

Analog Electronic Circuits
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Lecture - 34
Effect of Finite Output Resistance on The Basic Building Blocks - part 2

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Let us quickly revise what we were doing in the last class, where we concluded that the incremental equivalent of a mass transistor operating in saturation is not simply the simple-minded voltage controlled current source that we have been using so far. So, this is $g_m v_{GS}$, but also has r_o .

And r_o is basically $1 / \lambda I_D$ and just like the values of all the elements in the incremental network, they all depend on the operating point right, in this case capital I_D and of course, the properties of the device ok. And, then we started looking at all our circuits that we have looked at so far and examining the effect of output impedance on the performance of these circuits right.

And so, we will look at the voltage-controlled voltage source of the common drain amplifier. And, we found that well the effect is benign is just that this r_o appears in parallel with?

Student: r.

With the load resistance and therefore, all the formula involving R_L will just simply you have to replace R_L with R_L parallel with r_o . Then, we looked at the voltage controlled current source that is what we were looking at and with the voltage controlled current source, where this is v_i and this is R and this is r_o . One thing that we noticed is that the output resistance that you see this R_o is now, what is it now?

Student: $R \parallel g_m r_o + r_o + R$.

Sanity check if our R is 0, then we should see small r_o or small r_o that corresponds to the output impedance of the transistor. If R is infinite, what do you expect to see? It must become infinite because there is no place for current to flow alright. And, if small r_o is infinite, again we should expect to see infinity simply because there is no correction between the drain and the source.

So, basically the drain current is independent of the drain potential. And, an important simplification is if what you call $g_m R$ is much much larger than 1 and of course, $g_m r_o$ is also much much larger than 1, that is you know that is usually satisfied for a transistor operating in saturation. Then, it follows that the output resistance R_o is simply $g_m r_o R$. alright.

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The slide contains two circuit diagrams and their associated equations. The top diagram shows a transistor with a load resistor R and an output resistance r_o . The bottom diagram shows a similar circuit but with an input voltage v_i and an output voltage v_y . The equations derived are:

$$R_o \approx g_m r_o R \quad \text{if } g_m R \gg 1, g_m r_o \gg 1$$

$$v_y = \frac{g_m (R \parallel r_o)}{1 + g_m (R \parallel r_o)} v_i + v_x$$

NPTEL logo is visible in the top right corner of the slide.

The next thing that I would like to draw your attention to is what happens to the potential at the source. So, this is v_i , this is r_o ok. So, if we apply v_x there, I mean to find r_o what did we do? We set v_i to, how do we find r_o ?

Student: v_i .

We set v_i to 0, applied v_x and found.

Student: Current.

Found the current flowing. So, if we want to find the source potential let us call that v_y incrementally, what will it be? I mean if r_o is infinity, the question I am asking now is what is v_y ? Ok. So, first simple things first, if r_o tends to infinity, what comment can you make about v_y ?

It will be $g_m R / (1 + g_m R) v_i$.

And, why is that? Because, you know v_x evidently does not, if r_o tends to infinity, then v_x does not influence the current in the transistor which therefore, means that it cannot influence the potential at the source because the current in the transistor is what flows to the source right ok.

Now, since v_y basically needs to be found when r_o is not infinity, well the easiest thing to do is to use; how will you find v_y , in general when r_o is not infinite? So, if so, use superposition so, v_y is both a linear function of v_i and v_x . So, with v_y v_i you should be able to basically $g_m(R/r_o)/(1 + g_m(R/r_o)) v_i$ + plus *something into* v_x . It is something into v_x that we should be trying to find ok, alright. So, what is that something into v_x ? How will you figure that out? Well, you set v_i to 0.

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So, this becomes, what is the value of that current source?

Student: $g_m v_y$.

So, well we write we know what v_x , what is the current flowing there? We know this already. You do not have to do all the analysis that you did earlier, we know what that current flowing is, what is that current flowing? v_y/r_o .

Ok, I mean that is v_y is unknown in terms of known quantities what; we have already done this analysis.

Student: $v_x R / (r_o g_m R + R + r_o)$.

The current flowing there is? That is what R_o is known for. But we have done it just now and basically the output resistance is known to be the capital R_o which is the long form is this character there correct. So, the current flowing is? V_x/R_o .

R_o , where does that current flow through the resistor R . So, what comment can you make about v_y ?

It is nothing but $R/r_o v_x$.

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if $\beta_m R \gg 1, \beta_m r_o \gg 1$
 $\Rightarrow R_o \approx \beta_m r_o R$

What is v_y ? $\Rightarrow r_o \rightarrow \infty, v_y = \frac{\beta_m R}{1 + \beta_m R} v_i$

$v_y = \frac{\beta_m (R || r_o)}{1 + \beta_m (R || r_o)} v_i + \frac{R}{\beta_m r_o + R + r_o} v_x$

$R \rightarrow \infty$

$\frac{v_y}{v_x} = \frac{1}{1 + \beta_m r_o} \approx \frac{1}{\beta_m r_o}$

What is r_o ? It is $\beta_m R r_o + R + r_o$. If this is v_i and the transistor has got some β_m and some r_o , our analysis shows that V_y has v_y is given by this expression. This part makes sense because it is simply the gain of the voltage follower.

Now, let us take a minute to understand. I mean with you know this we had the expression right. Again, let us do simple things first. Let us know if R tends to infinity what is the expression we get? So, in other words v_y by v_x becomes.

If R tends to infinity, in other words the source is open, what do we get?

Student: $1/\beta_m r_o$.

$1/\beta_m r_o$. Now, why does it make intuitive sense? It does not make intuitive sense.

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The slide shows a circuit diagram at the top left with a voltage source v_i and a resistor R in series. The output voltage is v_y . To the right, the equation is $v_y = \frac{g_m(R||r_o)}{1+g_m(R||r_o)} v_i + \frac{R}{g_m r_o + R + r_o} v_x$. The first term is circled in green. Below it, the approximation $\frac{v_y}{v_x} = \frac{1}{1+g_m r_o} \approx \frac{1}{g_m r_o}$ is shown. At the bottom, a transistor circuit is shown with gate voltage V_g , drain voltage $V_D + v_x$, and source voltage $V_S + v_y$. A current source I_{ref} is connected to the source. The NPTEL logo is in the top right corner.

So, in other words in English what this means is that if I take a transistor operating in saturation and let us say there is some gate potential, absolute potential; not incremental potential mind you. And, let us say this was carrying some current I_{ref} and let us say the drain potential capital V_D was such that the transistor is operating in saturation. So, there will be some source potential V_S .

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The slide is similar to the previous one but with a small change in the drain voltage. The transistor circuit diagram now shows the drain voltage as $V_D + \Delta V$ and the source voltage as $V_S + \frac{\Delta V}{g_m r_o}$. The current source is I_{ref} . The equation $\Delta i_D = g_m \Delta v_{gs}$ is written in red. The NPTEL logo is in the top right corner.

Now, if this increased by a small amount of V_D or a small amount ΔV , what comment can you make? What is our analysis telling us? What comment can you make about the source

potential? Yeah. So, basically will it go up or go down? Go up by $\Delta V/g_m r_o$. That is what our analysis is telling us is equivalent to correct ok. The question is why does this make sense? Ok, V_S should increase fine, but how much is V_S increasing by?

We have increased the drain potential by?

Student: ΔV .

But the source potential is increasing not by ΔV , but by a much smaller value. Why does that make sense? See, if you have a transistor, the incremental current in the drain is related to both the how is the incremental current in the drain you know, what is the incremental current in the drain? Is it dependent on both?

Student: V_{GS} .

V_{GS} and V_{DS} . So, Δi_D is nothing but g_m times the change in the gate source.

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The whiteboard content includes the following equations and diagrams:

$$v_y = \frac{g_m(R||r_o)}{1+g_m(R||r_o)}v_i + \frac{R}{g_m r_o + R + r_o}v_x$$

$$\frac{v_y}{v_x} = \frac{1}{1+g_m r_o} \approx \frac{1}{g_m r_o}$$

$$i_D = g_m v_{GS} + \frac{v_{DS}}{r_o}$$

$$0 = g_m v_{GS} + \frac{v_{DS}}{r_o}$$

where $v_{DS} = V_D - V_S$.

Change in the gate source voltage, let me say I should be careful here small i_D is g_m times small v_{GS} , drain source voltage yes. So, what is this? So, the small i_D is $g_m v_{GS} + V_{DS}/r_o$. Is this clear, yes, no?

Student: Yes.

Ok. So, we have gone and changed the drain potential, not the V_{DS} , we have gone only and changed the drain potential, but because there is a current source in the source, what comment can you make about this small i_D ? There is a current source in the source. So, what is the current in the transistor?

Student: Constant.

Remains constant and is equal to capital I_{ref} . So, our act of increasing the drain potential has resulted in no change in the.

Student: Current.

In the current through the transistor. So, a small i_D must be.

Student: 0.

0 is that clear. So, this basically means that $g_m v_{GS}$. What does small v_{GS} represent again?

The change in the gate source voltage plus small v_{DS}/r_o . Where v_{DS} represents the change in the drain source voltage. So, what is the incremental gate, what is the change in the gate voltage? Minus the change in the source voltage. What is the change in the gate voltage?

Student: 0.

0, the gate is at a constant potential.

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The whiteboard content includes the following equations and diagrams:

$$v_y = \frac{g_m(R \parallel r_o)}{1 + g_m(R \parallel r_o)} v_i + \frac{R}{g_m r_o + R + r_o} v_{cs}$$

Below this, a circuit diagram shows a MOSFET with a current source I_{ref} and a resistor r_o at the source. The drain voltage is $V_D + \Delta V$ and the source voltage is $V_S + \Delta V$. The equations shown are:

$$i_d = g_m v_{gs} + \frac{v_{cs}}{r_o}$$

$$0 = g_m v_{gs} + \frac{v_{cs}}{r_o}$$

$$= -g_m v_{gs} + \frac{\Delta V - v_{cs}}{r_o} \leftarrow \Delta V$$

$$\times \frac{\Delta V}{r_o g_m}$$

So, basically this is nothing, but $-g_m$ times the change in the source voltage plus, what is v_{DS} ? The change in the drain voltage which is $\Delta V - v_s/r_o$. So, what is happening? You yank up the source, I mean you yank up the drain potential right, but the potential the current through the transistor is remaining the same ok. So, if you increase the gate the drain potential and if the current has to remain the same, I mean if the source was grounded if you increase the drain potential by ΔV , how much would the current through the transistor increase by?

If the source was grounded or at a constant potential, if you increase the drain potential by ΔV ; how much would the current increase by?

Student: $\Delta V/r_o$.

Now, that current now flows through the impedance at the source right which is basically you know very large and causes the gate potential to increase or decrease. It causes the gate potential, what is that current source doing there in the source? It is maintaining the constant current by virtue of what?

Student: gate source.

It is a part of a negative feedback loop, that is keeping the gate source voltage, it is doing whatever it takes to keep the current constant. So, if I increase the current in the if I attempt to increase the V_{DS} by some amount right ok, what is it doing? It is going and changing the source voltage to do whatever it takes to keep the current unchanged. So, which of these you

know I mean which of these quantities v_{GS} versus V_{DS} which of them has got a larger influence on the drain source current? In a transistor operating in saturation. v_{GS} right and this g_m and $1/r_o$ are basically quantifying the relative influences of v_{GS} and v_{DS} on the drain current correct and the saying that the gain of the transistor $g_m r_o$ is a large number is simply another way of saying that the influence of the gate source voltage on the drain current is $g_m r_o$ times larger than the influence of?

Student: Drain source.

The drain source voltage on the drain current, is this clear? That $g_m r_o$ is just simply a number that quantifies how much more effective v_{GS} is in changing the drain current, when compared to V_{DS} . Ideally of course, that number $g_m r_o$ must be?

Student: Infinite.

Infinite which basically means that V_{DS} has absolutely no influence or g_m I mean v_{GS} is infinitely more influential on the drain current I_D than the V_{DS} correct ok. So, the drain current is like what you people do right, gate source voltage is like you know the influence of your friend right, drain source voltage is the influence of your teachers and your parents ok alright. So, you can see that the influence of your friends is a lot more than your parents can give you biglecture right, but you know your friends say something you know saying let us go to; let us go to a restaurant outside and immediately you are gone right.

So, basically you can see that the influence of the gate source voltage is much more than the influence of the drain source voltage right. And, the further away you are from home, the influence of the drain source voltage becomes smaller and smaller right ok. When you go to America, the 0 influence is right.

So, basically what this equation is telling us is that if I increase the drain source voltage by the drain voltage by ΔV right the current will tend to increase will tend to increase, but negative feedback is kicking the source voltage to keep the current increase to make the current increase to 0 right. When the source voltage increases what is happening?

Student: V_{DS} .

The V_{DS} of course, is there, but that is not that secondary, what is more important is that.

Student: v_{GS} .

The v_{GS} is decreasing and that is a lot more important because the v_{GS} has got a lot more influence on the?

Student: Drain current.

The drain current is more than the drain source voltage. So, if you increase the so, therefore the source potential changes intuitively therefore, must be; what comment can you make? ΔV is the current drain voltage. So, the source potential increases, what comment can we make about the increase in the potential of the source given the fact that the v_{GS} has got a lot more influence on the drain current.

Do you understand the question? We have increased the drain potential right, the current in the transistor is not changing correctly, that is only possible when the source potential.

Increases to compensate for the potential increase in the current due to the increased drain voltage. The question I am asking is intuitively, what comment can we make about the magnitude of the increase in this in the source voltage? Will it be ΔV ?

Student: Less than ΔV .

It will be way less than ΔV because you know due to increase in the potential of the drain source voltage for all practical purposes has increased by how much? If the source potential is changing very little, what for all practical purposes if I increase the drain voltage by ΔV , how much is V_{DS} increased by?

Student: ΔV .

Approximately.

Student: ΔV .

ΔV . So, what is the current increase due to the increased drain source voltage? I mean the drain source voltage will induce a change in current of $\Delta V/r_o$, but that is nullified by a decrease in the v_{GS} .

In the gate source voltage is correct. So, how much must the source voltage increase to nullify a current increase of $\Delta V/r_o$? The drain the V_{DS} here, this V_s is very very small

compared to ΔV that we all agree, because $g_m r_o$ is very very large right. So, basically it takes only a small change in the source voltage to nullify this increase in the drain potential. So, that V_s is very small compared to?

Student: ΔV .

So, this current is approximately $\Delta V/r_o$.

And so, therefore for the V_s say the change in the source voltage needed to nullify this current $\Delta V/r_o$ is given by how much? The change in current is $\Delta V/r_o$ and that divided by g_m must give you the magnitude of the increase in the source voltage needed to nullify this increase in current. So, that is why it makes sense that when R tends to infinity this v_y/v_x is approximately $1/g_m r_o$ ok alright. I mean one thing is to say well you know I did the math and this is the expression I got ok alright, but it is always good to stare at the equations and figure out why those equations make sense alright.

And of course, I mean if I mean and if R tends to infinity of course, that this is definitely true, but if this is also true if $g_m r$ is much larger than 1, which is what you would do to make a good voltage controlled.

Yeah, which is the same as saying. Say If you want to make a voltage controlled current source with the transconductance right is independent largely independent of the properties of the transistor right, then that is only possible when that $g_m r$ is much much larger than 1, under those circumstances you can see that the incremental voltage y will be $v_i g_m r / (g_m r + 1)$, approximately $1/g_m r_o$ times the voltage at the drain, is this clear? Ok. So, that was the discussion about the voltage controlled current source.

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The slide features a whiteboard with a circuit diagram and handwritten notes. The circuit diagram, labeled 'CCVS:', shows an input voltage source v_s in series with a resistor R_S connected to the base of a transistor. The emitter is grounded. The collector is connected to a resistor R and a load resistor R_L in parallel. The output voltage v_o is measured across R_L . The transistor's output resistance is denoted as r_o . Handwritten notes in pink and black ink include the expression $\approx \frac{\Delta V}{V_o}$, and the conditions $g_m R_S, g_m R_L \gg 1$ and $g_m r_o \gg 1$. The NPTEL logo is visible in the top right corner of the whiteboard. Below the whiteboard, a person in a blue shirt is seated at a desk, looking at a laptop.

Now, the current controlled voltage source is and it is R , this is R_L , this is v_o . So, if you want the properties of this voltage controlled current source to be largely independent of the device parameters, what all constraints must be met? Ok. Let us start with an easier question. If g_m tends to infinity, what will be the input resistance? How is the negative feedback loop working? First of all, you recognize that this is a negative feedback loop ok. So, how is the negative feedback loop working? What is it compared to R ok. So, the output voltage does not change irrespective of the load resistance you know provided the g_m is infinite. And therefore, the output resistance is 0, you understand ok alright. So, basically if g_m tends to infinity of course, the input resistance and the output resistance are both 0. And, what is the trans con trans impedance? What is the trans impedance?

Student: R .

So, and if you want to make the trans impedance you know largely independent of the g_m of the transistor, what all constraints must be satisfied?

Student: g_m greater than $1/R_S$.

So, $g_m R_S$ and $g_m R_L$ must be greater than 1.

So, now if we add an r_o , if the transistor had an output resistance r_o ; what comment can we make?

Student: It will be the R_L .

It will be in parallel with R_L . So, is it benign or is it, you know, a serious problem?

Student: No.

Yeah, for all practical purposes we remember that g_m you know if $g_m R_L$ is much much larger than 1. And, in any case $g_m r_o$ is much much larger than 1 because the transistor is operating in saturation. So, to first order output resistance of the transistor has a minor effect or almost no effect ok.

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The slide displays a circuit diagram of a common gate amplifier. At the top, a current source i_b is connected to a resistor R_s . The circuit includes a transistor with an output resistance r_o and a load resistor R_L . The input resistance is calculated as $R_{in} = \frac{1}{g_m} \parallel r_o \approx R_{in}$. The slide also features an NPTEL logo in the top right corner and a small video inset of the lecturer in the bottom left corner.

Now, the last controlled source is the current control current source and again it's mostly a rehash of what we have seen earlier. So, this is the common gate amplifier, this is R_s , this is the output current i_o and now the transistor has got an output resistance r_o . So, what comment can you make about the input resistance? What is it before without r_o ? What is the input resistance without r_o ?

Student: $1/g_m$.

Now it is going to be $1/g_m$ in parallel with r_o . So, what comment can you make if this is a serious problem or if it's not? Which will dominate $1/g_m$ is going to be much smaller than r_o . So, basically the input resistance is for all practical purposes going to remain unchanged, ok.

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What about the output resistance, sorry this is capital R_o and this is i_o . So, what comment can we make about the output resistance R_{out} ? We have done this you know at least; it will be blue in the face is just simply g_m .

Here symbols are different, that is all, what is the output resistance seeing now?

Student: $g_m R_s r_o + R_s + r_o$.

And, again as we basically. So, if $g_m R_s$ is much much larger than 1, then basically the output resistance is $g_m R_s g_m r_o R_s$. So, basically this is a bad current source with an output resistance of R_s by using the current controlled current source, you can make its output resistance look much better and it enhances the output resistance by a factor of $g_m r_o$. Does it make sense? Ok, alright. So, I will leave it as an exercise to figure i_{out}/i_{in} , without doing the math can somebody tell me what the approximate output i_{out} will be or do we anticipate a big change because of that small r_o . That is all I am asking you whether we anticipate a big change due to the? It will not be a big change, because input resistance is roughly the same. So, the input current going in is also going to be the same as it was without that small r_o and so, whatever current goes into the source must come out through the drain right. So, basically i_{out} will be the same approximately $g_m R_s / (g_m R_s + 1)$. So, with that we have completed the analysis of the effect of finite output resistance on the performance of all the four basic series alright.

So, we will continue in the next class.