


**Analog Electronic Circuits**  
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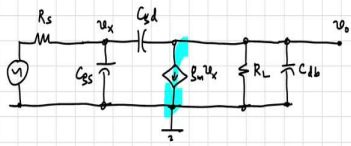
**Lecture - 63**  
**The Common-Source Amplifier with Parasitic Capacitances**


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Lecture 30

$$\frac{v_o}{v_s} = \frac{-g_m R_L \left(1 - s \frac{C_{gd}}{g_m}\right)}{1 + s \left\{ R_s \left[ C_{gs} + C_{gd} (1 + g_m R_s) \right] + R_L (C_{gd} + C_{db}) \right\} + s^2 (C_{gs} C_{gd} + C_{gd} C_{db} + C_{db} C_{gs}) R_s R_L}$$





So, in the last class we were looking at the frequency response of the common source amplifier. We did a lot of algebra and finally, came up with this messy looking expression and we are trying to make sense of the various terms. This of course, makes sense that simply the response of the common source amplifier to at DC and that says you very well know it is  $-g_m R_L$ .

We also recognize that even though there are three capacitors, the denominator polynomial is only second order and that is because there is a capacitor. Finally, we are also happy that the denominator all the coefficients of the denominator have the same sign which means that the system is stable right and you know; that means, all this effort you know so far was worthwhile, because if this is not stable then we might as well pack our bags and go home correct alright.

So, the next thing is that we looked at the numerator and we noticed that strangely there seems to be a 0 and the 0 happens to be in the right half plane. And, yesterday we saw the intuition why 0 must occur and that is because you can see that there are two paths from the

input from that node  $v_x$  to the output. And, whenever there are two paths, multiple paths from input to output you always have some complex frequency at which the outputs of the two paths interfere destructively and therefore, cause a null in the transfer function right.

And, here you can see that the path gains are you know one is through  $C_{gd}$  which attempts to pull the output voltage up whereas, the trans conductance  $g_m$  attempts to pull the voltage down right. So, we see that there is a 0 and because the signs of the two paths are opposite, it is now intuitively clear why you must have a right half plane 0 alright ok. So, now it's or it's the turn to look at the denominator right.

So, at DC of course, the denominator evaluates to 1, as you know slightly higher frequencies right. So, basically not DC, but not you know very high; which of these two terms in the denominator do you think will dominate?

Which term will dominate? So, at low frequencies therefore, the denominator will largely be determined by the constant and the  $s$  term right, but the  $s^2$  term will only kick in at much higher frequencies right. So, let us now try and see if we can make any sense of why this term is there in the first place and you know why it is of the form it is. So, so, at low frequency therefore, you know what low frequency means, what is the meaning of low frequency?

Like any like anything else right there is you know there is low basically make you know makes no sense unless its compared I mean something is compared to something right. So, what is compared to what now? Ok, I mean ok can we comment about the frequency dependent currents and the frequency independent current? So, we have various elements in this network, we have capacitors, resistors and so on right.

At DC all the current is flowing in the elements in the branches which are whose behaviour is frequency independent right. As you keep increasing frequency what happens? So, basically it will be as frequency keeps increasing, these capacitors which are open at DC start to draw current right. So, basically low frequency another way of thinking about low frequency is basically the frequencies at which the capacitor currents are very small compared to the you know the currents in the frequency independent branches right, in this case resistors and the control source GMPs. Is this clear? Alright. So, if we can at low frequencies if we can neglect

the first order, if we can neglect the capacitor currents, what volt I mean is approximately what is  $v_x$ ?

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NPTEL

$$\frac{v_o}{v_s} = \frac{-g_m R_L \left(1 - \frac{s C_{gd}}{g_m}\right)}{1 + s \left\{ R_s \left[ C_{gs} + C_{gd} (1 + g_m R_L) \right] + R_L (C_{gd} + C_{db}) \right\} + s^2 (C_{gs} C_{gd} + C_{gd} C_{db} + C_{db} C_{gs}) R_s R_L}$$

Handwritten notes in pink:

- Current through  $C_{gs}$ :  $s C_{gs} v_i$
- Current through  $C_{gd}$ :  $s C_{gd} (1 + g_m R_L) v_i$
- Output voltage:  $v_o \approx -g_m R_L v_i$

$v_x$  is approximately  $v_i$  ok. And, what is  $v_o$ ? It is approximately  $-g_m R_L v_i$  ok. So, what comment can you make about the current through the capacitance? The voltage across the capacitor is  $v_i$ . So, what is the current through the capacitance?

Student:  $C_{gs}$ .

Yeah. I mean in the Laplace domain it is simply nothing but.

Student:  $s C_{gs}$ .

Alright. So, and what is the current through  $C_{gd}$ ?

Student:  $s C_{gs}$ .

What is the voltage across  $C_{gd}$ ?

Student:  $(1 + g_m R_L) v_i$ .

So, what comment can you make about the current through  $C_{gd}$ ?

Student:  $s C_{gd}$ .

It is?

Student:  $(1 + g_m R_L)v_i$ .

Alright. So, if you look at the impedance at DC of course, the impedance looking here is infinitely right, but as frequency changes I mean current is flowing. So, there is basically an impedance frequency dependent you know voltage to current relationship. And so, what is the approach I mean; so, at low frequencies therefore, what comment can we make about the impedance looking in? We have applied a voltage which is  $v_i$ . The current flowing is.

Student:  $-s C_{gs} v_i + s C_{gd} (1 + g_m R_L)v_i$ .

So, this basically is the current. So, what is the impedance looking in?

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The slide shows a handwritten transfer function and a corresponding circuit diagram. The transfer function is:

$$\frac{v_o}{v_s} = \frac{-g_m R_L \left(1 - \frac{s C_{gd}}{g_m}\right)}{1 + s \left\{ R_s \left[ C_{gs} + C_{gd} (1 + g_m R_L) \right] + R_L (C_{gd} + C_{db}) \right\} + s^2 (C_{gs} C_{gd} + C_{gd} C_{db} + C_{db} C_{gs}) R_s R_L}$$

The circuit diagram below it shows an input voltage source  $v_i$  connected to a resistor  $R_s$ . Following  $R_s$ , there is a node with voltage  $v_x$ . A capacitor  $C_{gs}$  is connected between this node and ground. A dependent current source  $g_m v_x$  is connected between the node and ground. A capacitor  $C_{gd}$  is connected between the node and the output node. The output node is connected to a load resistor  $R_L$  and a capacitor  $C_{db}$  to ground. The output voltage is  $v_o$ . Handwritten annotations in pink include  $v_i$ ,  $v_x$ ,  $v_o$ , and the transfer function itself.

So, the impedance looking in therefore, is  $1/C_{gs} C_{gd} (1 + g_m R_L s)$ . So, what is this, what is this, what does this element look like? Are they in parallel or in a series manner? Ok.  $C_1 + C_2$ . Ok. So, you basically say so, it looks like a capacitance of  $C_{gs}/(1 + C_g)$  I mean  $C_{gs} + C_{gd} (1 + g_m R_L)$ .

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$$\frac{V_o}{V_s} = \frac{-g_m R_L (1 - s C_{gd})}{1 + s \left\{ R_s \left[ C_{gs} + C_{gd}(1 + g_m R_L) \right] + R_L (C_{gd} + C_L) \right\} + s^2 (C_{gs} C_{gd} + C_{gd} C_L + C_L C_{gs}) R_s R_L}$$

$$\frac{V_x(s)}{V_i(s)} = \frac{1}{1 + s R_s \left\{ C_{gs} + C_{gd}(1 + g_m R_L) \right\}}$$

NPTEL

So, what comment can you make about the transfer function from  $v_i$  ok. We know that this is only approximate alright. So, what is it, what is the, what will be a better approximation to that voltage there? Looking in impedance here, basically this is  $v_i$ , this is  $R_s$ , looking in here is a capacitance of value  $C_{gs} + C_{gd}(1 + g_m R_L)$ . So, what comment can you make about the voltage here now? Yeah. This voltage, this voltage that  $v_i$  is you know, was a 0th order approximation. To, a better approximation to the voltage there is, which is valid at low frequency must therefore, be  $V_x(s)$  must be nothing, but or  $V_x/V_i$  is nothing but  $(1/(1 + s R_s)) (C_{gs} + C_{gd})(1 + g_m R_L)$ .

Now, that is the  $V_x$ . So, now we are interested in finding I mean of course, at DC the output voltage is  $-g_m R_L v_i$ , but we are interested in finding the voltage at I mean the transfer function there at low frequencies correct. Ok. Not at DC, but slightly removed from DC ok and that is simply the transfer function. I mean we know the transfer function from  $V_i$  to  $V_x$  is this. We now, therefore, need to find the transfer function between  $V_x$  and  $V_o$  right. So, if we know the transfer function from  $V_x$  to  $V_o$ , then you multiply the two transfer functions and therefore, you will be able to get  $V_i$  to  $V$ . Is that clear?

If you have a network, if you know this voltage, you can remove the entire portion to the left and replace it by a voltage source  $V_x$  correctly and it is a voltage source. So, the fact that you are drawing current from that voltage source, does not affect the transfer function from  $V_x$  to

$V_o$  right and we have already accounted for the current flowing through  $C_{gd}$ ; that is already coming in here. Is this clear?

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So, basically all that we are doing is we are replacing if you want to look at it that way; let us assume we have  $V_x$  here and you have  $C_{gd}$ , this is  $g_m v_x$ , this is  $R_L$  and this is  $C_{db}$ . This is the circuit we have analyzed correctly. We know  $V_x$  already. How do we know  $V_x$ ? Because we have accounted for the loading of the rest of the circuit following that  $V_x$  node. Correct.

So, this is what we are trying to find.  $V_o/V_x$  alright. So, again what you call the first order approximation is you have  $g_m v_x$  being you know what do you call pulling current out like this ok. And so, what comment can we make about the time constant across  $R_L$  now, which is larger? This  $V_x$  or  $-g_m R_L V_x$ ?

$-g_m R_L V_x$  is a much larger voltage than  $V_x$ . So, for all practical purposes therefore, what comment can you make about this voltage? It is very small compared to this voltage. This is swinging a lot. This is all swinging very little right. So, what comment can we make about the time constant across I mean across the capacitors across the capacitor  $C_{db}$ ? Or, in other words, what will I mean? First of all, if you find the transfer function from  $V_x$  to  $V_o$ , what do you expect to see? What order do we expect to see?

First order, why? If you want to find the transfer function, what is the order of this? Yesterday, we discussed that the order of the system is independent of where the input is

connected right. So, if we are so, the denominator polynomial of this transfer function should be independent of  $V_x$  right. So, if you short  $V_x$ , you will. I mean you should get the, it does not matter where you put  $V_x$ .

So, what comment can you make about the denominator polynomial of this of course, you can evaluate it, but what I am asking is what comment can we make about the denominator polynomial? What order will it be? So, basically if you set  $V_x$  to 0, this becomes a short circuit  $g_m V_x$  becomes open right.

So, basically you have and the time constant that the circuit is associated with therefore, is  $(R_L (C_{db} + C_{gd}))$  right. So, therefore,  $V_o/V_x$  must therefore, be equal to the denominator polynomial must be  $1 + s R_L/(C_{db} + C_{gd})$ , correct. And, I mean do we expect to see a numerator or do we expect to see a numerator polynomial or not?

Yes, why? So, from  $V_x$  to  $V_o$  there is a path through  $C_{gd}$  and there from  $V_x$  to  $V_o$  there is another path through  $g_m$  right. So, basically you expect to see a numerator polynomial. So, what will this be? What will the DC gain be  $-g_m R_L$  and you will have a right half plane  $1 - s g_m/C_{gd}$  correct. So, basically this is telling you that if this was driven by a perfect voltage source  $V_x$ .

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$$\frac{V_o(s)}{V_i(s)} \cong \frac{1}{1 + s R_L \{ C_{db} + C_{gd}(1 + g_m R_L) \}} \cong \frac{1}{1 + \frac{s}{P_1}}$$

$$\frac{V_o}{V_x} = \frac{-g_m R_L \left(1 - \frac{s C_{gd}}{g_m}\right)}{1 + s R_L (C_{db} + C_{gd})}$$

$$\frac{V_o}{V_i} \cong \frac{-g_m R_L \left(1 - \frac{s C_{gd}}{g_m}\right)}{\left(1 + \frac{s}{P_1}\right) \left(1 + \frac{s}{P_2}\right)} \times \frac{-g_m R_L \left(1 - \frac{s C_{gd}}{g_m}\right)}{1 + s \left(\frac{1}{P_1} + \frac{1}{P_2}\right)}$$

time constant @  $V_x$       time constant @  $Q$

Sorry thank you,  $s C_{gd}/g_m$  yeah. So, this is just telling you that if this was the voltage source  $V_x$ , the transfer function from  $V_x$  to  $V_o$  is simply this character and is this is this approximate



or is this exact? This is exactly right. There is no approximation here at all. The only approximation is coming because input to  $V_x$  is approximated because you know we said that you know at low frequency well you know the output is approximately  $-g_m R_L$  and so on.

So, at low frequency therefore,  $V_o$  by  $V_i$  is approximately what?  $-g_m R_L (1 - s C_{gd}/g_m)/(1 + s/P_1) (1 + s/P_2)$  alright. So, you can see therefore, that ok which is approximately for small  $s$  this is what  $-g_m R_L (1 - s C_{gd}/g_m)/(1 + s(1/P_1 + 1/P_2))$

Now, what is  $1/P_1$ ? This is the time constant associated with at node  $V_x$  and this is the time constant associated with at  $V$  alright. So, basically I mean and of course, there is 0 alright. So, this is a good approximation to the frequency response at low frequency.

So, what is the so, therefore, we see clearly that the we of course, get the DC gain and the 0 and at low frequency the denominator polynomial must approximately look like this. So, what is this  $1$  by  $P_1$ ? What is the time constant at node  $V_x$ ? Well, that corresponds to charging the effective capacitance here right through a resistor  $R_s$ . So, what is the time constant there?

Student:  $R_s$ .

What is the effective capacitance at that node  $V_x$ ? It is to think of it as just  $C_{gs} + C_{gd}$ , but it is not, it is more than  $C_{gd}$ . Why?

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The slide shows a circuit diagram of a common-emitter amplifier. The input signal  $V_i$  is applied through a resistor  $R_s$  to the base of a BJT. The base-emitter junction is modeled with a capacitance  $C_{gs}$ . The collector-emitter junction is modeled with a capacitance  $C_{gd}$ . The collector is connected to a load resistor  $R_L$  and a collector-base junction with capacitance  $C_{cb}$ . The output voltage is  $V_o$ . The current gain is  $\beta_m$  and the transconductance is  $g_m$ .

The transfer function is given as:

$$\frac{V_o}{V_i} = \frac{-g_m R_L}{1 + s R_s \{C_{gs} + C_{gd}(1 + g_m R_L)\} + s^2 (C_{gs} C_{gd} + C_{gd} C_{cb} + C_{cb} C_{gs}) R_s R_L}$$

The low-frequency approximation is shown as:

$$\frac{V_o}{V_i} \approx \frac{-g_m R_L (1 - \frac{s C_{gd}}{g_m})}{1 + \frac{s}{P_1}}$$



I mean so, if you have a capacitor C and you apply a voltage v correctly, the looking in capacitance is obviously, the current flowing is  $s C_v$ . But, if this is moving in the opposite direction as you know if this voltage is  $-A_v$  correct. So, this voltage is only moving by v, but the capacitor somehow is going down the net voltage across the capacitance even though you have applied only  $v; s(1 + A_v)$  alright.

So, this capacitance C when looking in here appears as if it has been multiplied by a factor  $(1 + A)$  right and this is often what is called the Miller effect or this is called Miller multiplication alright. So, this is an example of the evidence of Miller multiplication alright.

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The slide contains the following equations and notes:

$$\frac{v_o}{v_s} = \frac{g_m}{1 + s \left\{ R_s \left[ C_{gs} + C_{gd}(1 + g_m R_L) \right] + R_L (C_{gd} + C_{db}) \right\} + s^2 (C_{gs} C_{gd} + C_{gd} C_{db} + C_{db} C_{gs}) R_s R_L}$$

Miller multiplied

$$\frac{V_x(s)}{V_i(s)} \cong \frac{1}{1 + s R_s \left\{ C_{gs} + C_{gd}(1 + g_m R_L) \right\}} \cong \frac{1}{1 + \frac{s}{p_1}}$$

$$\frac{V_o}{V_x} = \frac{-g_m R_L (1 - \frac{s C_{gd}}{g_m})}{1 + s R_L (C_{db} + C_{gd})}$$

So,  $C_{gd}$  is Miller multiplied right. So, the effective capacitance that you see looking in is not  $C_{gs} + C_{gd}$ , but  $C_{gs} + 1 + g_m R_L C_{gd}$ . And therefore, the time constant associated with that first node is nothing but  $R_s$  the effective capacitance which is  $C_{gs} + C_{gd}$ .

Student:  $1 + g_m R_L R_s$ ,

That is the time constant at that node. What is the time constant at the second node? It is nothing but what is it?

Student:  $R_L C_{db} + C_{gd}$ .

So, that is the time constant at the second alright. So, basically you can see that the first two terms make sense right. There is the intuition behind why these terms look the way they are.

So, the last term of course, you know have no great intuition to offer right. It basically you know I mean this is what now I would call think like how you do in gate right.

This coefficient must have dimensions of square right. So, it must be you know multiply two resistors and two capacitors right. Your coaching class forgot to tell you which two capacitors. So, you could be you do not be partial to anyone, you just basically take all the three capacitors and then multiply them up two at a time right.

And, then you see only two resistors in the picture. So, this  $R_s$  and  $R$  ok alright. So, what do you call; so, that is the. So, now, when you see a picture like this, I mean you know you should be able to simply write out the transfer function right without having to sit and rewrite all the KCL, KVL equations.

So, the key point therefore, is that you know if you drive the common source amplifier with the resistance you know  $R_s$ , then the if you want to approximate the common source response by a first order system, then to a very first order you can basically see that the  $C_{gd}/g_m$  right. What comment can you make you know when you compare it to the denominator pole, the numerator which will be much larger?

This time constant in the denominator or this time constant? You know that  $g_m R_L$  is much larger than 1. So, what comment can you make about the time constant here versus the time constant there, which will be larger? I mean  $C_{gd}/g_m$  or this monster expression in the denominator?

The denominator is expected to be much larger, correct and as a consequence therefore, you know if you want to approximate you know the frequency response of a common source amplifier to first order.

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The slide displays a circuit diagram of a common source amplifier. The input is  $V_i$  and the output is  $V_o$ . The circuit includes a gate-source capacitor  $C_{gs}$ , a gate-drain capacitor  $C_{gd}$ , a load resistor  $R_L$ , and a drain-bulk capacitor  $C_{db}$ . The transfer function is derived as follows:

$$\frac{V_o}{V_i} = \frac{-g_m R_L \left(1 - \frac{s C_{gd}}{s_m}\right)}{\left(1 + \frac{s}{p_1}\right) \left(1 + \frac{s}{p_2}\right)} \approx \frac{-g_m R_L \left(1 - \frac{s C_{gd}}{s_m}\right)}{1 + s \left(\frac{1}{p_1} + \frac{1}{p_2}\right)}$$

Handwritten annotations indicate that  $\frac{1}{p_1}$  is the time constant at  $V_i$  and  $\frac{1}{p_2}$  is the time constant at  $V_o$ . The final expression for the bandwidth  $\omega_o$  is:

$$\omega_o = \frac{1}{R_S \{C_{gs} + (1 + g_m R_L) C_{gd}\} + R_L (C_{gd} + C_{db})}$$

You can safely say that this is  $-g_m R_L / (1 + \omega_o)$ , where  $\omega_o$  is  $1/R_S (C_{gs} + (1 + g_m R_L) C_{gd} + R_L (C_{gd} + C_{db}))$ , alright. In many cases you know the load that you will see is not purely a resistor, you also let us say you are trying to load another amplifier, then its input capacitance also becomes a part of  $C_{db}$  correct. So, basically you will see that you know right where the bandwidth is dependent on you know  $R_S R_L$  and basically this is the time constant sum of the effective bandwidth is basically the sum of the reciprocal of the sum of the time constant.

So, basically the bandwidth is simply inversely proportional to you find all the time constants everywhere you add them all up right to first order, it's the bandwidth of the system is simply  $1/g_m / (\tau_1 + \tau_2)$ , where  $\tau_1$  and  $\tau_2$  are the time constants. Ok, alright.

So, this covers the common source amplifier, but we will be coming back to the full-blown expression later when we cover two stage op-amp. So, it is important to be aware that we are not going to sit and rewrite and rederive the expression all over again. We are now in a position to simply write out the expression because most of these terms are, you know, pretty self-explanatory or we have some way of remembering alright. Ok. So, that was the common source amplifier.