

Analog Electronic Circuits
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Lecture - 35
Effect of Finite Output Resistance on the Basic Building Blocks - Part 3

(Refer Slide Time: 00:22)

Lecture 17

CCCS

$$R_{out} = g_m r_o R_s + r_o + R_s$$

Alright, good morning, welcome to Analog Electronic Circuits this is lecture 17. In the last class, we were looking at the current controlled current source and we found that when the transistor has got a finite output resistance r_o the output resistance looking in. What is R_{out} ? It is basically $g_m r_o R_s + r_o + R_s$.

Now, today let us do the following : the last thing standing is to figure out what the input impedance is.

(Refer Slide Time: 01:41)

When $R_L = 0$, $R_{in} = \frac{1}{g_m} \parallel r_o = \frac{r_o}{1 + g_m r_o}$

$V_{test} = \frac{V_x}{1 + g_m r_o}$

$V_{test2} = \frac{i_{test} r_o}{1 + g_m r_o}$

$V_{test} = \frac{i_{test} R_L}{1 + g_m r_o} + \frac{i_{test} r_o}{1 + g_m r_o} = \frac{R_L + r_o}{1 + g_m r_o} \cdot i_{test}$

$R_{in} = \frac{R_L + r_o}{1 + g_m r_o} \approx \frac{R_L + r_o}{g_m r_o} = \frac{R_L}{g_m r_o}$

So, earlier we have seen what should happen when R_L is 0. When R_L is 0, what comment can you make about R_{in} ? We know this already. It is $1 / g_m$ parallel r_o , which is basically $r_o / (1 + g_m r_o)$. Now, the question is what is the difference between this result and what we are looking for? Now, we have?

Student: R_{in} .

So, how will we find R_{in} ? You can either put a voltage source to measure current or current source to measure voltage. What do you think is more appropriate in this case?

Student: Current source.

Current source: why? The sun rises in the east, you can apply KCL, you can apply KVL, you can apply both. All these are correct statements, but now why does it make sense to push? I mean, I claim that it is better to push a current and measure the voltage rather than put a voltage and measure the current. In both cases, finally, if you have done the math right, you will get the same answer. What does it make sense to do in this case? When does it make sense to push current and measure the voltage as opposed to apply voltage and measure the current?

Student: When things are in series.

What is in series here with what?

Student: R_L .

R_L is in series with that transistor right. So, it makes more sense to put a current source and measure the voltage, right? And we can do that, but we can also do another thing. Using our analysis, so that we did yesterday. So, this is i_{test} . So, what is without any doubt regardless of the value of g_m and r_o , what is the voltage across R_L ?

Student: $i_{test} cR_L$. There is no doubt about this at all, correct? So, basically yesterday we did the following. If we put a V_x here and we did not we had left that R_s as I mean that resistance in the source was infinite. What if I call this V_{test1} , what is V_{test1} ? We saw this yesterday, so please do not annoy me and then stare at me as if you have never seen this before. This is nothing but $V_x / (g_m r_o + 1)$. We have also seen this other situation where what is V_{test2} ?

Student: i_{test} into $1/g_m$.

It is simply the current pushing in multiplied by the looking in impedance and what is looking in impedance? It is right staring at you right in the face, right. It is basically $i_{test} \times 1/g_m$ parallel r_o which is $i_{test} r_o / (1 + g_m r)$.

Given that you know these two results can we basically say what we test this?

Student: $i_{test} R_L$.

So, this basically $i_{test} R_L$ because this voltage is nothing but?

Student: $i_{test} R_L$.

Remember that you can always remove a component and replace it with a voltage source whose value is exactly the same as they drop across that component. What theorem is that?

Student: Substitution.

Substitution, right. So, that is basically $i_{test} R_L / (1 + g_m r_o) + i_{test} r_o / (1 + g_m r)$. So, this is nothing but R_L plus r_o over 1 plus $g_m r_o$. Does that make sense? Sanity check. So, what is the input resistance therefore? What is the input resistance? $R_L + r_o / (1 + g_m r_o)$, sanity check. $R_L = 0$ we must get back $1/g_m$ parallel R_o which evidently is true. Any other Sanity check is if r_o tends to infinity, R_n must be equal to $1/g_m$. Why does it make sense?

If r_o tends to infinity the expression tells us that the input resistance is $1/g_m$. Now, question I am asking you is why does it make intuitive sense? That is I mean there is no current in r_o and

the answer is $1 / g_m$ is basically saying that this is the equation I am going to describe the equation to you in words. Right, that is not what I am asking for. Why does the result make sense? That is the key point is that if the output impedance r_o is infinite, then whatever happens at the drain has no influence on the drain potential, has no influence on the current in the transistor period, correct? So, evidently the voltage of the source cannot be dependent on whatever you do at the drain like if you put a voltage source on a resistor. I do not know, put an elephant, it does not matter to you the potential at the source remains unchanged, right.

So, whatever you put in the drain circuit if r_o is infinite has absolutely no influence on what happens at the source which is why it makes sense that what is the impedance in the drain does not matter as far as R_n is concerned if r_o is infinite. If r_o is not infinite, then what is this formula telling us? It depends on?

Student: Drain.

Whatever you do at the drain right and I can basically this is if $g_m r_o$ is much larger than 1 which is typically true, then what do you expect to see? How does it simplify to if $g_m r_o$ is much much larger than 1? R_L / g_m does not even have the dimensions of resistance. What does this reduce to if $g_m r_o$ is much much larger than 1. What can you neglect here? $R_L + r_o / g_m r_o$ which is nothing but $A + B/C$ is nothing but $A / C + B / C$ is so difficult to do $R_L / g_m r_o + 1$. Does it make sense? Right. And so basically, we see that if you put if there is a resistance R_L in the drain when looking into the source it is going to be divided by this factor $g_m r_o$ ok and that is in addition to the $1 / g_m$ that you get ok. A common misconception is that their impedance is always equal to $1/g_m$. A very easy way of debunking that myth is basically to see what happens when R_L is made infinitely, right. If you open the drain what is the input impedance?

Student: Infinity.

Infinity why?

Student: No current.

With no current it can flow. Right. So, clearly the looking impedance must depend on the drain potential I mean and must depend on what you do at the drain, and all that the transistor

is doing is making sure that the drain potential the drain impedance when looked at from the source is reduced by this factor $g_m r_o$. Is this clear? Alright.

(Refer Slide Time: 12:09)

The whiteboard contains the following equations:

$$V_{test} = \frac{i_{test} R_L}{1 + g_m r_o} + \frac{i_{test} r_o}{1 + g_m r_o} = \frac{R_L + r_o}{1 + g_m r_o} \cdot i_{test}$$

$$R_{in} = \frac{R_L + r_o}{1 + g_m r_o} \approx \frac{R_L + r_o}{g_m r_o} = \underbrace{\frac{R_L}{g_m r_o}}_{\beta r_o} + \underbrace{\frac{1}{g_m}}_{r_{in}}$$

The NPTEL logo is visible in the top right corner of the whiteboard. The lecturer is visible in the bottom right corner of the video frame.

So, with that we are done with the 4 controlled sources.