

Analog Electronic Circuits
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Lecture - 66
Stability of Negative Feedback System the First Order Forward Amplifier

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$$\frac{1}{\left(1 + \frac{s}{p_1}\right)\left(1 + \frac{s}{p_2}\right)} \approx \frac{1}{1 + s\left(\frac{1}{p_1} + \frac{1}{p_2}\right)}$$

STABILITY OF NEGATIVE FEEDBACK AMPLIFIERS

$$\frac{V_o}{V_i} = \frac{1}{f} \cdot \frac{LG(s)}{1 + LG(s)}$$

$$LG(s) = A(s)f$$

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Alright, and you know before we start drawing circuits let us basically get back to our basic block diagram or the negative feedback loop we will do it at the block level. So, now we understand that this is not just a frequency independent gain this is a frequency dependent gain $A(s)$ and the closed loop gain V_o/V_i is nothing but,

$$\frac{V_o}{V_i} = \frac{1}{f} \frac{LG(s)}{1 + LG(s)}$$

This loop gain now is a function of s , and is simply nothing but $LG(s) = A(s)f$.

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STABILITY OF NEGATIVE FEEDBACK AMPLIFIERS

$$\frac{V_o}{V_i} = \frac{1}{f} \cdot \frac{LG(s)}{1 + LG(s)} = \frac{1}{f} \cdot \frac{1}{1 + \frac{1}{LG(s)}}$$

$$LG(s) = A(s)f$$

First-Order: $f A(s) = \frac{A_o f}{(1 + \frac{s}{\omega_o})} = LG(s)$ | Bandwidth of LG(s) = ω_o

$$\frac{V_o}{V_i} = \frac{1}{f} \cdot \frac{1}{1 + \frac{1}{\frac{A_o f}{1 + \frac{s}{\omega_o}}}} \approx \frac{1}{f} \cdot \frac{1}{1 + \frac{s}{A_o f \omega_o}} \quad \left. \begin{array}{l} A_o f \gg 1 \\ \text{Unconditionally} \\ \text{stable} \end{array} \right\}$$

So, let us start with the simplest possible system you know, let us say a forward amplifier which is first order. So, $A(s)$ is nothing but A_o . It is DC gain, and there will be a single pole. So, let us call that s/ω_o , in a common source amplifier that ω_o will correspond to $1/(R_s C_{gs} + 1 + C_{gd} A)$ etcetera, right, ok.

So, but at this point at the system level we just say it is good enough to model it by a single pole with bandwidth ω . Now, what comment can we make about the loop gain function? So, the loop gain function is simply this multiplied by f , correct. And it is actually easier to write this as $1/f (1/(1 + 1/LG(s)))$.

So, V_o/V_i therefore, is nothing but,

$$\frac{V_o}{V_i} = \frac{1}{f} \frac{1}{1 + \frac{1}{\frac{A_o f}{1 + \frac{s}{A_o f \omega_o}}}}$$

So, if $A_o f$ is somewhat large ok, then this is approximately equal to,

$$\frac{V_o}{V_i} = \frac{1}{f} \frac{1}{1 + \frac{s}{A_o f \omega_o}}$$

In reality I am missing a DC gain term which is basically you know $A_o f / (1 + A_o f)$ and there is going to be a small correction even in the in that frequency that correction will be of the form $A_o f / (1 + A_o f)$ all the I am going to I mean to first order they are all close to 1. So, first question. So, whenever you take a system with memory and put it in a feedback loop we

worry about the stability of the closed loop system, correct, ok. So, what comment can we make about the system, is it stable or not? Yeah, so, remember that we would have arranged the negative feedback loop. So, that $A_o f$ is you know is basically positive, right. So, clearly the pole is in the left half s plane, alright. So, this is stable and you know is it only stable for some particular values of $A_o f$ or is it stable for all values of $A_o f$ as long as the sign of $A_o f$ is positive?

So, it is basically therefore, it is called unconditionally stable in the sense that if you have DC negative feedback then regardless of the magnitude of the loop gain at DC you will the system will be stable, right, ok. So, what is the bandwidth I mean now let us get a little more. So, we are happy that this closed loop system is stable.

Now, let us take a look at the bandwidth of the loop gain function a 3 dB bandwidth of the loop gain function, what is this? Power gain, ok. So, what if the power gain becomes half what happens to the voltage gain? 1 by root, ok. So, what is the DC gain of the loop gain function? This is the loop gain function. What is the DC gain of the loop gain function?

Student: $A_o f$.

So, basically what at what frequency does this magnitude of the loop gain function fall to $\frac{1}{\sqrt{2}}$ its value at DC at?

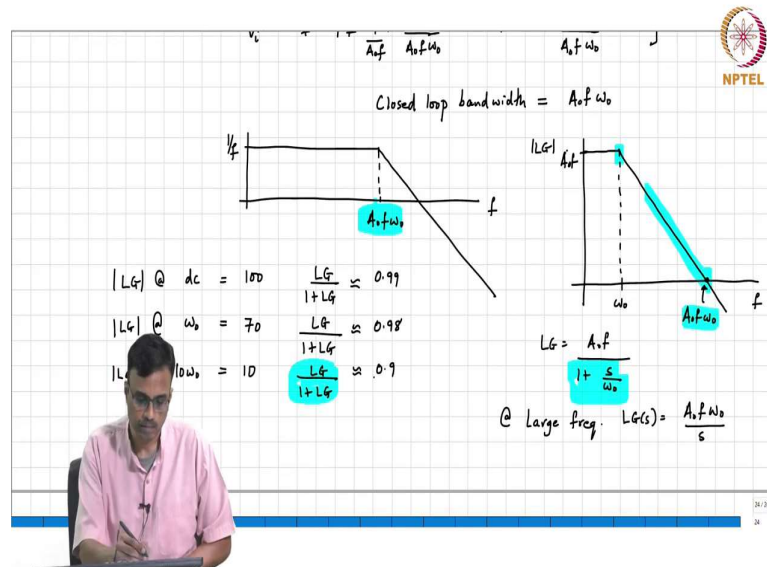
Student: ω_o .

So, the bandwidth of the loop gain function is ω_o , ok. But what comment can and roughly I mean colloquially right, when somebody says the bandwidth of this amplifier is I do not know 10 MHz. What do you understand by that statement? In other words you know, do you think if somebody gave you a bandwidth you know signal with I mean an amplifier with a bandwidth of 10 MHz and said you know go give me a lot of gain at 40 MHz, right.

What would you say? Yeah so, basically you know loosely speaking the bandwidth is a number that quantifies you know the frequency range over which you know the system is useful, correct. So, I mean if you basically say you know I have a 5 5 5 MHz bandwidth you know or your amplifier and somebody said you know I need to process 25 MHz signal then you say well, this amplifier is simply not suitable right, because the bandwidth is only 5 MHz, ok, alright.

So, that is you know that is what we understand by bandwidth. So, you know using that connotation this loop gain function only has a bandwidth at of ω_0 . So, what I mean this would tend to indicate that you know beyond ω_0 the loop gain function is falling off you know very rapidly at 20 dB per decade, right. So, you might think that you know you know beyond ω_0 this feedback loop is simply not working because the loop gain is falling off, alright.

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However, if you look at the closed loop bandwidth, what is the closed loop bandwidth? $A_0 f \omega_0$, alright, ok. So, what is this telling us? The closed loop bandwidth right, ok, is $A_0 f \omega_0$ which is telling us that I mean the what you call if you plot the closed loop magnitude response this is frequency you plot the bode plot this is this corresponds to $1/f$ and you have -20 dB/decade and this basically is $A_0 f \omega_0$, right. And what is the feedback loop trying to do essentially? What is the forward amplifier or what is the negative feedback loop trying to accomplish? It is trying to make sure that $V_o f$ is equal to what is the feedback loop trying to do? It is trying to make sure that a $V_o f = V_i$, alright. And if you and that will only happen if the loop gains.

If the loop gain is very large, right. So, but here we see that the loop gain is starting to fall off even at ω_0 , right. But you would think that the feedback loop has stopped becoming effective beyond ω_0 , right. However, as we see the closed loop bandwidth is $A_0 f \omega_0$ correct, which is telling us that no the feedback loop is still effective you know at frequencies much larger than ω_0 , right. And it only stops becoming effective around frequencies $A_0 f \omega_0$.

So, intuitively why does this make sense? Ok. So, basically notice that the closed loop gain right, is $LG/(1 + LG)$. And what is the loop gain at DC? What is the loop gain at DC? $A_o f$, right. So, what is $(1 + \text{loop gain})$? It is $(1 + A_o f)$. So, a loop gain/ $(1 + \text{loop gain})$ at DC is almost 1, right.

Now, let us say the loop gain at DC is just let me take a number. So, let us say the magnitude of the loop gain at DC is say 100, right. So, loop gain/ $(1 + \text{loop gain})$ is approximately 0.99 actually ok, close to 1. Now, let us say we look at the bandwidth of the loop bandwidth of the loop gain function is ω_o , correct. So, what comment can you make about the magnitude of the loop gain at ω_o ?

Student: 70.

Roughly 70, right, ok. It is actually a complex number; it is $70 \angle 45^\circ$, right. But you know that number is so much larger than 1 that what comment can you make about loop gain/ $(1 + \text{loop gain})$. You know, let us say something like 0.98 ok, alright. So, even though the loop gain has fallen from 100 to 70 which is a 30 percent change this number loop gain/ $(1 + \text{loop gain})$ is.

Student: 0.98.

Now, let us take the magnitude of the loop gain at $10 \omega_o$. What is the magnitude of the loop gain at $10 \omega_o$ roughly?

Student: 10.

Estimate is approximately 10, right. And what is the angle of the loop gain at $10 \omega_o$? Has it been roughly about you know you know - 90 degrees? But again we will get an estimate we will just hike it, right. loop gain/ $(1 + \text{loop gain})$ is approximately $10/11$ which is about 0.9, correct.

So, you can see that even though the magnitude of the loop gain has fallen from 100 to 10 this number loop gain/ $(1 + \text{loop gain})$ has only fallen by 10 percent, right, ok. So, when will this party stop? This is basically saying that you know I mean you can keep reducing loop gain and we do not seem to see much effect in the magnitude of the closed loop gain.

So, when will you know when this will start falling? I mean in other words when will this stop becoming close to 1? Yeah, so, basically when the magnitude of the loop gain starts

becoming comparable to 1 is when this quantity $\text{loop gain}/(1 + \text{loop gain})$ will no longer be 1, correct.

So, therefore, so what is therefore, the you know so what is therefore, the resolution of that surprising or seemingly surprising fact that the we the loop gain has started falling off at ω_o already. But and therefore, we originally thought that the feedback is breaking even at ω_o , right.

And therefore, the loop apparently seems to be I mean should not work, but it seems like, but at the loop is actually working the feedback is working quite nicely even up to frequencies as large as $A_o f \omega_o$.

So, what is the so, now, since we have seen this argument, what is why is this happening? So, it is true that the magnitude of the loop gain is falling dramatically beyond the loop gains bandwidth which is ω_o . However, the magnitude of the loop gain we started off with was way larger than 1 right, and the closed loop gain depends on this quantity $\text{loop gain}/(1 + \text{loop gain})$. So, here as long as the magnitude of the loop gain is sufficiently large compared to 1 the what do you call the feedback loop is still effective, ok, alright. So, basically and that frequency and at what and at what frequency does the magnitude of the loop gain fall to fall to 1?

So, let us say this is f this is the magnitude of the loop gain function in a log scale this is $A_o f$ at DC and where is 3 dB bandwidth of the loop gain? ω_o beyond ω_o it falls at 20 dB per decade, right. So, at what frequency does the magnitude of the loop gain become 1? Quick analysis, ok.

Before you keep sitting and wasting your time writing long equations, let me ask you one thing. At frequencies far away from the I mean first of all is this unity gain frequency going to be close to ω_o far away from ω_o ? Far away, at frequencies far away from ω_o which of these terms dominates?

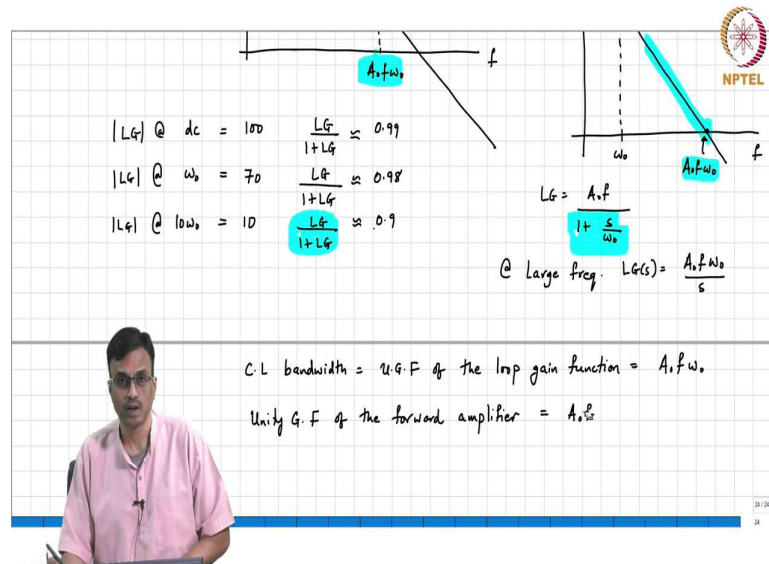
Student: s/ω_o .

So, at large frequency $LG(s)$ is nothing but $A_o f \omega_o/s$, correct, ok. So, now, take a look at and tell me what frequency is the unity gain frequency of the loop gain?

Student: $A_o f \omega_o$.

So, the unity gain so, basically the unity gain frequency of the loop gain function also happens to be the closed loop bandwidth of the system, right.

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Ok. So, is the unity gain frequency of the loop gain function which is the same as $A_0 F \omega$, ok. Next observation that I would like to make. What is the unity gain frequency of the forward amplifier?

Student: $A_0 f$.

$A_0 f$ where is f coming from the forward amplifier? Which is the forward amplifier? Which is the forward amplifier? What is the transfer function? So, what is the unity gain frequency of the forward amplifier?

Student: $A_0 \omega$.

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$|L(s)| @ \omega_0 = 70 \quad \frac{L(s)}{1+L(s)} \approx 0.98$
 $|L(s)| @ 10\omega_0 = 10 \quad \frac{L(s)}{1+L(s)} \approx 0.9$
 $L(s) = \frac{A_0 f}{1 + \frac{s}{\omega_0}}$
 @ Large freq. $L(s) = \frac{A_0 f \omega_0}{s}$

C.L bandwidth = U.G.F of the loop gain function = $A_0 f \omega_0$
 Unity G.F of the forward amplifier = $A_0 \omega_0$
 C.L gain @ dc = $\frac{1}{f}$

$(C.L BW) \times (C.L gain) = A_0 \omega_0 = UGB \text{ of the forward amplifier}$

$A_0 \omega_0$, alright. So, what is the closed loop gain at DC approximately? What is the closed loop DC gain?

Closed loop DC gain $1/f$, alright. Now, what does this mean? Stare at those two and tell me. So, the product of the closed loop bandwidth and the closed loop gain is $A_0 \omega_0$. Do you follow, ok which is nothing but the UGB of the forward amplifier alright. So, basically if you want to get a higher closed loop gain the closed loop bandwidth that you get is going to be small and the product of the two happens to be equal to the unity gain frequency of the forward amplifier.

This has led many people to believe that the gain bandwidth product is a constant right. That is only true. I mean that is what happens if you basically do not if you simply mug up a formula without thinking about it right.

This is only true under these conditions that with the formula was derived under which the formula would derive correctly that is forward amplifier is you know first order system a feedback block is basically memory less. So, that the loop gain function is the first order system in which case then it is true that if you attempt to increase the closed loop gain you will basically reduce the closed loop it will result in a smaller closed loop bandwidth. Does it make sense to people? Alright.