Analog Electronic Circuits
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Lecture - 49
Half-Circuit Analysis

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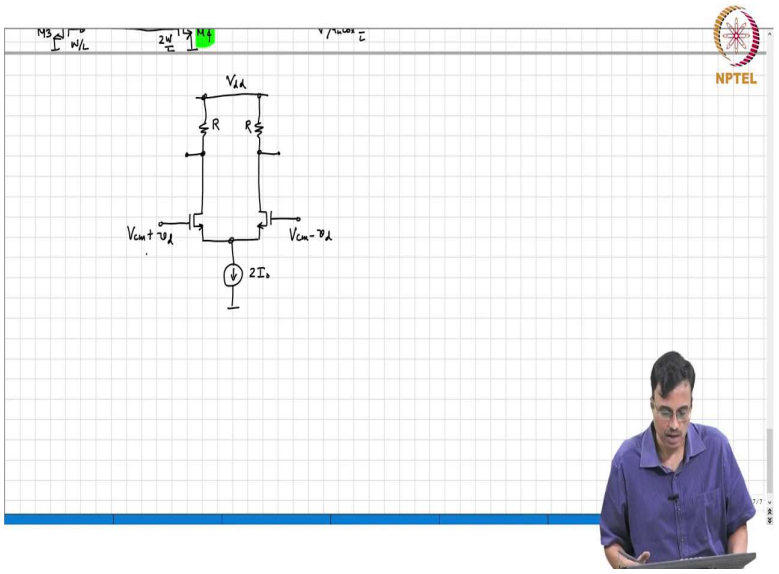

So, this is $V_{cm} + v_d$, $V_{cm} - v_d$. We found the operating point and therefore, now we are ready to do small signal analysis. Of course, the easiest thing to do is simply replace every transistor with its incremental equivalent and then you know go and do the math.

However, we can do something smarter here and what enables me to do that is to claim that we can do things in a much simpler manner. And you know what we have discussed so far namely simply blindly going and replacing every transistor with its incremental equivalent works.

There is nothing wrong with that, but one thing we are not exploiting that would work for any circuit. In this circuit there is also something special that we have not exploited and what is that we have not exploited? When you stare at the picture it is symmetric. So, whenever there is symmetry in a circuit or whenever there is symmetry in anything, you can always exploit it to?

To simplify analysis right and you know break it up into simpler problems. Right and that is you know we will use symmetry to simplify our lives ok. And so, you know that clearly the circuit is symmetric and so, in the small signal equivalent what happens, if we draw the small signal equivalent what happens to V_{dd}?

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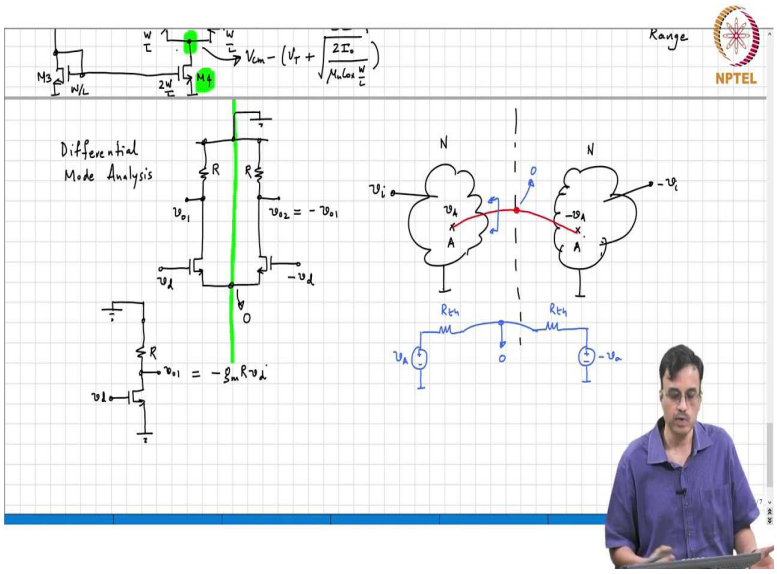

Grounded, V_{cm} goes away, this becomes - V_{dd}. What happens to 2 I_o? The $2I_o$ is often also called the tail current source. So, this is the small signal equivalent where now the transistors are assumed to be representations of their small signal equivalent circuits. Now, we need to analyze this. So, we see that there is an axis of symmetry ok and whenever there is an axis of symmetry you can always simplify life right and let me illustrate.

So, let's say you have a circuit like this, some arbitrary linear circuit, and you let's say there is a copy and paste of that one here. So, this is exactly equivalent. So, let say we have a node A here and a node the corresponding I mean this is an axis of symmetry the circuits are not connected at this point to identical circuits. Ok

Let's say we excite one with v_i the other one with - v_i. Alright let say let say ground. So, what comment can you make about the voltage small signal voltage v_A? On the left side versus the right side. The polarities on the is the same circuit, same circuit linear circuit excited with negative of the excitation. So, therefore, by linearity the voltages at the corresponding nodes in the left and the right will be exactly identical in amplitude, but 180 degrees in phase, ok. So, if I now join this node to that node what comment can I make about that voltage. So, if

you look at the Thevenin equivalent at point A what will the Thevenin equivalent be for the circuit on the left? Come on quick people there will be some v_A and in series with R_{th} for the circuit on the right.

It will be R_{th} and - v_i and therefore, when you join these two that voltage will be 0. So, it is not merely enough if simply the polarities of the two voltages are equal and opposite right the polarities opposite in the magnitudes are equal it must all it is also important that both the Thevenin impedances are exactly the same you understand. So, because the circuits are because of symmetry the circuits are exactly identical and therefore, the Thevenin equivalent is the same and therefore, if you join these two nodes the net voltage the voltage at the middle will be 0. Ok.

So, now, let's see. So, this is what I will call the differential mode analysis. And to do that we recognize that there is an axis of symmetry: the two sides are being excited by equivalent opposite voltages. So, this node will be based on our discussion just now. What comment can we make about that common source node? Why is it called a common source node?

The source is common to both sides. So, what comment can we make about the incremental voltage at the common source node? It will be 0. Ok If that is 0 and what comment can you make about say we let us call this v_{o1} and v_{o2} there will be equivalent opposite. So, there is no point in solving the left side and the right side. You can only solve one side and the other voltages on the other side will simply be the negative of this. So, we can only therefore work with half of the entire circuit and since this half came with differential mode analysis this is called the differential mode half circuit. What should we do with the source therefore?

Source must be ground. So, how does this look? I mean this is very familiar. This is nothing but the common source amplifier. So, v_{o1} must be $-g_m R v_d$. Alright So, V_{o2} is therefore $+g_m Rv_d$.

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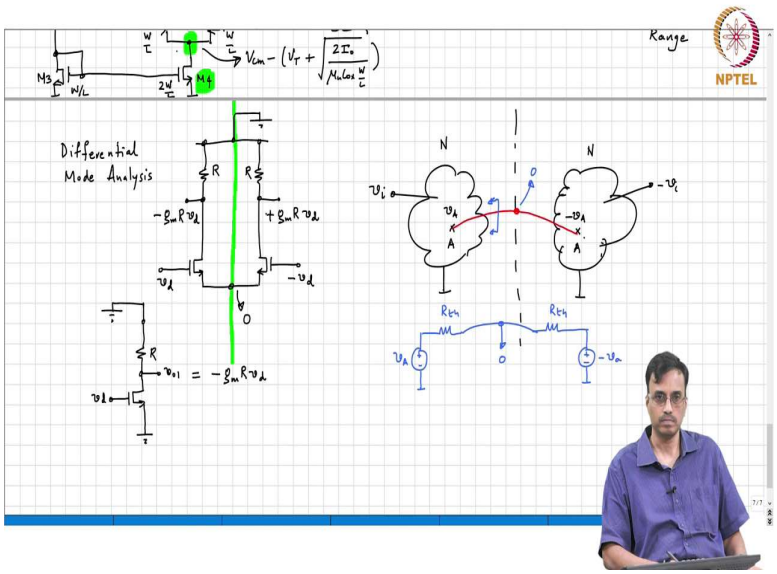

So, basically this is the incremental voltage is g_m R v_d this is - g_m R v_d.

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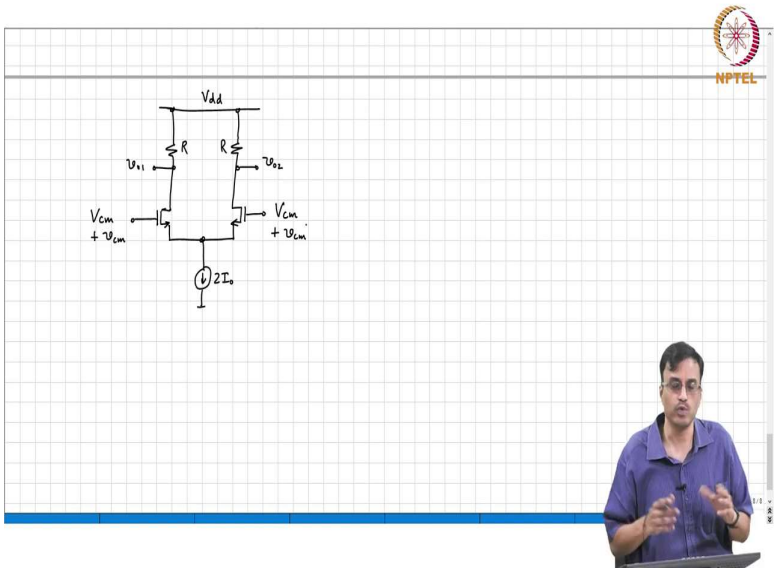

Now, let us remember that when we derived the circuit, we said that our circuit only responds to R. It only responds to the difference between v_x and v_y it does not respond if v_x and v_y change by the same amount. In other words if the common mode changes by some amount the output should not change. So, I mean here you know we should verify that fact correct and to verify it again what should we do this was V_{cm} if the differential mode was 0 this was V this was that is just simply V_{cm} on both sides. So, how do we verify the fact that our

outputs here v_{o1} and v_{o2} do not depend on do not change if both the inputs are changed by the same amount how will we verify it?

So, basically, we apply a small change. We move both these sides up by the same amount and check to see what v_{o1} and v_{o2} are right. What will we find or what should we find? Let me rephrase the question again the output which are our outputs which are our outputs either v_{o1} or v_{o2} depending on whether we want you know positive polarity or negative polarity anyone. Let's say v_{o1} ok when we started off, we said that this v_{o1} must only depend on the difference between v_x and v_y, which corresponds to the differential mode part of the signal and it should not depend on the common mode voltage right and we are trying to verify that using analysis.

So, to that end what we are doing we are keeping the differential mode 0 correct do you understand ok. And we are increasing both v_x and v_y by the same amount alright and the question I am asking you people is what should we expect to find for v_{o1}? We should expect to find 0 because the output is not expected to depend on any change in the common mode voltage. Is this clear, people.

Student: Yes sir.

Alright now so therefore, we now expected to do analysis of this circuit except that now both sides are pushed up by the same amount alright. So, again we exploit symmetry; the circuit is absolutely symmetric about this point.

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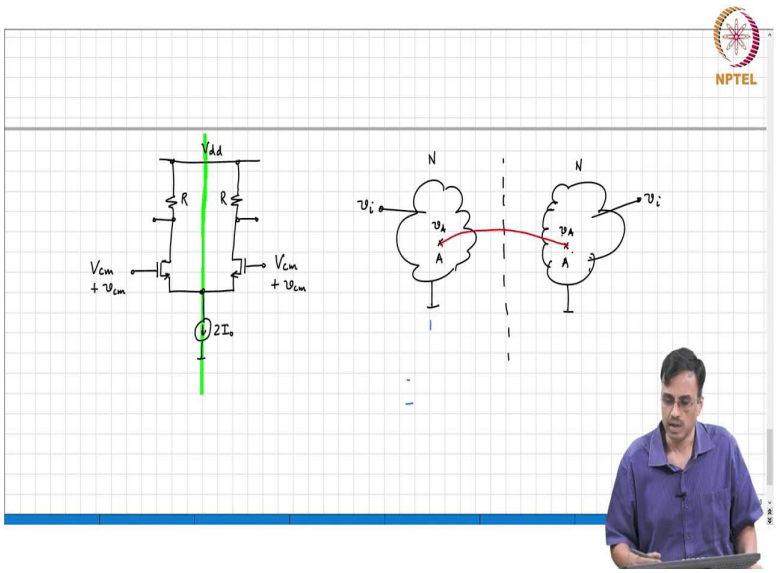

So, if we draw the circuit now it is basically so, what are we doing we are taking two identical networks we are and. Now, we are applying the same potential v_i to both sides ok and let say you connect let say there was a connection like this. What comment can you make about the connection? What comment can we make about the current through that connection?

Student: 0.

0, right and why? Because well by symmetry you know v_A will be the same as v_A on the other side and therefore, no current can flow.

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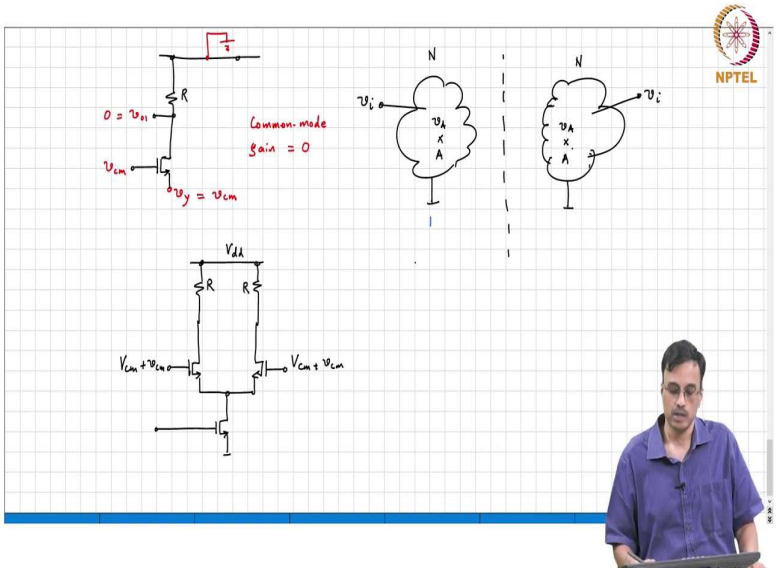

And therefore, this can be no current can flow you can remove that connection right no current flows for any value of v_i. So, you remove that connection and therefore, you can only analyze one half on the other side you get exactly the same. So, that is now again half circuit analysis, but now it is coming for common mode excitation. So, that half circuit is called what you would call a common mode half circuit correct?

So, what will you do with V_{dd}? What should we do with the ground as usual, current source open circuit and then we can forget about what happens to this wire that is the one which is connecting both the halves. So, this can be removed so, this incremental v_{o1} is what we are interested in fine. Are these clear people? Alright so, this circuit is also familiar to us. So,

what is the incremental and this V_{cm} and V_{CM} goes away this is only v_{cm}. So, what comment can you make?

Well, there is no source resistance. So, what comment can you make about v_y there small signal? So, v_{o1} must be equal to 0. So, what comment can you make about v_{o2}? That is also 0. So, this confirms what you call the original hypothesis when we derived the circuit that such a circuit would only be dependent on the difference between v_x and v_y alright. But of course, there are the practical non-idealities.

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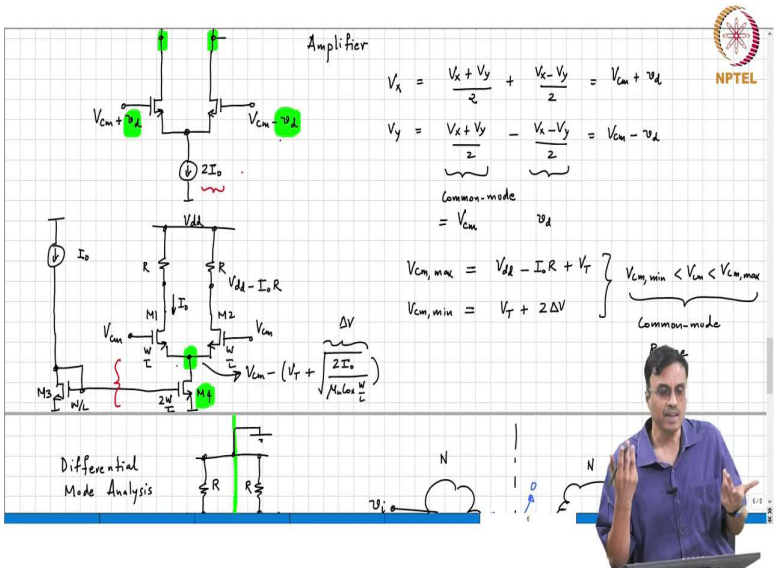

So, we assume that this current source is ideal in our analysis as we have seen here. The current source is not often realized by using some kind of current mirror, correct? So, what will be the small signal equivalent of this current source?

In practice this is not really going to be an open circuit, it is going to be common mode gain. The common mode gain is nothing but 0 the differential mode gain is nothing but - g_m R ok. So, in practice the resistance is going to be some current source right. So, this is V_{CM} + v_{cm} and this is R correct.

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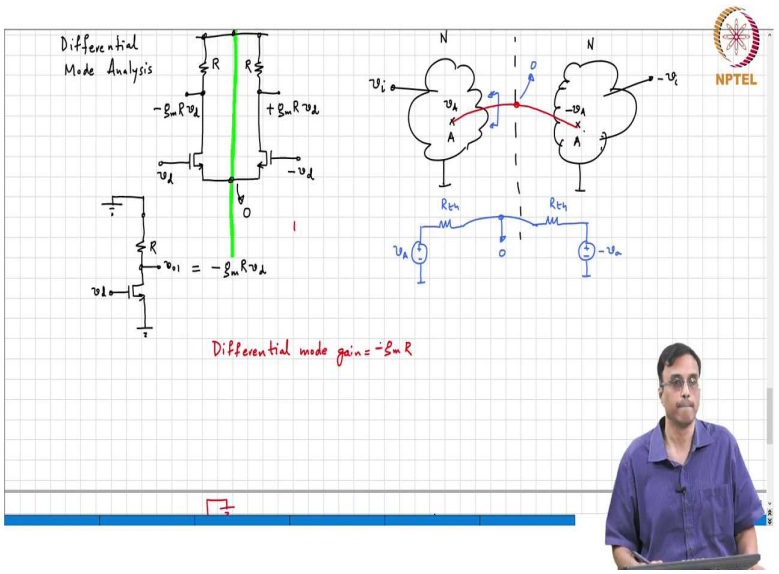

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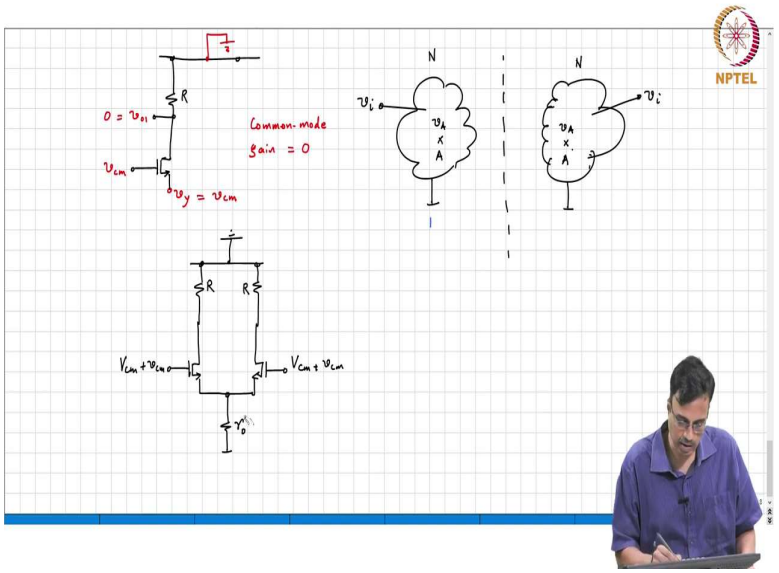

So, in an incremental network what should we replace V_{dd} of course? Will it be grounded very well? What should we replace the tail current source with?

Student: R_o.

So, let us call this some r o right which is the output resistance of that current source. Ok. If you want to make a better current source, what will we do? What will replace that transistor

single transistor by a cascode current source which increases the output resistance by a factor g_m R. So, anyway whatever that output resistance is, it is ok.

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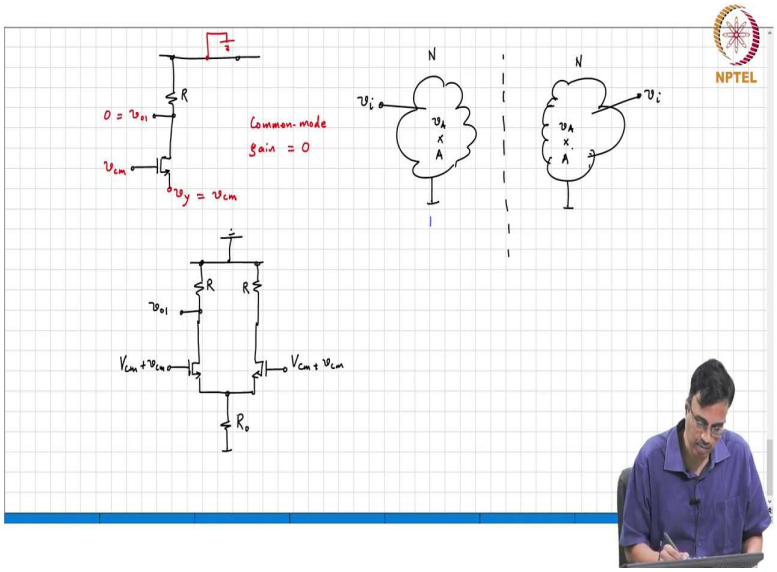

Let us call that R_o to stand for you know appropriately you can. So, now, to analyze this again we see symmetry and we want to find out what this v_{o1} is. So, what comment can you make about how we will analyze this circuit? Can we exploit half circuit analysis?

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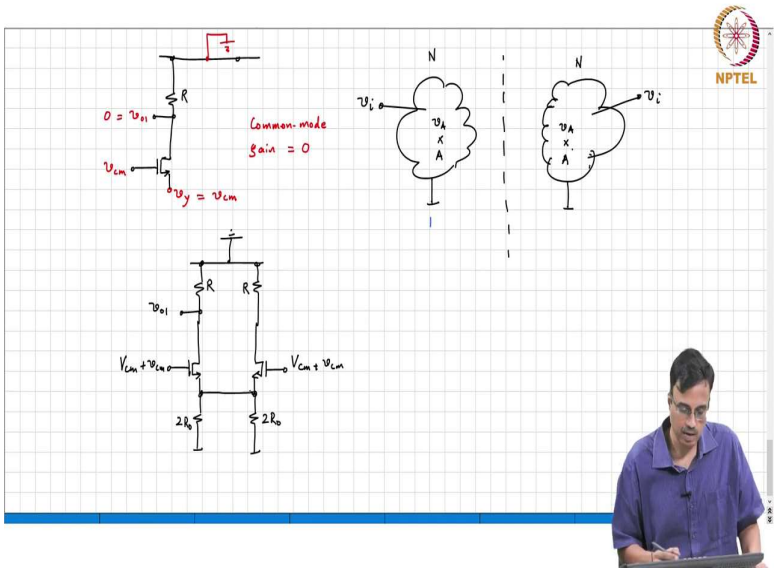

Ok, well then yes, I remember that half circuit analysis means that we have two identical circuits and we have something going from one side to another. So, this you can always split as $2R_o//2R_o$ and the half circuit analysis basically means that you get rid of this half ok.

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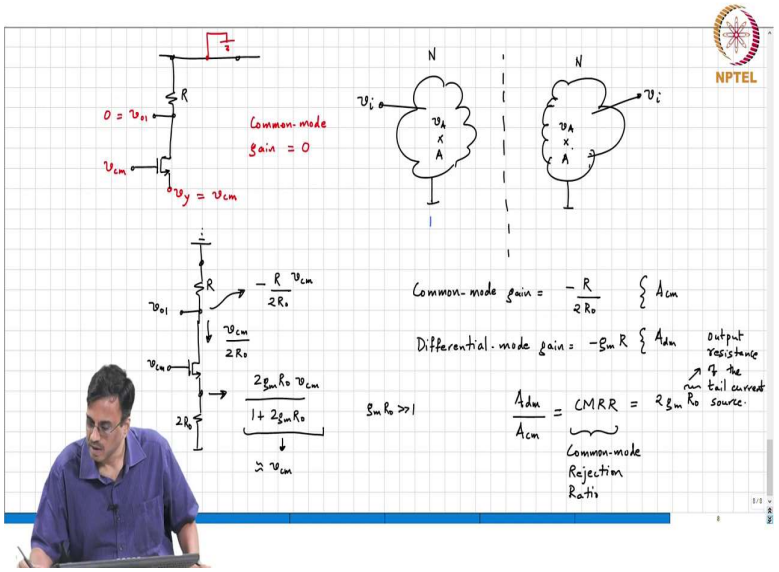

So, what comment can you make about v_{o1}? What is this incremental voltage? Now, you should be able to tell me quickly it is, $2\ g_m\ R_o/(1 + 2g_m\ R_o)$. So, what is the current flowing there?

That is just this divided by $2R_o$. So, basically and what comment can we make about $g_m\ R_o$? Is it a small number or large number?

Student: Large number.

It is a large number. So, what comment can we make about this quantity? Yeah, that is roughly $2\ g_m\ R_o$. So, what comment can you make about this whole expression? This is approximately v_{cm}. So, what current can you flow through the transistor?

Student: $v_{cm}/2$ R.

What comment can you make about the incremental voltage there? Therefore, v_{o1} $-R/2R_o$ v_{cm}. If the tail current shows, what is the bottom line? I mean ok this is the analysis. So, what does this mean? What is this telling us? So, ideally, we were hoping if the current source was ideal, the differential pair the output of the differential amplifier would only depend on the

difference between the two inputs v_x and v_y right with no dependence on the common mode component whatsoever. Unfortunately, in practice the current source is going to have a finite output impedance. We do know how to make the impedance better, but it will always remain finite ok. And the consequence of that finite impedance is that the gain for common mode signals is not 0. Alright it is this quantity is - $R/2R_o$. You know and if you know and what is this is ideally supposed to be 0, but it is not 0. I mean see one thing you must realize that in life and in engineering nothing that is supposed to be 0 will be 0, nothing that is supposed to be infinity will be infinity.

So, the common mode gain in practice therefore, will be v_o common mode gain which is ideally supposed to be 0 will be now by 2 R_o ok and recall what is the differential mode gain. The differential gain is - g_m R. Right so, this is often called A_{cm} for common mode and this is A_{dm} for differential mode. So, the ratio of the differential mode gain to the common mode gain is which ideally should have been infinite. It should not have responded to common mode signals at all. Unfortunately, because the current source is not ideal, this common mode rejection ratio (CMRR), which is ideally supposed to be infinite, will be nothing but 2 g_m R_o and this R_o is the output resistance of the tail current source.

So, again if we see that nothing in the world which is supposed to be infinite will be infinite. So, that gain that common mode rejection ratio which was supposed to be infinite is not quite infinite. We only hope that it can be a large number. Right If you use a you know current source with an output resistance of R_o then this will be of the common mode rejection will be of the order of g_m R_o.

If g_m R_o is say 50 then the common mode rejection is you know is its 100, but yeah is roughly of the same order correct. Now, if you put a cascode current source what comment can you make about the common mode rejection ratio? It will be of the order of g_m R_o^2 and so on. So, to summarize the properties of the differential pair this is $2I_o$.

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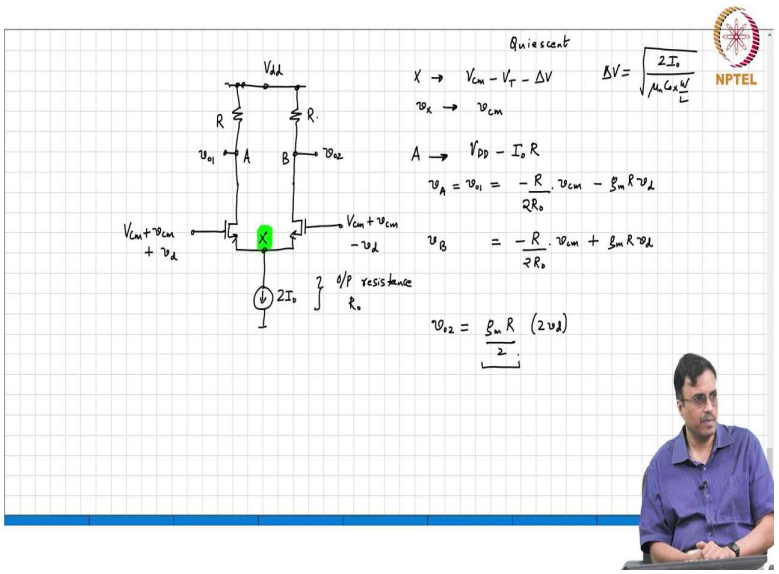

And let us assume that has an output resistance of R_o the input voltage is capital V_{CM} + some v_{cm} + some v_d where the v_{cm} models the change in the average voltage. This is - v_d. That is R, that is R, that is V_{dd}, v_{o1}, v_{o2}. So, the small signal the quiescent voltage at node lets us call this A and B and X. So, the quiescent voltage at X is what? V_{cm} - V_t - ΔV and what is ΔV?

Student: $\sqrt{2I_o}$

Yeah, it's nothing but $\sqrt{2I_o}/\mu_n$ C_{ox} W/L. So, the incremental voltage at x is what? What will be the incremental voltage at v_x? Is it simple?

Student: v_{cm}.

How did you get v_{cm}? So, I mean what do we do? Well again we go back to symmetry we find the individual voltages right and when you put both of them, I mean. So, you can think of this as the following if you want analyze what happens when there is a common mode change as well as a differential mode change simply by superposition you can consider only the common mode first only the differential mode next and use superposition to add up the potentials that you get in both half circuits. Right so, basically in the differential mode half circuit what comment did we make about that node X?

Student: 0.

0 in the common mode half circuit. What did we approximately V_{cm} because the incremental gain was 1 so, what comment can we make in the total circuit small v_x therefore, is v_{cm} small v_{cm} alright at nodes A and B what are the quiescent voltages or let me make let me write the quiescent voltage at node A V_{DD} - I_oR which is the same as the quiescent node voltage at node B ok. And what comment can we make about the incremental voltage at node A which is what we call v_{o1} which is again, it is the sum of the the voltage due to common mode excitation and the voltage due to differential mode excitation so, in the common mode half circuit v_A was -R/2 R_o v_{cm} and what is the differential voltage. -g_m r V_d. What about small v_B therefore, with just closing your eyes, and the common mode voltage will be the same. Does it make sense? So, this is all that there is to the differential pair with resistive load the gain that you can get therefore, is if you take either v_{o1} or v_{o2} the gain that you get is basically for the differential I mean remember that the difference between v_x and v_y is what is the difference between v_x and v_y is 2 v_d. So, the output voltage v_{o2} for example, is nothing but $g_mR/2V_d$ right. So, the differential pair is therefore, giving you a gain of $g_mR/2$ for the difference between the terminals v_x and v_y ok, but this is I mean this is basically doing the job of a common source amplifier except that it is attempting to only amplify the difference between v_x and v_y without ideally reacting to the to the common mode component.

In other words, if v_x and v_y change by the same amount. Ideally the differential pair output is not supposed to react to the change. However, in practice there will be a small change due to the common mode component. So, ideally, to see what is coming next, I mean this gain g_m R/2 of course, if you know cannot be very large right. So, just like how in a common source amplifier. You cannot make that g_m R you know if you attempt to keep making if you make that load resistance very large what comment can we make?

For a given supply voltage as you keep making R larger and larger the you know you will eventually end up you know you cannot get too greedy the eventually the transistor will go into the triode region correct and the only way to fix that problem was you know if you wanted to keep a resistive load you have to keep increasing the power supply voltage here you know again here also we see the same the same problem if you want to get more gain by keeping while keeping the transistors and 2 I_o the tail current source the same. What will you do? The formula tells us that it is $g_mR/2$. So, you go on increasing R. So, what happens as you go on increasing R. the transistors will eventually move into the triode region and the only way to fix that problem is to keep increasing the supply voltage. To increase the gain by a

factor of 100 you would have to increase the supply voltage also roughly by the same factor. So, what was the fix for that problem earlier?

Student: R

So, we used an active load. So, the same thing we will do here does it make sense alright. So, I will take a look at the motivation for trying to introduce the active load. It is very very similar to what we did for the common source amplifier. And tomorrow we will take a look at how we enhance the gain of the differential pair with an active load.

Thank you very much.