Analog Electronic Circuits
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Lecture - 48
The Differential Amplifier

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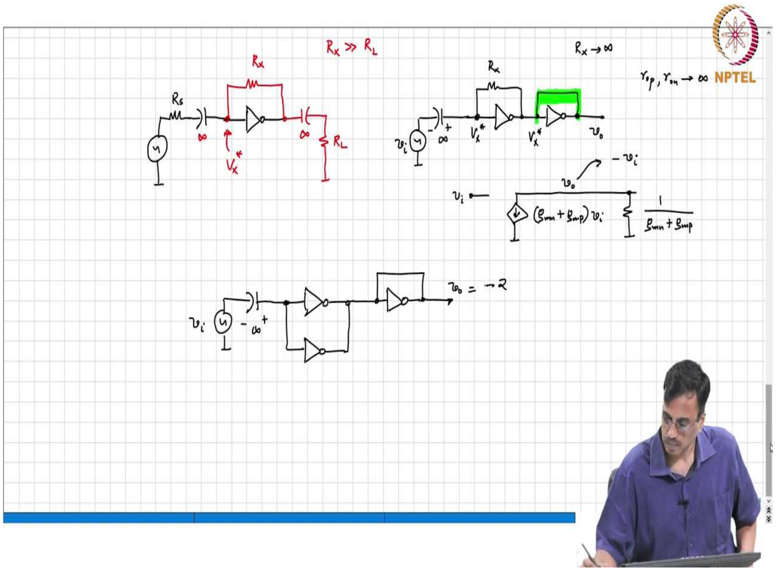

So, alright. So, now, that the principles are covered, we can move on to the next topic. Any questions?

Student: Yeah, on the right side of the green of the above plain, so, infinite capacitors will be connected in series with R_s.

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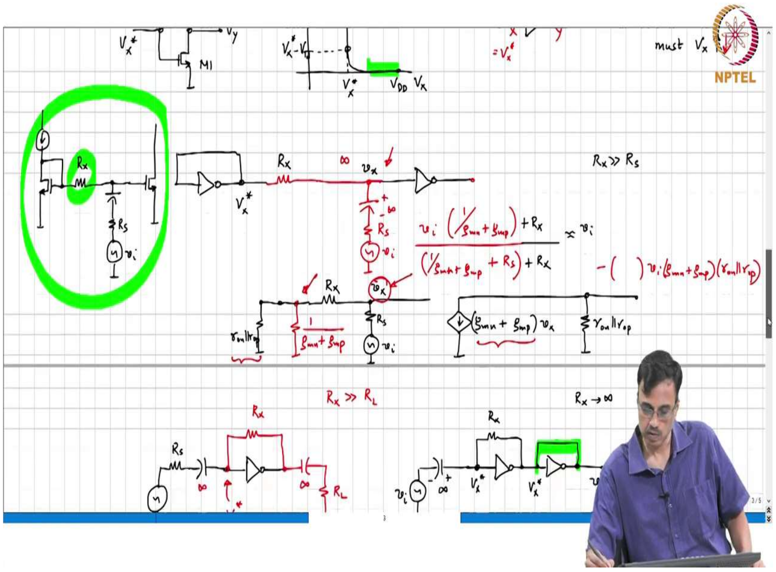

Sorry indeed thank you very much, sorry. Indeed yeah, thank you I hope that is fine thank you, very good.

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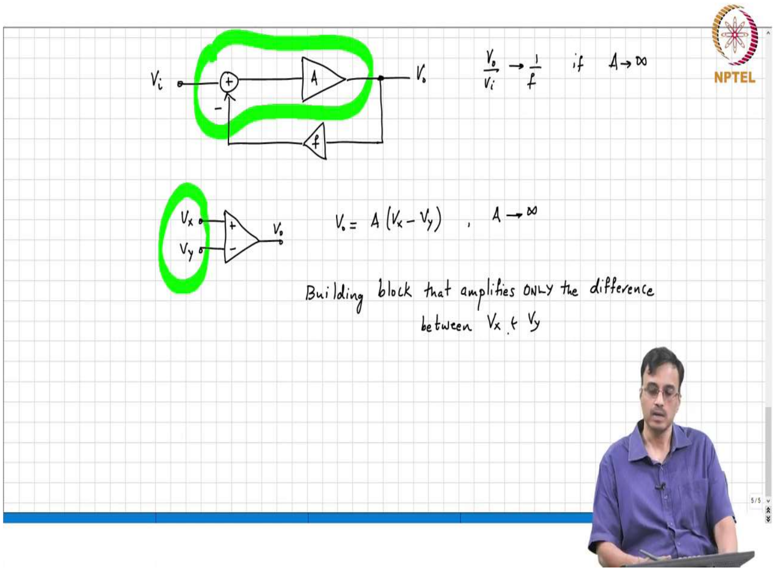

$\frac{V_o}{V_i} \to \frac{1}{f}$ if $A \to \infty$

$V_o = A(V_x - V_y)$, $A \to \infty$

Building block that amplifies ONLY the difference between V_x & V_y

So, now let us move on to the next topic which will motivate me. Remember that you know when we wanted to make a good voltage amplifier thing like that, we basically wanted to use feedback to make the input output properties of the amplifier independent of the properties of the active elements, correct.

And the ideal thing would be of course, therefore, if we look at that to go back to our negative feedback block diagram. So, we have some input and we have some output which we feedback and we said that the closed loop V_o/V_i, will tend to 1/f, if A tends to infinite. And then we said we actually will be able to sell that part as a building block where users can go and put their favourite feedback factor and get whatever closed loop gain they want. So, and therefore, what the operational amplifier, this was the operational amplifier and what the operational amplifier was doing was basically taking 2 voltages V_x and V_y and generating $V_o = V_x - V_o$ is $A(V_x - V_y)$ where A tends to be infinite.

So, the first step in the process therefore, I mean presumably with you know finite lambda of the transistors or non-zero lambda for the transistors its not possible to get a large gain with one stage, right. I mean which amplifier do you think is the most useful candidate to get a large gain among all the common sources, right? And perhaps with an active load. So, that you get you know as much gain as possible when in a small supply. Now, but here and you know presumably with one stage you cannot get you know I do not know I mean typical gains for an operational amplifier. So, maybe 10^4, 10^5 that kind of thing.

And therefore, it's not possible to get it in one stage. So, you have to cascade stages and on top of it you know here the common source amplifier we saw gave us gain. So, presumably we need to cascade several of these common source amplifiers to get I mean, if each common source amplifier can give us only a gain of say 100 then and if you want 10^4, then you have to at least cascade you have to cascade 2 stages, right.

If on the other hand the single stage gives only a gain of 25, then you have to cascade. I do not know more stages. So, it's a given that we have to cascade high gain stages to get a really high gain for the operational amplifier, right? However, there is a slight difference between the common source amplifier we know right and what the operational amplifier is supposed to be doing is the following. The operational amplifier is amplifying only the difference between V_x and V_y, right and our common source amplifier that with the common source amplifiers that we have seen so, far are all are just amplifying one input that small signal input V_i that we are giving you know they are just amplifying that with presumably a large gain.

If you want an even larger gain you take another common source amplifier and cascade it with the first one, correct. But what is called for we also need therefore; a building block whose job is to only amplify the difference between two voltages does it make sense? Right. And the motivation now you know why we need this is because in an operational amplifier that amplifier is only concerned about the difference between V_x and V_y. So, in other words whether V_x is you know if V_x and V_i are 1 millivolt and 0 or whether V_x and V_i are 100 point 100 volts + 1 millivolt and 100 volts the output of the operational amplifier must be the same, ok. So, in other words what we are interested in is finding a building block that amplifies only the difference between two voltages as I will call that V_x and V_y, alright.

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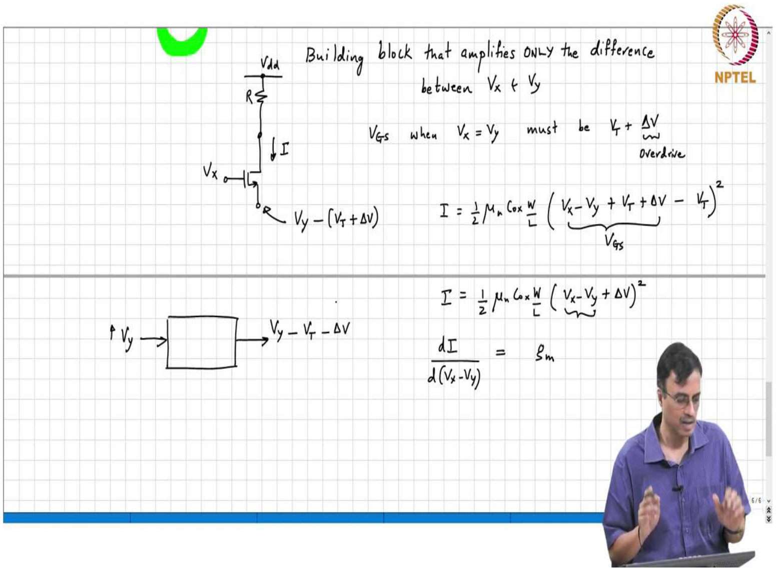

Building block that amplifies ONLY the difference between V_x & V_y

V_{GS} when $V_x = V_y$ must be $V_T + \underbrace{\Delta V}_{\text{Overdrive}}$

$$I = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(\underbrace{V_x - V_y + V_T + \Delta V}_{V_{GS}} - V_T\right)^2$$

$$I = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(\underbrace{V_x - V_y} + \Delta V\right)^2$$

$$\frac{dI}{d(V_x - V_y)} = g_m$$

So, and again. So, what is the simplest thing? So, we already know the common source amplifier, we know the transistor how it behaves, right. The one I mean one way to think about it is to say that well the transistor responds only to the difference between the gate voltage and the source voltage. So, hey you know how about you know why not put V_x here and V_y there for sure this only depends on V_x - V_y because if V_x and V_y both increase by exactly the same amount the transistor does not know anything I mean it just since it only reacts to V_x - V_y, right. But there is a small catch here: remember that in a good negative feedback loop what comment can you make about the difference between V_x and V_y?

I mean let us assume that we make this all work, right and we indeed have a great operational amplifier, right. Once you embed the operational amplifier inside a negative feedback loop what did we discuss would be the difference between the voltages V_x and V_y?

Student: negative feedback.

Negative feedback is going to force that difference to be ideally 0, in reality it is going to be a very very small voltage where that smallness depends on how large the loop gain is, is that clear? So, if we succeeded in making the operational amplifier after all, we should expect that V_x and V_y will be very close to each other, ideally they will be identical. So, if V_x and V_y are very close to each other and then you know you put one at the gate and one at the source what comment can you make about the transistor?

If V_x and V_y are identical, what comment can we make about the region of operation of the transistor that is cut off, right? So, that is basically evidently a contradiction, right because we want high gain between a high incremental gain, I mean V_x - V_y to be amplified by a large factor, but if V_x and V_y are the same the transistor is actually off, ok, alright. So, in other words, what we need. So, what we need definitely is that the transistor must not be off when $V_x = V_y$ the transistor must be in saturation carrying some current, correct. So, in other words the gate source voltage when V_x is equal to V_y must be it must be carrying some current which basically means that, must be some V_T + some overdrive, right, let me not confuse you with ΔV what do we call this, ok, let me call this, this is the overdrive voltage.

Does it make sense? So, when $V_x = V_y$ we want to make sure if the gate voltage is the transistor is alive and is actually carrying some current so that the gate source voltage is greater than the threshold by an amount ΔV, is that clear? So, then if.

So, therefore, what comment can you make about this voltage therefore, when V_x must be when V_x is equal to V_y V_{GS} must be equal to $V_T + \Delta V$. So, what voltage must be put at the. So, this voltage must be $V_y - V_T + \Delta V$, correct, alright. So, under those circumstances what comment can you make about the drain current of the transistor? I will be equal to ½ $\mu_n C_{ox}$ W/L V_{GS} which is $V_x - V_y + V_T + \Delta V - V_T$. So, this is V_{GS}, this is V_T^2, alright. So, which is therefore, ½ $\mu_n C_{ox}$ W/L $(V_x - V_y + \Delta V)^2$, alright. So, now what comment can you make about if V_x - V_y is a small quantity, right, ideally we know that is going to be almost 0. So, what comment can we make about $dI/d(V_x - V_y)$? This is a small quantity. So, now, if V_x - V_y changes by a little bit without writing any equations, that is nothing but? What is that quantity $\mu_n C_{ox}$ W/L ΔV is the g_m of that, I mean this is basically saying I am changing the gate source voltage by a small amount, what is the change in the drain current? It is nothing but, g_m V_x - V_y, is this clear?

Student: (Refer Time: 13:57).

Is this clear now? So, at this point. So, when if at the source rather than applying V_y, we applied V_y - V_T + ΔV, you know, we are the transistor still responds to only V_x - V_y, alright, and for small changes in V_x - V_y, the change in the drain current of the transistor is g_m V_x - V_y, to get a large gain which is only dependent on V_x - V_y therefore, what will be continue to do?

How will we convert this change in drain current into a change in voltage, why are they not. This is g_m, ok fine, how will we convert this small difference into a voltage? Where will we put the resistor in the drain, I mean this is exactly the same thing as a common source amplifier except that, there we had the source connected to ground and we applied the voltage to be amplified at the gate, now what are we doing? We recognize the fact that, we recognize the fact that the source, the transistor only responds to the difference between the gate and the source voltage.

So, we apply one of the voltages at the gate, the other one at the source, if you directly apply the other voltage at the source. If I find that we end up with a contradiction because the transistor is going to be off. So, to keep the transistor alive what we must apply at the source is not the second voltage V_y itself, it must be -V_T, ok. Now the question is how do I generate V_y - V_T - ΔV? What is the simplest single transistor circuit that you can think of? You want to put V_y there, what you want is V_y - V_T - ΔV, what single transistor, single transistor circuit can you think of that can accomplish this?

In your mind go over all the single transistor circuits that you know, if we need a box, ok, there is a, this is a box we want to get V_y - V_T - ΔV, correct. So, if V_y changes a little bit what comment can you make about the output? This is a box, ok, we are trying to figure out what is inside the box, please answer the question that you are asked, what is the output? V_T - ΔV, ok now if I change the input by a small amount, how much will the output change by the same small amount, correct. So, what is the incremental gain?

Student:1.

So, can you think of a simple single transistor circuit which has an incremental gain of 1? The common drain amplifier, is this clear? So, basically how do we generate this?

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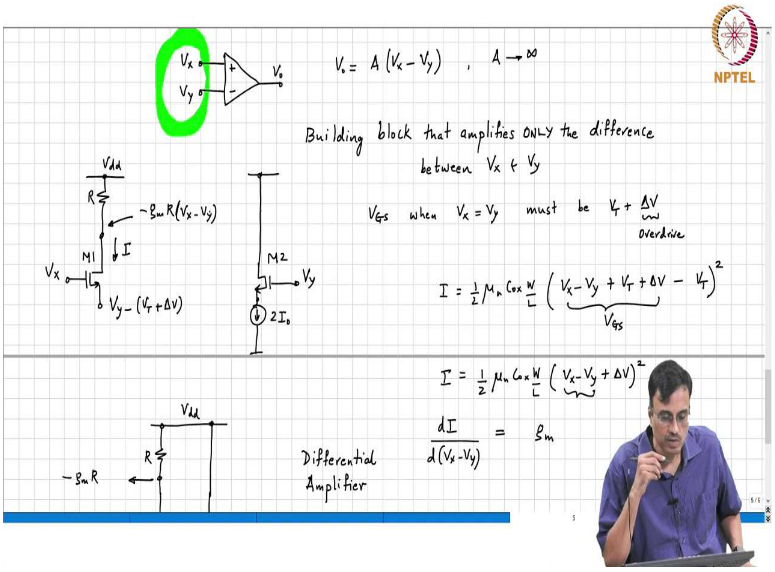

$V_o = A(V_x - V_y)$, $A \to \infty$

Building block that amplifies ONLY the difference between V_x + V_y

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Differential Amplifier

This guy here, well we take, let me just call that to I_o for this turns out that it becomes convenient, nothing else we could have called it I_o, ok. So, what happens now? What should we do? What is that voltage? Assume R_o is infinite. Yeah. So, basically the voltage drop across the gate source is dependent on that, on the size of the transistor and the current that you are pulling through it, correct. So, the voltage at the source of the second transistor M2 is some V_y - some V_{GS} of M2, ok alright. And I will choose the size of M 2 to be equal to M 1, right, and so, what should I do now? What is the next step? So, what I do is basically connect this, there ok.

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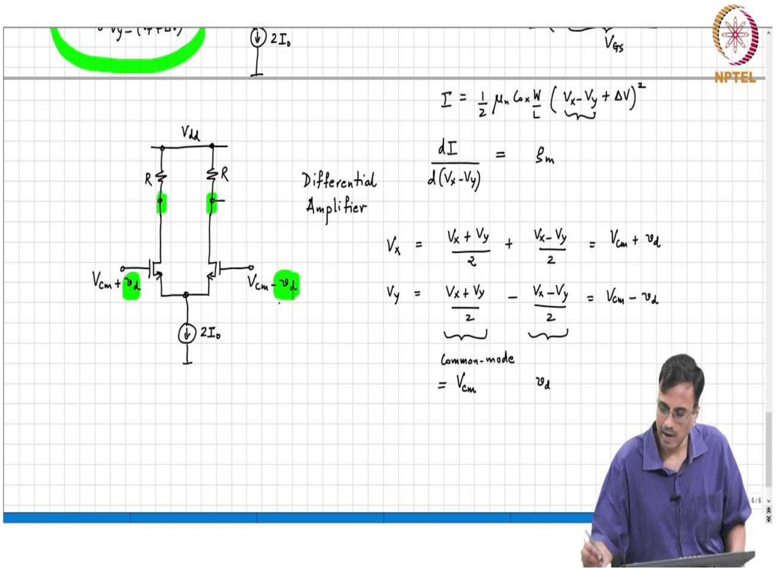

So, if I draw it slightly differently, basically the same thing drawn in a neater fashion, is 2 I_o, this is V_x and this is V_y, ok. And you put a resistance R there, right. So, this will give you, what is the incremental voltage? If V_x - V_y is very small, some - g_m R, right, if you want positive g_m R what will you do?

So, basically, I mean this whole situation is symmetric with respect to, if you put an R there, then it is symmetric with respect to whether you want to find (V_x - V_y) A or (V_y - V_x)A. So, if you put a resistance both drains, then you will have the same out incremental output with opposite sign depending on which drain you tap, ok. So, this is the so-called differential plane or differential amplifier. So, this part you understood, what is the incremental voltage here? So, -g_m R, what is the incremental voltage there?

Ok, let us go back here, ok. So, this voltage is, I mean if this voltage was V_y - V_T - ΔV, what will be the incremental voltage here? -g_m R (V_x - V_y), correct. So, that will give you the negative of, it will give an amplifier, here is an amplified version of the negative of V_x - V_y, ok. So, now, if you want a positive version, what will we do?

I mean notice the stuff below, it is perfectly symmetric, right. So, basically instead of putting the resistor on the left side, if you put it on the right side, then what will you get? You put on the right side what will you get? Now what is the difference between? So, let us say you just saw this part, what is the difference between that guy and this guy here? We are just interchanging x and y, correct, do you understand? So, if you take the output at the drain on the right-hand side, you will basically get something which is proportional to negative of V_y - V_x which is simply the inverted version of V_x - V_y, is this clear? So, as the intuition behind this, how we are able to take only an amplifier, only the difference between V_x and V_y, ok.

So, if V_x and V_y both go up by 1000 volts, alright, as long as you are able to keep all the transistors in saturation, nothing in principle should happen to the incremental output because the transistor only reacts to difference between V_x and V_y, is that clear, ok. So, now, let us start doing careful analysis, now a little bit of notation. So, V_x and V_y are expected to be very close to each other, I mean they will be same only if the gain of the amplifier is, of the operational amplifier is infinite, in reality the gain is you know never infinite, it's only a very large number so, that V_x are and V_y are going to be different, but the difference is going to be small. So, it's more convenient to express the outputs, the inputs V_x and V_y as $(V_x + V_y)/2$ + $(V_x - V_y)/2$ and $V_x + V_y)/2 + (V_x - V_y)/2$ ok.

So, instead of saying 10.01 volts and 10 volts you say, well 10.05 volts is the average of the 2 and 1 is a small amount more than the average, the other one is small amount less than the average, ok and this has got nothing to do with circuits, this is just a way of expressing 2 numbers in a different form, ok. So, this is called, I mean this is the part which is common to both V_x and V_y.

So, this is called the common mode component, ok, which is often called V_{cm} and this is the differential mode component which is called V, we are going to, I mean you can call it V dm, but I drop the m. So, this is nothing but $V_{cm} + v_d$, we deliberately write the differential mode as in smaller cases just to keep reminding ourselves that the difference between the average can be anything, right, but the difference is always small, ok and this is $V_{cm} - v_d$, alright. So, in other words this is $V_{cm} + v_d$, this is $V_{cm} - v_d$, alright and what we are interested in is how the voltages say here and here depend on v_d, ok, we already saw that it does not depend on V_{cm}, it only depends on v_d. So, well you know. So, I mean if you think about it as a circuit problem, circuit analysis problem v_d is a small signal, right and one side it is $V_{cm} + v_d$, other side it is $V_{cm} - v_d$ to find the response to v_d, what do we do? v_d is a small signal, here is the non-linear circuit, what will we do?

What are we trying to do? First job is to find the operating point. So, to find the operating point what will you do? You say you set the small signal component to 0, correct and then find the node potential.

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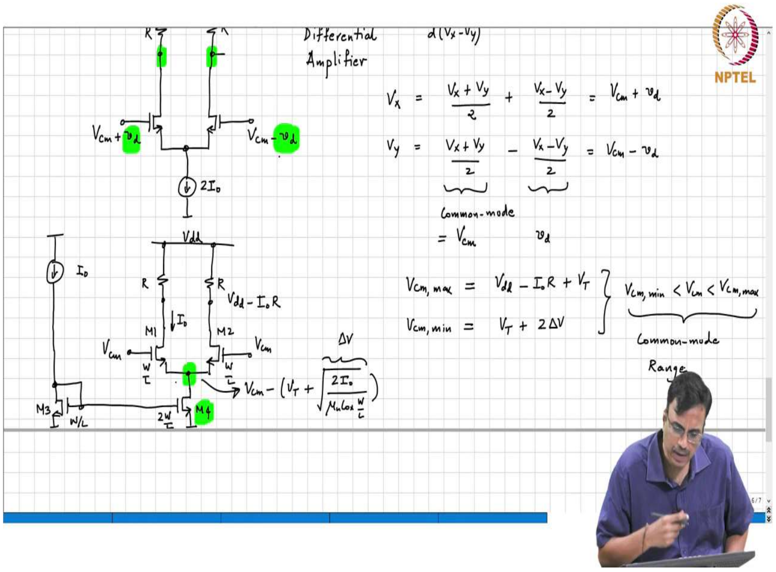

$d(V_x - V_y)$

$$V_x = \frac{V_x+V_y}{2} + \frac{V_x-V_y}{2} = V_{cm} + v_d$$

$$V_y = \underbrace{\frac{V_x+V_y}{2}}_{\substack{\text{Common-mode} \\ = V_{cm}}} - \underbrace{\frac{V_x-V_y}{2}}_{v_d} = V_{cm} - v_d$$

$$\left.\begin{aligned} V_{cm,max} &= V_{dd} - I_oR + V_T \\ V_{cm,min} &= V_T + 2\Delta V \end{aligned}\right\} \underbrace{V_{cm,min} < V_{cm} < V_{cm,max}}_{\text{Common-mode Range}}$$

So, basically the operating point calculation is very straightforward, this is some capital V_{cm}, this is $2I_o$, this is R, R and V_{dd}. So, to find the operating point, I mean what is the first thing that strikes you when you look at the circuit? It is symmetric, right? So, basically what will be the current flowing through M1?

So, the current flowing here is I_o. So, what will be the. So, what is the quiescent voltage of the drains? V_{dd} - I_o R, what is the quiescent voltage at the source? V_{cm} - V_{GS}. Which is V_T + ΔV, which is basically square root of $2I_o$ by that μ_n C_{ox} blah blah blah, is that clear? Ok. So, first question, what is the largest V_{cm} that will make sure that the transistors M1 and M2 are in saturation? By the way, why do we want them, why do we want M1 to and M2 to operate in saturation?

You will get a large gain, incremental gain. So, the maximum, so; obviously, as you keep increasing V_{cm}, right, are the transistors M1 and M2 going deeper into saturation or closer towards the triode region. Closer towards the triode region because the drain potential is remaining constant, the gate potential is going on increasing. So, they are getting closer and closer to the triode region. So, there must be a maximum input common mode voltage you can use before the transistors go into the triode region. So, what is that V_{cm} max? It is V_{dd} - I_o R + V_T and how are we going to realize this current source?

Student: current mirror.

Current mirror with what? So, you put a, alright. So, for example. So, here is you know M3, M4, let me call this 2 I_o alright, this I will call ΔV, we call this W/L, this is 2 W/L. So, when V_{cm} is becoming lower and lower, what happens? The drain potential of M4 is $V_{cm} - V_T - \Delta V$, correct. So, as M as V_{cm} goes on decreasing, M1 and M2 are ok, but M4 is going towards the triode region. So, what is the. So, as you keep lowering the input common mode voltage which the transistor gets into, it has the potential to get into trouble with M4. So, what is the minimum common mode voltage that you can use before something bad happens?

It is ΔV, is that clear? Ok. So, this is ΔV. So, then what is the minimum common mode voltage, V_{cm}, the source voltage is at ΔV. So, what must the gate voltage of M1 and M2 be so that the source voltage is at ΔV.

So, the gate source voltage of M1 and M2 is nothing but $V_T + \Delta V$. So, the source is at ΔV. So, what is the minimum common mode input voltage? $V_T + 2\ \Delta V$. So, therefore, the actual V_{cm} must be greater than a certain minimum and less than a certain maximum, this is called the common mode, range alright, ok. So, if the input common mode voltage is in the common mode range, right then both M1 and M2 are guaranteed to operate in saturation, ok.

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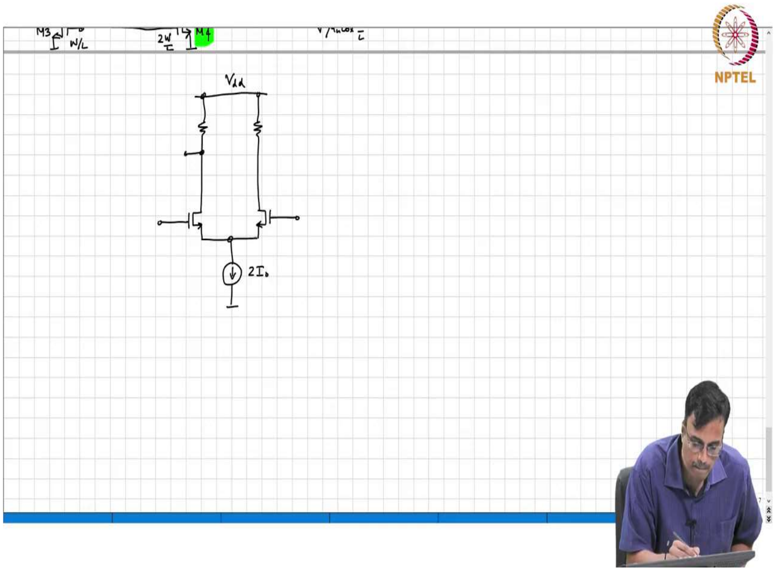

So, that basically is the, we have done, before we are done with the operating point. So, let us revert to our original.