Analog Electronic Circuits
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Lecture - 05
Linear and Nonlinear Two-ports and the incremental-y Matrix

Alright folks, good morning and welcome to Analog Electronic Circuits this is lecture 3. In the last class we basically saw how one can handle networks with two terminal elements which are non-linear. Today we will look at the other class of elements that we see namely two-ports, right, where you have two port network, which can potentially be non-linear.

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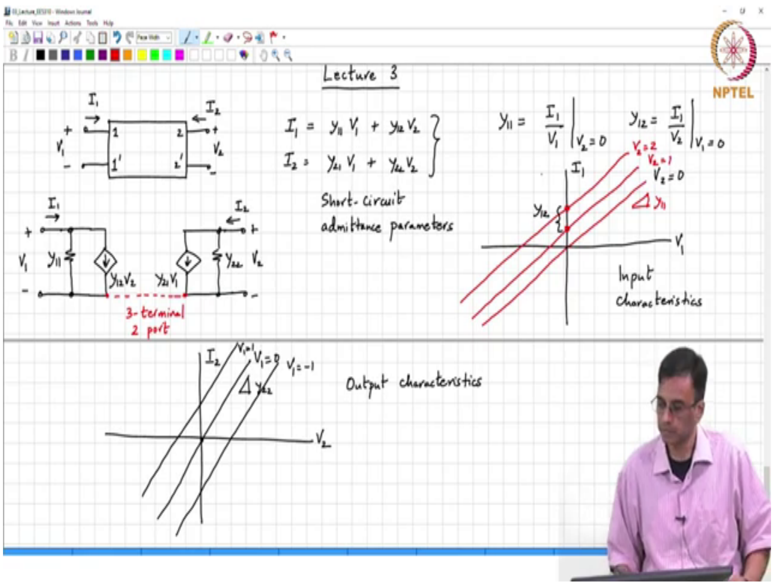

But before that let me quickly review two port networks which are linear. So, let us say you have a box, a linear network, with two ports. And of course, there could be a whole lot of stuff inside you do not know what is inside. However, as far as the outside world is concerned you have just access through two ports.

And the many ways of describing this the behaviour of this box as far as the external world is concerned and you have seen this in your prior classes and what comment can you make, I mean how can we model this linear two port?

Ok. If you have a two terminal network or two terminal a box with two terminals as far as the external world is concerned, how can we model this a linear two terminal box?

You have a linear two terminal box, right. A box with two terminals coming out you are told it is linear it does not have any sources inside. How can you model this this box?

Student: Resistance.

It is a you model it as a resistance right, assuming there is no memory inside you basically can model it as a resistance right as a linear resistance alright. Now, if you have a two port with two pairs of terminals coming what how can you model it?

Yeah, you already learnt in your earlier classes a variety of a parameter sets that can be used to describe the behaviour of these two terminals or this I mean this two-port network as far as the external world is concerned right. All of you have you heard of the parameter sets. What is the commonly used parameter sets that are used to describe a linear two port network.

Yes, you.

Are you familiar with this or you are not?

Student: Yes sir.

Ok. So, what are the commonly used two ports parameters?

Student: Admittance parameters, impedance parameters.

The admittance parameters, the impedance parameters, the scattering parameters, the hybrid parameters and all sorts of stuff. Here we can take, any one of them they are all equivalent as you have a given one set you can actually derive another set the algebra may be involved, but otherwise it is straight forward ok.

So, let us take for example, the y parameter set. So, when you describe a two-port network using Y parameters what exactly what are the independent variables and what are the dependent variables?

Student: V_1.

V_1 and V_2 the port voltages are.

Student: Independent.

Independent and the port currents are?

Student: Dependent.

Dependent alright. So, and because it is a linear network the port current I_1 is a linear function of V_1 and V_2 and the constants of proportionality therefore, have to have dimensions of admittance, right? So, that is basically,

$$I_1 = Y_{11} V_1 + Y_{12} V_2,$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

Ok, and given a box, how will you find say Y_{11}?

Student: I_1 over V_1.

So, Y_{11} as you can see from these equations is nothing but I_1 over V_1 under the condition that,

Student: V_2.

$V_2 = 0$ and when you say V_2 equal to 0, what I mean what is it?

Student: Short circuit.

It is a short circuit. So, these are therefore, called short circuit admittance parameters. Does that make sense? Ok, similarly Y_{12} is nothing but.

Student: I_1 over V_2.

I_1 over V_2, when?

Student: $V_1 = 0$.

$V_1 = 0$. So, Y_{11} is the admittance seen at port 1 when port 2 is shorted and similarly Y_{22} is the admittance scene at port 2 when port 1 is shorted and what does Y_{12} quantify?

Y_{12} quantifies the effect of a voltage excitation at port 2 on the short circuit current in.

Student: Port 1.

Port 1 and likewise Y_{21} quantifies the short circuit current in port 2 when you apply a voltage V_1 at port 1, correct, ok. So, well as they say a picture is worth 1000 words. So, I mean whenever you write the equations it is often quite useful to draw a picture.

But of course, if you had only a box with two terminals drawing the picture is easy. What picture will we draw if you want to demonstrate its characteristics? If you have a two terminal element what picture will we draw to demonstrate its characteristics?

Student: linear.

We just draw it is a linear if we further assume that the two terminal box is linear then there is the characteristics of that box are nothing but you plot I versus V or V versus I ok. Now, what comment can if you want to show these equations graphically what do you think we will do?

Student: one variable.

Ok. So, clearly I_1 is a function of two variables V_1 and V_2 and to show this I_1 graphically as a function of V_1 and V_2 we would have to draw a 3 D plot unfortunately the paper is only.

Student: 2 D.

2 D and therefore you fix one variable. So, what we do is basically say I will plot I_1 as a function of V_1 for?

Student: V_2.

For several values of V_2 correct and if I do a make such a plot what will I get.

Student: Straight line.

So, for let us say for V_2 equal to 0 what will you get?

Student: straight line

You will get a straight line passing through the origin. So, just for argument's sake I want to draw a line like that what is the slope what is the slope of this line? This slope is nothing but Y_1 ok. Now, so, this corresponds to $V_2 = 0$. Now, for V_2 say is equal to 1 volt what do we see?

It will be the resulting so called curve will be parallel to this line except that it will be either shifted upward or downward depending on the sign of Y_1 right. So, $V_2 = 1$, $V_2 = 2$ alright. So, for various values of V_2 you will get a family of.

Student: curves.

Of curves in this particular case because the two port is linear there will be a family of lines where the spacing between the lines will be identical increments in V_2 alright. So, this therefore, is going to be Y_{12}, does it make sense people ok.

So, these are what are called the I mean typically in this part of the world we write from left to right. So, whenever we have a two port, I mean one port is the input and one port is the output. So, which port do you think will be the input?

Student: Left.

Yeah, basically stuff flows like this in this part of the world. So, the left port it is called the input port. So, these since these curves capture the behaviour of the input port these are what are called the input characteristics of the two port right. So, in other words rather than giving out Y_{11}, Y_{12}, Y_{21} and Y_{22}, you might as well supply the user with these with this picture right and from this from the input characteristics what all can the user infer?

Student: Y_{11}.

Y_{11} and.

Student: Y_{12}.

Y_{12} alright ok.

So, now the obvious thing is what other characteristics do we need to draw? Well, analogously we have the output characteristics which basically have we plot I_2 versus V_2 for different values of.

Student: V_1.

V_1. So, I am just going to draw some pictures. So, this is going to pass through the origin. So, this is for $V_1 = 0$ and what is the slope? Slope is Y_{22} and again you will get a family of lines which are parallel to each other and so, say $V_1 = -1$ and say $V_1 = 1$ and ok.

And again the all these lines are parallel to each other and for equal increments in V_1 there will be parallel the distance between these 2 adjacent lines will be identical ok. So, these are the output characteristics right. Another way of depicting the same two port right, not in graphical form, but in circuit element form or like a network is to how do we I mean how do we express these in terms of network elements?

Basically, you can draw you can represent these equations in this form. So, this is Y_{11} and what should we do? This Y_{11} and admittance so, is it a resistance or a conductance?

Student: Conductance.

It is a conductance then what?

Student: Current source.

What kind of current source?

Student: It is a voltage controlled current source.

So, it is a voltage controlled current source and must it flow down or must it flow up?

Student: Down.

Down and what is the value of that voltage controlled current source?

Yes.

Student: Y_{12}.

Y_{12}.

Student: V_2.

V_2. And similarly for port 2, this is Y_{21} V_1 and Y_{22}, does that make sense people? Alright. So, the next question I would like to ask you is what is the minimum number of terminals needed to make a two port network?

Student: Three terminals.

You need.

Student: Three.

Three terminals because one terminal can be shared between the input and output ports ok. In that case then what happens is that these two terminal, I mean nodes will be the same and this will be for a 3-terminal 2 port. Is that clear, alright.

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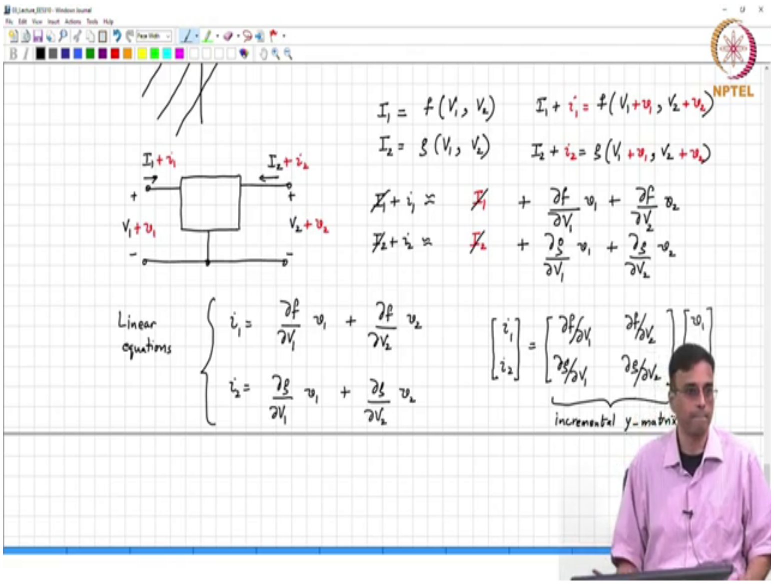

Having refreshed our memory about linear two ports, now let us consider the simplest case of what happens when the two port is non-linear. When I say the simplest non-linear two port it also has three terminals, ok. And so, we apply some voltages at the ports and some currents result. So, which are the independent variables and which are the dependent variables?

Student: Voltage is independent.

The way we have drawn it voltage is independent and the currents are?

Student: Dependent.

This is exactly the same thing that we did with the linear two port right. I mean there is no need there is no nothing wholly about using voltages as independent variables, we could have as well used currents as independent variables in which case the voltages have become dependent, but since we chose to go that way for the linear two port we will also do that for the non-linear two port. So, now what comment can we make about I_1?

Student: Non-linear function.

It is a non-linear function of some function f(V_1 and V_2), ok and similarly I_2 is a some other non-linear function g(V_1,V_2) correct.

$$I_1 = f(V_1, V_2)$$

$$I_2 = g(V_1, V_2)$$

And so, if you embed this non-linear two port in a bigger network, correct? What comment can we make about finding the branch voltages and the currents?

Yes you.

Student: Kirchhoff's laws.

Student: Kirchhoff's laws.

Yeah. So, to basically if you embed this network this non-linear two port in a larger network like we discovered in the case of a two terminal non-linear elements the Kirchhoff's voltage and current laws will become coupled a set of coupled non-linear equations and that is just as difficult to solve as the as in the previous case and therefore, we have to resort to we have to resort to a numerical solution, right.

However, if we increased the exciting terminal voltages by small amounts right ok. What comment can we make about the currents? They will also change by small amounts and again we will come back to this definition of what small means, but at this point let us assume that they change by small amounts. So, consequently what comment can we make?

$$I_1 + i_1 = f(V_1 + v_1, V_2 + v_2)$$

$$I_2 + i_2 = g(V_1 + v_1, V_2 + v_2)$$

Ok.

So, if you subtract these two equations what do you get? What can we say about i_1? We can simply expand the second equation or the equations on the right in the following way.

$$I_1 + i_1 = f\left(V_1, V_2\right) + \frac{\partial f}{\partial V_1} v_1 + \frac{\partial f}{\partial V_2} v_2$$

$$I_2 + i_2 = g(V_1, V_2) + \frac{\partial g}{\partial V_1} v_1 + \frac{\partial g}{\partial V_2} v_2$$

And this of course, we know to be what is that quantity?

What is a $f(V_1, V_2)$?

That is nothing but I_1, similarly what is $g(V_1, V_2)$?

Student: I_2.

I_2 correct. So, therefore, this guy and that guy go away. Likewise, those two terms go away. And what do we have?

$$i_1 = \frac{\partial f}{\partial V_1} v_1 + \frac{\partial f}{\partial V_2} v_2$$

$$i_2 = \frac{\partial g}{\partial V_1} v_1 + \frac{\partial g}{\partial V_2} v_2$$

Does that make sense people, ok. So, in other words we can write the incremental changes in the currents as in matrix for. Are these linear or non-linear?

Student: Linear.

These are a set of linear equations where even though the two port is non-linear, for small changes in the exciting voltages the changes in currents can be related to the changes in port voltages using a set of linear equations, right? Again, this is not much of a surprise this is exactly what we saw in the one terminal case. In the one terminal case we the I-V characteristic of in general is a is non-linear around the operating point we were able to approximate the curve by a straight line.

Student: Straight line.

Correct, now I_1 is a function of V_1 and V_2. So, it is basically a surface in 3 D, and around an operating point we are approximating the surface by a plane. So, these are the linear equations and using our experience with the two terminal non-linear elements where we called the ratio of the change in current to the change in voltage as what do you call that?

Student: Incremental conductance.

Huh?

Student: Incremental conductance.

The incremental conductance or incremental admittance.

Now, what we what are we going to call this matrix?

Student: admittance matrix.

Earlier we called it the admittance matrix, so the y matrix, now we should call it as.

Student: Incremental y matrix.

The incremental y matrix.