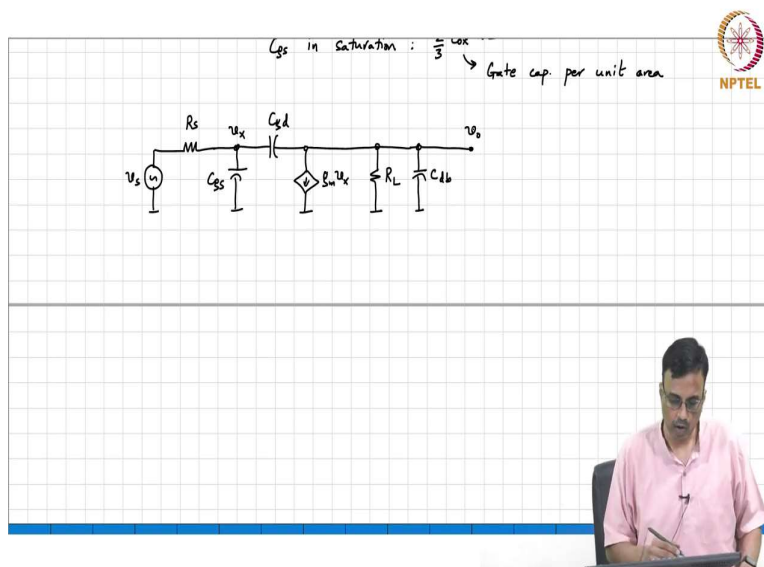


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**Lecture - 62**  
**The Common-Source Amplifier with Parasitic Capacitances**

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And the first I mean the biasing all remains the same, right. So, it is only may, it only makes sense to look at the incremental models of these amplifiers and see what effect the capacitances have on the performance of the amplifier. So, the first and simplest one is of course, the common source amplifier. I am going to draw only the equal, small signal equivalent.

So, this is  $C_{gs}$ , this is  $C_{gd}$ . Let us call that  $v_x$ , this is  $g_m v_x$ , this is  $r_o$ , this is  $C_{db}$ . And this  $r_o$  and  $R_L$ , I am going to club together into one resistance load resistance  $R_L$ , right. And this is  $v_o$ , ok. Now, how do we, so how do we write? So, we can think of this, I am going to replace this as if it is not an equivalent.

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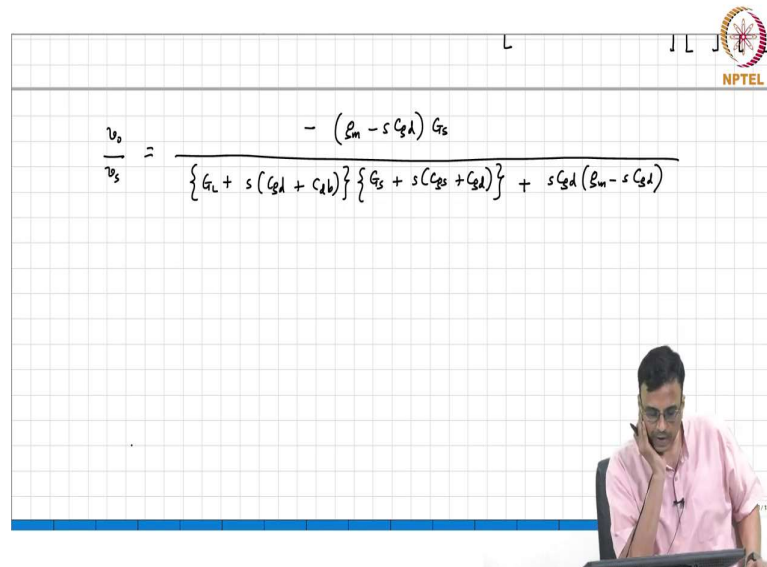
So, this is  $v_s/G_s$ , right.  $G_s$  is  $1/R_s$ . I am going to write everything in terms of conductances. So, this is  $G_s$ , this is  $G_L$ , right and I will be using  $G_s$  and  $G_L$  interchangeably depending on what is more convenient to, right. So, there should be no confusion, ok.

So, how do we find  $v_o$ ? Well, we write the nodal equations and solve them. Well, solve those KCL equations. Well, it is easiest done in matrix form. So, what are the unknowns that we are trying to find? We have two unknown  $v_x$  and  $v_o$ , and this is  $G_s v_s$  and 0, alright.

So, what are the terms in the matrix?  $G_s + s C_{gs} + C_{gd}$ , alright. This is  $-s C_{gd} - s C_{gd}$ ,  $G_L + C_{gd} + C_{gd} + C_{db}$ , alright. Then, where does  $g_m$  show up? Which row will it show up and which column?

First column, alright. So, what is  $v_o$  therefore? This is nothing but you find the determinant of the matrix, ok. So, this is basically  $G_L + s C_{gd} + C_{db} G_s + s C_{gs} + C_{gd} - s C_{gd} g_m - s C_{gd} +$ , right, yeah, ok. And what is the numerator?


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$$\frac{v_o}{v_s} = \frac{-(g_m - sC_{gd})G_s}{\{G_L + s(C_{gd} + C_{db})\} \{G_s + s(C_{gs} + C_{gd})\} + sC_{gd}(g_m - sC_{gd})}$$

So,  $v_o/v_s$  therefore, seems like a terribly boring exercise, but I am sorry I cannot make it any more interesting. I will, we will solve the math and then we will take a look at it and figure out why it makes intuitive sense. But before that we have to actually do algebra. So, please help me out here. So, the denominator, what can we do?

First thing, see it is always easier to write it as DC gain multiplied by some; what is the DC gain? Sanity check. As  $s$  equal to 0, what should we get? I mean look at the stare at the circuit. What should we get it as say at DC?  $-g_m R_L$ . So, what we would like to do is remove  $g_m$  out of here, and divide the denominator also by  $g_m$ , alright.


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$$\frac{v_o}{v_s} = \frac{-\frac{g_m}{g_m} (1 - s C_{gd}) G_s}{\left\{ \frac{G_L + s(C_{gd} + C_{db})}{g_m} \right\} \left\{ G_s + s(C_{gs} + C_{gd}) \right\} + \frac{s C_{gd} (1 - s C_{gd})}{g_m}}$$


And similarly why should we do that?


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$$\frac{v_o}{v_s} = \frac{-\frac{g_m}{g_m} (1 - s C_{gd})}{\left\{ 1 + \frac{s(C_{gd} + C_{db})}{G_L} \right\} \left\{ 1 + \frac{s(C_{gs} + C_{gd})}{G_s} \right\} + \frac{s C_{gd} (1 - s C_{gd})}{G_s G_L}}$$


I just moved the  $g_m$  outside here. And I will move  $G_L$  out of the whole  $G_s G_L$ , I will divide the denominator by  $G_s G_L$ . So, this becomes  $1/G_L$ , this becomes  $1/G_s$ , this becomes  $G_s G_L$ , ok. And therefore, the numerator I should get is  $1/G_L$ .

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$$\frac{v_o}{v_s} = \frac{-\beta_m R_L (1 - s C_{gd})}{\beta_m} \cdot \frac{\beta_m}{\beta_m} = \frac{-\beta_m R_L (1 - s C_{gd})}{\beta_m \left\{ 1 + s \frac{(C_{gd} + C_{db})}{G_L} \right\} \left\{ 1 + s \frac{(C_{gs} + C_{gd})}{G_s} \right\} + \frac{s C_{gd} (1 - s C_{gd})}{G_s G_L}}$$


$$= \frac{-\beta_m R_L (1 - s C_{gd})}{\beta_m \left\{ 1 + s \left[ \frac{C_{gd} + C_{db}}{G_L} + \frac{C_{gs} + C_{gd}}{G_s} \right] + \frac{s^2 (C_{gs} + C_{gd})(C_{gd} + C_{db})}{G_s G_L} + \frac{s C_{gd}}{G_s G_L} \right\}}$$


So, this gives us nothing but  $-\beta_m R_L$  a numerator term which reduces to 1 when  $s = 0$  and a denominator term which also reduces to 1 when  $s = 0$ . So, now let us simplify this stuff. So, this becomes  $1 + s \left( \frac{C_{gd} + C_{db}}{G_L} + \frac{C_{gs} + C_{gd}}{G_s} \right) + s^2 \frac{(C_{gs} + C_{gd})(C_{gd} + C_{db})}{G_s G_L} + \frac{s C_{gd}}{G_s G_L}$ . This is  $\beta_m - s C_d$  is it, but we removed the  $\beta_m$  common, is not it.

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$$\frac{v_o}{v_s} = \frac{-\beta_m R_L (1 - s C_{gd})}{\beta_m} \cdot \frac{\beta_m}{\beta_m} = \frac{-\beta_m R_L (1 - s C_{gd})}{\beta_m \left\{ 1 + s \frac{(C_{gd} + C_{db})}{G_L} \right\} \left\{ 1 + s \frac{(C_{gs} + C_{gd})}{G_s} \right\} + \frac{s C_{gd} (\beta_m - s C_{gd})}{G_s G_L}}$$

$$= \frac{-\beta_m R_L (1 - s C_{gd})}{\beta_m \left\{ 1 + s \left[ \frac{C_{gd} + C_{db}}{G_L} + \frac{C_{gs} + C_{gd}}{G_s} \right] + \frac{s^2 (C_{gs} + C_{gd})(C_{gd} + C_{db})}{G_s G_L} - \frac{s^2 C_{gd}^2}{G_s G_L} + \frac{s C_{gd} \beta_m}{G_s G_L} \right\}}$$


So, this is  $C_{gd} \beta_m / G_s G_L$ . I hope this is ok. Dimensionally it is consistent, ok. And then you have to subtract  $-s^2 C_{gd}^2 / G_s G_L$ . And God help us if you made a mistake, ok, alright.

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$$\frac{v_o}{v_s} = \frac{-g_m R_L (1 - s C_{gd})}{g_m} \div \left[ 1 + s \left\{ \frac{C_{gd} + C_{db}}{g_L} + \frac{C_{gs} + C_{gd}}{g_s} \right\} + \frac{s^2 (C_{gs} + C_{gd})(C_{gd} + C_{db})}{g_s g_L} - \frac{s^2 C_{gd}^2}{g_s g_L} \right]$$

$$C_{gs} R_s + C_{gd} (R_s + R_L + g_m R_L R_s) + C_{db} R_L$$

So, this is what I am going to simplify this further. This is  $C_{gd} + C_{gd}$ , let us say there is a  $C_{gs} R_s + C_{gd} R_s + R_L + g_m R_L R_s$ , ok. So, that basically takes care of these 3 terms,  $+ C_{db} R_L$ , ok. So, rather than do it this way, we can group terms with respect to  $R_s$ .

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$$\frac{v_o}{v_s} = \frac{-g_m R_L (1 - s C_{gd})}{g_m} \div \left[ 1 + s \left\{ R_s (C_{gs} + C_{gd} (1 + g_m R_s)) + R_L (C_{gd} + C_{db}) \right\} + \frac{s^2 (C_{gs} C_{gd} + C_{gd} C_{db} + C_{db} C_{gs}) R_s R_L}{g_s g_L} \right]$$

Then, what about this term? Well, the denominator is common, alright. So, if you multiply this out you basically see that the  $C_{gd}^2$  term gets cancelled. So, what you get is simply  $+ C_{gd}$ ,  $C_{db} + C + C_{db}$ .

Does it make sense folks? Alright. So, this is the transfer function from the input to the output. So, let us see if we can make sense of these terms, right? I am going to draw the picture again.

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Let us go back to our original circuit which is  $R_s$  here,  $R_L$  here and this is  $v_s$ , ok. So, now, I mean there is no denying that this expression looks, you know, kind of very intimidating, right? Let us now stare at this expression and see if we can get any insight into why these terms make sense. First thing, what order transfer function do we have?

It is a second order transfer function. First question is do you know if this makes sense? Why does it make sense? I mean how many capacitors are there? There are 3 capacitors, but still the transfer function is second order. Is it likely that we made a mistake or does it seem reasonable?

So, you can see that even though there are 3 capacitors and you would normally expect the 3 reactive elements you would normally expect to see a third order transfer function. But you can see that you have a one capacitor loop, so all the 3 capacitors are not, all the 3 capacitor voltages are not independent, right. Once you specify one the other two are; I mean the other two are once two are specified the third capacitor voltages.

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So, basically the point is that even though there are 3 energy storage elements, you have 3 of them connected in a loop. So, all the 3 you know capacitor voltages are not independent. And therefore, it is still a second order system, right. And in fact, how many energy storage elements are there here?

Student: 2.

Now, what is the transfer function from input to output?

Student: It is constant.

It is a constant, right. So, you can see that the number of energy storage elements is not necessarily equal to the order, right., ok So, with that in mind, so, because there is you know one capacitor loop where it makes sense that the order of the denominator is second order, alright, ok. So, the next question to ask is whether the system is stable or not, right? Does this represent a stable transfer function? So, for a system to be stable what comment can we make about the location of the poles?

They must be in the left half's plane, alright. So, where are the poles? Where is the location of the poles of this transfer function? Left half. Why? All the coefficients of the denominator have the same sign, correct. So, so far we have understood why this  $-g_m R_L$  will make sense, ok. We also understand why the denominators of a second order and we also recognize that the system is stable, ok. Now, let us stare at the numerator. What comment can we make



about the numerator? Where are the 0s? So, basically you can say that the numerator, this is of the form  $(1 - s)/\omega_z$ , right and this is now in the right half plane 0.

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The slide displays the transfer function  $\frac{v_o}{v_s} = \frac{-\beta_m R_L \left(1 - \frac{s C_{gd}}{s_w}\right)}{1 + s \left\{ R_s \left\{ C_{gs} + C_{gd} (1 + \beta_m R_L) \right\} + R_L (C_{gd} + C_{db}) \right\} + s^2 (C_{gs} C_{gd} + C_{gd} C_{db} + C_{db} C_{gs}) R_s R_L}$ . The term  $(1 - \frac{s C_{gd}}{s_w})$  is highlighted in blue and labeled as a right half plane zero. Below the equation is a circuit diagram of a common-emitter amplifier with a dependent current source  $\beta_m i_b$ , a load resistor  $R_L$ , and various capacitors  $C_{gs}$ ,  $C_{gd}$ , and  $C_{db}$ . The input voltage is  $v_s$  and the output voltage is  $v_o$ .

So, is a right half plane 0 problematic for stability? Ralf, what do you think? No, right. The 0s can be anywhere in the s-plane and therefore, there it is basically not a problem. Now, the question that we now have is first of all you know, why is there a 0? Let us take a look at the numerator and see why first of all, what is the intuition behind. First of all, why is there a 0? Right. And b, why is the 0 in the right half s-plane? Correct, ok.

So, in other words, can we stare at the circuit and basically say you know why there is a 0 in a transfer function, in the transfer function, ok. Let me ask you another question. So, can any of you give me an example of an instance, where 0s of a transfer function affect you in daily life?

You know, all of you own a phone, right and you are sitting in the hostel, ok. Suddenly, your friend calls you saying hey you know what happened you know you owe me like 3000 rupees, you know you borrowed it from me like long ago, then suddenly what happens? Signal is very bad, ha right, ok. And the call goes off.

So, why do you think this is happening? And it always happens only most of the time when you are inside a building, right? When you are in a wide open space it usually never happens,

correct. So, why do you think this is happening? Now, what do you do if you know your signal is lost? What do you instinctively do?

You just move your head this way and suddenly you get a signal and you can talk, right? So, what is going on? I remember this only happens when you are you know among either you know when you are in a building, right. Usually, it does not happen in a wide open space. So, what is happening?

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The slide displays the following transfer function:

$$\frac{v_o}{v_s} = \frac{-\beta_m R_L \left(1 - \frac{s C_{gd}}{\beta_m}\right)}{1 + s \left\{ R_s \left\{ C_{gs} + C_{gd} (1 + \beta_m R_s) \right\} + R_L (C_{gd} + C_{db}) \right\} + s^2 (C_{gs} C_{gd} + C_{gd} C_{db} + C_{db} C_{gs}) R_s R_L}$$

Below the equation is a circuit diagram with a voltage source  $v_s$ , resistor  $R_s$ , node  $v_x$ , capacitor  $C_{gs}$ , dependent current source  $\beta_m v_x$ , capacitor  $C_{gd}$ , load resistor  $R_L$ , and capacitor  $C_{db}$ . To the right is a ray diagram illustrating signal paths reflecting off a surface.

So, it turns out that, so, this is the base station, right. So, this is you know this is your hostel and this is you in your hostel, right. And so, you know even if you are standing by the window, I mean you know there is basically there is you know there is one path like that.

There is also another path like that. And then perhaps this is some tree there, right. So, a third path like that. And then on the tree there is a monkey, so it hits the monkey and then comes there, is a fourth path like that and so on. So, what you receive on your phone is basically you have signals from multiple paths, each path has its own delay and its own attenuation. I mean to call it gain would be wrong because I mean you know you are transmitting watts of power at the base station and you only receive a few micro watts, ok at best you know some even picowatts, ok.

So, so, all these are all you know this, so each of these paths has got different attenuation factors and different delays, right. So, if you are at a point, it can so happen that the signal

from all those paths interferes constructively, right where therefore, you get a huge you get a you know a large signal, right, ok. Alternatively, it could also happen that all the

All the paths basically interfere destructively. And therefore, there is no signal there, you understand, right. And which is why you just mean if you are not getting any signal, you just move your foot away and then suddenly you get the signal, alright. So, what is happening? How is it possible? If you move slightly what happens?

There the phase of each path now is different. See, for many numbers to add to 0, correct, those numbers must be related in a very special way, correct, ok. These are all complex numbers because they are all you know magnitude in phase, correct. If the if there is a null at a certain point, it must so happen only when, you know sound as some you know if you conditions which are must be you know pretty difficult to achieve, correct, ok.

But the moment you move away a little bit, that condition goes away because the phases are all now completely different. So, you know earlier they were cancelling perfectly, now they do not cancel so perfectly and therefore, that means that the signal you are receiving becomes higher, right? So, what is the moral of the story and what does it have to do with our common source amplifier? When you are in an open field this never happens. Why? There is only one path, alright, ok. So, if you are in the Thar Desert and you lose signal, that is not because there are 100 paths hitting your phone, right. If you are in the Thar Desert and you lose signal, it is because you have entered Pakistan, that is all, you understand, right. There is no signal at all. Moving your head this way will not help, ok, alright. So, what is the moral of the story therefore?

You can have 0s when there are multiple paths from input to output. So, whenever you have a system where there are multiple paths from input to output, you will always have some complex frequency at which the signal from both these paths cancels, alright, ok. So, with that background, now can you tell me why we can intuitively see why there are 0s in the transfer function. And b, why the 0s are in the right half plane. Why are there 0s?

There are two paths from the input to the output, what is how the signal gets from the input to the output through two paths? One is through via  $v_x C_{gd}$ , the other one is through  $g_m v_x$ , alright. So, since there are two paths, you know each path has some gain to the output, right at some complex frequency the two gains are equal and opposite. And therefore, you get a 0, correct. And now why so, you can see that here the gain is a function of  $C_{gd}$ , right. And here it

is a function of  $-g_m$ , correct. Because intuitively, if I yank  $v_x$  up, if I yank  $v_x$  up through  $C_{gd}$  will this node tend to go up or go down?

Student: Up, go up.

It will go up. If I throw  $g_m$ , what is  $g_m$  trying to do?

It is,  $g_m$  is trying to pull the voltage down, right. So, they are trying to do it in opposite directions which is why you have a 0 in the right half s plane, right. So, basically the bottom line therefore is that it makes, I mean now staring at the circuit we see why we should expect to see a 0? Because there are two paths for the signal from the input to the output, one is through  $C_{gd}$  and one is through  $g_m$ , ok.

And we also should be able to see why the 0 is in the right half s plane and that is because you know  $C_{gd}$  is trying to, through  $C_{gd}$  the output is you know is trying to go up, through  $g_m$  the output is trying to go down, alright. So, we understand this part. So, in the next class, we will take a look at the denominator and figure out whatever intuition we can form, and try to see if the individual terms in the denominator make any sense at all.