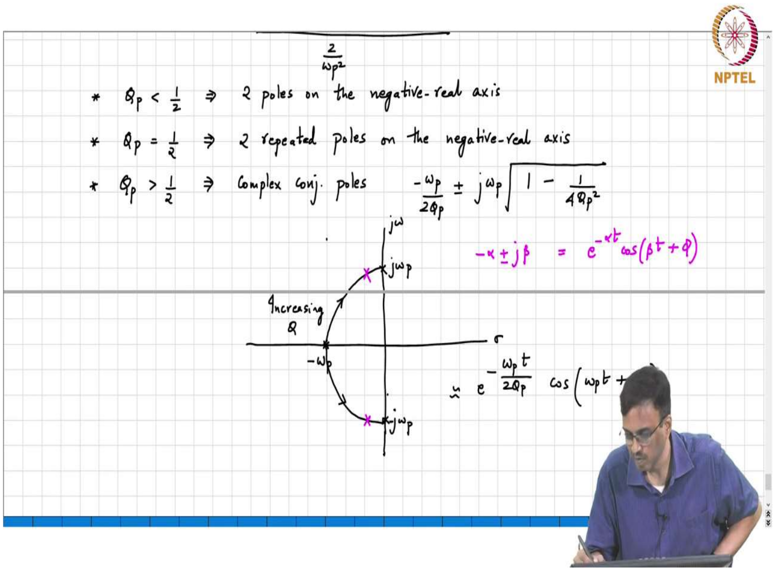

And finally, Q_P > ½ you will have a complex conjugate and where are the poles located? It is

$$\frac{-\omega_P}{2Q_P} \pm j\omega_P \sqrt{1 - \frac{1}{4Q_P^2}}$$

Now, let now I would like to draw your attention to the location of these poles right assuming they are complex. So, if I draw the complex plane this is σ and this is $j\omega$. So, for exactly $Q_P = ½$ where are the 2 poles?

Student: $-\omega$.

you will have 2 poles there. Now, as Q_P keeps increasing from beyond to values greater than half what comment can we make about the location of the poles? The complex conjugate of course, right as Q_P tends to infinity what comment can we make about the poles?

Student: $j\omega$.

They will be on the $j\omega$ axis where on the $j\omega$ axis $\pm j\omega_p$, correct. Ok for values $Q > ½$ as they as the Q tends to infinity if you plot the locus of the poles what will the locus do? Yeah of course, it breaks into 2 parts, correct ok Q_P equal to infinity we all know that it touches the $j\omega$ axis, but what about values less than Q_P?

It is complex that we know already alright I will give you a clue please look at the can you look at the location of the poles and find the magnitude of the complex number. The poles are complex quantities. What is the magnitude?

It is a circle. You can see that the magnitude of the poles is constant right, so it is not a parabola , it is not some weird shape, it is very clearly a circle. You understand. So, as the Q keeps increasing the poles basically move closer and closer to the j ω axis and as Q tends to infinity the poles will alright. Now, let us say we have some you know for some Q you have those 2 poles there. And again, as a quick refresher if you have a pole at if you have 2 poles at - α + - j β right what comment can you make about the impulse response of a system with poles at these frequencies?

So, if you have poles at $-\alpha \pm j\beta$ the impulse response will contain components of the form $e^{-\alpha t}$ $\cos(\beta t + \varphi)$, where that φ and you know the magnitude of this whole thing depends on you knowing other poles and all this other stuff. But this is the natural frequency that corresponds to a pole at, but corresponds to poles at $-\alpha \pm j\beta$ correct. So, in our case therefore, this in our example poles here will correspond to the - transient response which contains components $e^{-\omega\varphi t}/2Q_P$ cosine for large Q. What comment can you make about the imaginary part, what can be neglected? 1/4 Q_P is even, for example, if Q_P is φ you know 1/4 Q_P^2 is 100. So,$\sqrt{}$(1 -

1/100) is 0.995 right. So, basically it is pretty much ω_P, so this is approximately some $\omega_P t + \varphi$. Does it make sense? Alright. So, for large Q what comment can you make about this waveform I mean how this waveform looks for large Q?

For infinite Q sustained oscillation for you know for large Q it will take a large time to decay right and for very small Q it will decay fast right.

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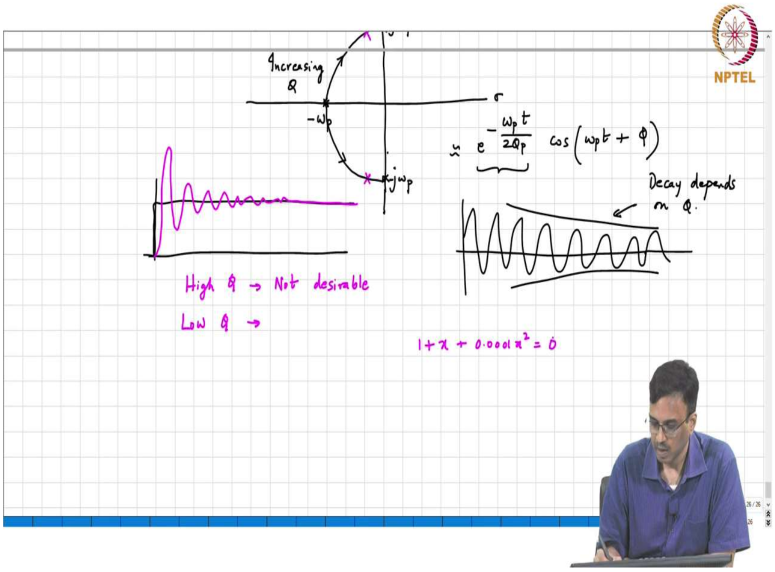

So, basically that the transient response will contain some kind of you know the impulse response therefore, will contain some kind of component like this where the decay of this envelope depends on Q. So, the larger the Q the longer it will take for the envelope to decrease right. So, therefore, if the impulse response contains something like this, what happens if you have a step to such a system with a large Q? Let us say this is a step. How is the step response loop? So, basically there will be a lot of overshoot and a lot of ringy right, it will take forever to die ok. Now, you know when you have an amplifier do you think this kind of step response is desirable?

Yeah, I mean when you mean such a response may be desirable in other situations definitely not for an amplifier ok. So, a ringing step response is definitely not desired in an amplifier and I mean and the classic example I can give you for this kind of ringy response is you know let us say you are traveling in a bus ok and the shock absorbers are messed up. And you are sitting in the last seat of the bus and the bus goes and hits a pothole and what do you do? What do you keep going up and down right?

And before you go before you know you manage to come to rest when the bus hits another pothole and therefore, you will keep jumping up and down like that right and. So, this goes on forever right. So, there is definitely not a case where the systems queue the quality factor of the of that of the natural frequencies of that mechanical system are very very very high ok. Whereas if you fix the shock absorbers what is it doing? It is basically doing the making sure the quality factor is low, so that you know the response does not have ringy. So, a high quality factor is not desired. Now, the question is ok well high Q is not desired, so maybe perhaps low Q is desired. So, what about low Q? If I make A much much smaller than half then what will happen?

So basically if Q is very small then you know for all practical purposes you can neglect this. I mean if Q is very small then you know this 1/Q is very large. So, for all practical purposes you can neglect that S^2 by ω p^2 and this way this looks like a single pole system with.

Student: 1.

So, let us say I have an equation 1 + x = 0 what are the roots? x = -1. Now, if I add 0.0001 x^2 how many roots are there?