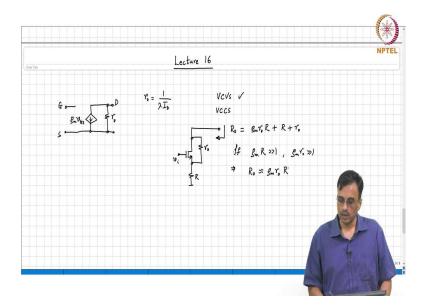
## Analog Electronic Circuits Prof. Shanthi Pavan Department of Electrical Engineering Indian Institute of Technology, Madras

## Lecture - 34 Effect of Finite Output Resistance on The Basic Building Blocks - part 2

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Let us quickly revise what we were doing in the last class, where we concluded that the incremental equivalent of a mass transistor operating in saturation is not simply the simple-minded voltage controlled current source that we have been using so far. So, this is  $g_m$   $v_{GS}$ , but also has  $r_o$ .

And  $r_o$  is basically 1 /  $\lambda$   $I_D$  and just like the values of all the elements in the incremental network, they all depend on the operating point right, in this case capital  $I_D$  and of course, the properties of the device ok. And, then we started looking at all our circuits that we have looked at so far and examining the effect of output impedance on the performance of these circuits right.

And so, we will look at the voltage-controlled voltage source of the common drain amplifier. And, we found that well the effect is benign is just that this  $r_0$  appears in parallel with?

Student: r.

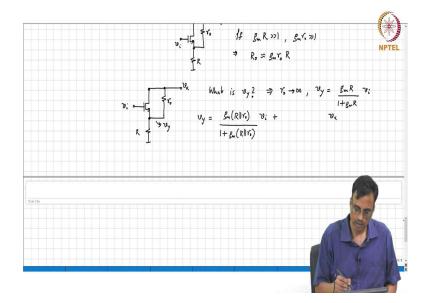
With the load resistance and therefore, all the formula involving  $R_L$  will just simply you have to replace  $R_L$  with  $R_L$  parallel with  $r_o$ . Then, we looked at the voltage controlled current source that is what we were looking at and with the voltage controlled current source, where this is  $v_i$  and this is R and this is  $r_o$ . One thing that we noticed is that the output resistance that you see this  $R_o$  is now, what is it now?

Student:  $R g_m r_o + r_o + R$ .

Sanity check if our R is 0, then we should see small  $r_o$  or small  $r_o$  that corresponds to the output impedance of the transistor. If R is infinite, what do you expect to see? It must become infinite because there is no place for current to flow alright. And, if small  $r_o$  is infinite, again we should expect to see infinity simply because there is no correction between the drain and the source.

So, basically the drain current is independent of the drain potential. And, an important simplification is if what you call  $g_m$  R is much much larger than 1 and of course,  $g_m$   $r_o$  is also much much larger than 1, that is you know that is usually satisfied for a transistor operating in saturation. Then, it follows that the output resistance  $R_o$  is simply  $g_m$   $r_o$  R. alright.

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The next thing that I would like to draw your attention to is what happens to the potential at the source. So, this is  $v_i$ , this is  $r_o$  ok. So, if we apply  $v_x$  there, I mean to find  $r_o$  what did we do? We set  $v_i$  to, how do we find  $r_o$ ?

Student: v<sub>i</sub>.

We set  $v_i$  to 0, applied  $v_x$  and found.

Student: Current.

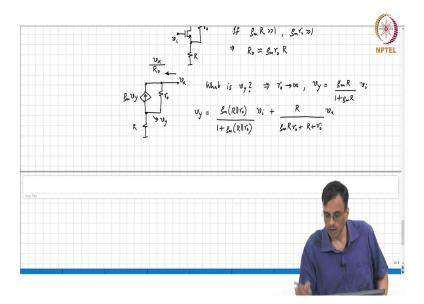
Found the current flowing. So, if we want to find the source potential let us call that v<sub>v</sub> incrementally, what will it be? I mean if r<sub>o</sub> is infinity, the question I am asking now is what is v<sub>v</sub>? Ok. So, first simple things first, if r if tends to infinity, what comment can you make about  $v_v$ ?

It will be  $g_m R/(1 + g_m R) v_i$ .

And, why is that? Because, you know  $v_x$  evidently does not, if  $r_o$  tends to infinity, then  $v_x$ does not influence the current in the transistor which therefore, means that it cannot influence the potential at the source because the current in the transistor is what flows to the source right ok.

Now, since  $v_v$  basically needs to be found when  $r_o$  is not infinity, well the easiest thing to do is to use; how will you find  $v_y$ , in general when  $r_0$  is not infinite? So, if so, use superposition so,  $v_y$  is both a linear function of  $v_i$  and  $v_x$ . So, with  $v_y$   $v_i$  you should be able to basically  $g_m(R//r_o)/(1+g_m(R//r_o))$   $v_i$  + plus something into  $v_x$ . It is something into  $v_x$  that we should be trying to find ok, alright. So, what is that something into v<sub>x</sub>? How will you figure that out? Well, you set  $v_i$  to 0.

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So, this becomes, what is the value of that current source?

Student:  $g_m v_y$ .

So, well we write we know what  $v_x$ , what is the current flowing there? We know this already. You do not have to do all the analysis that you did earlier, we know what that current flowing is, what is that current flowing?  $v_y/r_o$ .

Ok, I mean that is  $v_y$  is unknown in terms of known quantities what; we have already done this analysis.

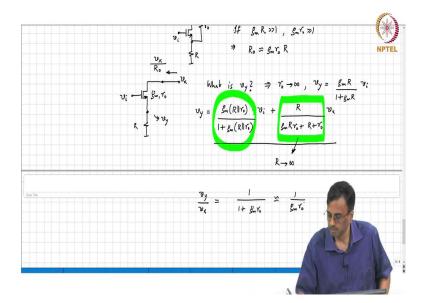
Student:  $v_x R/(r_o g_m R + R + r_o)$ .

The current flowing there is? That is what  $R_o$  is known for. But we have done it just now and basically the output resistance is known to be the capital  $R_o$  which is the long form is this character there correct. So, the current flowing is?  $V_x/R_o$ .

 $R_{\text{o}}$ , where does that current flow through the resistor R. So, what comment can you make about  $v_{\text{v}}$ ?

It is nothing but  $R/r_o v_x$ .

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What is  $r_o$ ? It is  $g_m R r_o + R + r_o$ . If this is  $v_i$  and the transistor has got some  $g_m$  and some  $r_o$ , our analysis shows that  $V_y$  has  $v_y$  is given by this expression. This part makes sense because it is simply the gain of the voltage follower.

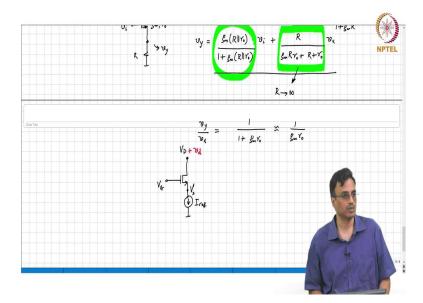
Now, let us take a minute to understand. I mean with you know this we had the expression right. Again, let us do simple things first. Let us know if R tends to infinity what is the expression we get? So, in other words  $v_y$  by  $v_x$  becomes.

If R tends to infinity, in other words the source is open, what do we get?

Student:  $1/g_m r_o$ .

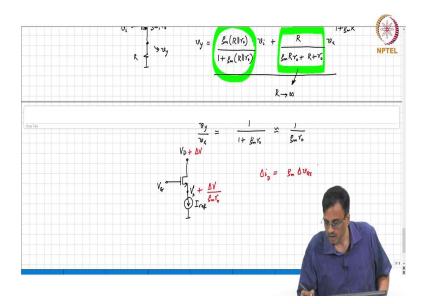
 $1/g_m r_o$ . Now, why does it make intuitive sense? It does not make intuitive sense.

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So, in other words in English what this means is that if I take a transistor operating in saturation and let us say there is some gate potential, absolute potential; not incremental potential mind you. And, let us say this was carrying some current  $I_{ref}$  and let us say the drain potential capital  $V_D$  was such that the transistor is operating in saturation. So, there will be some source potential  $V_S$ .

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Now, if this increased by a small amount of  $V_D$  or a small amount  $\Delta V$ , what comment can you make? What is our analysis telling us? What comment can you make about the source

potential? Yeah. So, basically will it go up or go down? Go up by  $\Delta V/g_m$  r. That is what our analysis is telling us is equivalent to correct ok. The question is why does this make sense? Ok,  $V_s$  should increase fine, but how much is  $V_s$  increasing by?

We have increased the drain potential by?

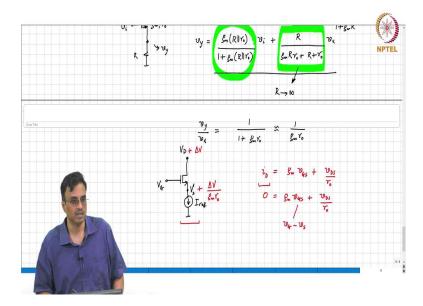
Student:  $\Delta V$ .

But the source potential is increasing not by  $\Delta V$ , but by a much smaller value. Why does that make sense? See, if you have a transistor, the incremental current in the drain is related to both the how is the incremental current in the drain you know, what is the incremental current in the drain? Is it dependent on both?

Student: V<sub>GS</sub>.

 $V_{\text{GS}}$  and  $V_{\text{DS}}.$  So,  $\Delta i_{\text{D}}$  is nothing but  $g_{\text{m}}$  times the change in the gate source.

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Change in the gate source voltage, let me say I should be careful here small  $i_D$  is  $g_m$  times small  $v_{GS}$ , drain source voltage yes. So, what is this? So, the small  $i_D$  is  $g_m$   $v_{GS} + V_{DS}/r_o$ . Is this clear, yes, no?

Student: Yes.

Ok. So, we have gone and changed the drain potential, not the V<sub>DS</sub>, we have gone only and

changed the drain potential, but because there is a current source in the source, what

comment can you make about this small i<sub>D</sub>? There is a current source in the source. So, what

is the current in the transistor?

Student: Constant.

Remains constant and is equal to capital I<sub>ref</sub>. So, our act of increasing the drain potential has

resulted in no change in the.

Student: Current.

In the current through the transistor. So, a small  $i_D$  must be.

Student: 0.

0 is that clear. So, this basically means that  $g_{\scriptscriptstyle m}\,v_{\scriptscriptstyle GS}.$  What does small  $_{\scriptscriptstyle GS}$  represent again?

The change in the gate source voltage plus small  $v_{DS}/r_o$ . Where  $v_{DS}$  represents the change in

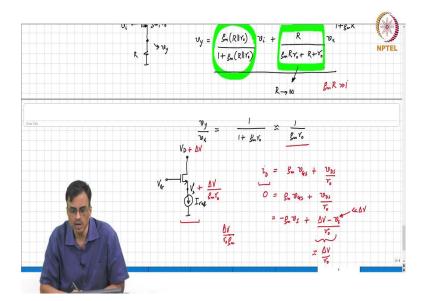
the drain source voltage. So, what is the incremental gate, what is the change in the gate

voltage? Minus the change in the source voltage. What is the change in the gate voltage?

Student: 0.

0, the gate is at a constant potential.

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So, basically this is nothing, but -  $g_m$  times the change in the source voltage plus, what is  $v_{DS}$ ? The change in the drain voltage which is  $\Delta V$  - $v_s/r_o$ . So, what is happening? You yank up the source, I mean you yank up the drain potential right, but the potential the current through the transistor is remaining the same ok. So, if you increase the gate the drain potential and if the current has to remain the same, I mean if the source was grounded if you increase the drain potential by  $\Delta V$ , how much would the current through the transistor increase by?

If the source was grounded or at a constant potential, if you increase the drain potential by  $\Delta V$ ; how much would the current increase by?

Student:  $\Delta V/r_o$ .

Now, that current now flows through the impedance at the source right which is basically you know very large and causes the gate potential to increase or decrease. It causes the gate potential, what is that current source doing there in the source? It is maintaining the constant current by virtue of what?

Student: gate source.

It is a part of a negative feedback loop, that is keeping the gate source voltage, it is doing whatever it takes to keep the current constant. So, if I increase the current in the if I attempt to increase the  $V_{DS}$  by some amount right ok, what is it doing? It is going and changing the source voltage to do whatever it takes to keep the current unchanged. So, which of these you

know I mean which of these quantities  $v_{GS}$  versus  $V_{DS}$  which of them has got a larger

influence on the drain source current? In a transistor operating in saturation. v<sub>GS</sub> right and this

 $g_{\text{m}}$  and  $1/r_{\text{o}}$  are basically quantifying the relative influences of  $v_{\text{GS}}$  and  $v_{\text{DS}}$  on the drain current

correct and the saying that the gain of the transistor g<sub>m</sub> r<sub>o</sub> is a large number is simply another

way of saying that the influence of the gate source voltage on the drain current is g<sub>m</sub> r<sub>o</sub> times

larger than the influence of?

Student: Drain source.

The drain source voltage on the drain current, is this clear? That g<sub>m</sub> r<sub>o</sub> is just simply a number

that quantifies how much more effective  $_{\text{GS}}$  is in changing the drain current, when compared

to  $V_{DS}$ . Ideally of course, that number  $g_m r_o$  must be?

Student: Infinite.

Infinite which basically means that  $V_{\text{DS}}$  has absolutely no influence or  $g_{\text{m}}$  I mean  $_{\text{GS}}$  is

infinitely more influential on the drain current I<sub>D</sub> than the V<sub>DS</sub> correct ok. So, the drain current

is like what you people do right, gate source voltage is like you know the influence of your

friend right, drain source voltage is the influence of your teachers and your parents ok alright.

So, you can see that the influence of your friends is a lot more than your parents can give you

biglecture right, but you know your friends say something you know saying let us go to; let

us go to a restaurant outside and immediately you are gone right.

So, basically you can see that the influence of the gate source voltage is much more than the

influence of the drain source voltage right. And, the further away you are from home, the

influence of the drain source voltage becomes smaller and smaller right ok. When you go to

America, the 0 influence is right.

So, basically what this equation is telling us is that if I increase the drain source voltage by

the drain voltage by  $\Delta V$  right the current will tend to increase will tend to increase, but

negative feedback is kicking the source voltage to keep the current increase to make the

current increase to 0 right. When the source voltage increases what is happening?

Student: V<sub>DS</sub>.

The V<sub>DS</sub> of course, is there, but that is not that secondary, what is more important is that.

Student: v<sub>GS</sub>.

The v<sub>GS</sub> is decreasing and that is a lot more important because the v<sub>GS</sub> has got a lot more

influence on the?

Student: Drain current.

The drain current is more than the drain source voltage. So, if you increase the so, therefore

the source potential changes intuitively therefore, must be; what comment can you make?  $\Delta V$ 

is the current drain voltage. So, the source potential increases, what comment can we make

about the increase in the potential of the source given the fact that the v<sub>GS</sub> has got a lot more

influence on the drain current.

Do you understand the question? We have increased the drain potential right, the current in

the transistor is not changing correctly, that is only possible when the source potential.

Increases to compensate for the potential increase in the current due to the increased drain

voltage. The question I am asking is intuitively, what comment can we make about the

magnitude of the increase in this in the source voltage? Will it be  $\Delta V$ ?

Student: Less than  $\Delta V$ .

It will be way less than  $\Delta V$  because you know due to increase in the potential of the drain

source voltage for all practical purposes has increased by how much? If the source potential

is changing very little, what for all practical purposes if I increase the drain voltage by  $\Delta V$ ,

how much is  $V_{DS}$  increased by?

Student:  $\Delta V$ .

Approximately.

Student:  $\Delta V$ .

 $\Delta V$ . So, what is the current increase due to the increased drain source voltage? I mean the

drain source voltage will induce a change in current of  $\Delta V/r_o$ , but that is nullified by a

decrease in the  $v_{GS}$ .

In the gate source voltage is correct. So, how much must the source voltage increase to

nullify a current increase of  $\Delta V/r_o$ ? The drain the  $V_{DS}$  here, this  $V_S$  is very very small

compared to  $\Delta V$  that we all agree, because  $g_m r_o$  is very very large right. So, basically it takes only a small change in the source voltage to nullify this increase in the drain potential. So, that  $V_S$  is very small compared to?

Student:  $\Delta V$ .

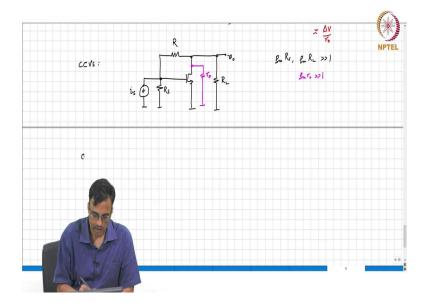
So, this current is approximately  $\Delta V/r_o$ .

And so, therefore for the V<sub>s</sub> say the change in the source voltage needed to nullify this current  $\Delta V/r_o$  is given by how much? The change in current is  $\Delta V/r_o$  and that divided by  $g_m$ must give you the magnitude of the increase in the source voltage needed to nullify this increase in current. So, that is why it makes sense that when R tends to infinity this  $v_y/v_x$  is approximately  $1/g_m r_o$  ok alright. I mean one thing is to say well you know I did the math and this is the expression I got ok alright, but it is always good to stare at the equations and figure out why those equations make sense alright.

And of course, I mean if I mean and if R tends to infinity of course, that this is definitely true, but if this is also true if  $g_m$  r is much larger than 1, which is what you would do to make a good voltage controlled.

Yeah, which is the same as saying. Say If you want to make a voltage controlled current source with the transconductance right is independent largely independent of the properties of the transistor right, then that is only possible when that  $g_m$  r is much much larger than 1, under those circumstances you can see that the incremental voltage y will be  $v_i g_m r/(g_m r + 1)$ ), approximately  $1/g_m$   $r_o$  times the voltage at the drain, is this clear? Ok. So, that was the discussion about the voltage controlled current source.

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Now, the current controlled voltage source is and it is R, this is  $R_L$ , this is  $v_o$ . So, if you want the properties of this voltage controlled current source to be largely independent of the device parameters, what all constraints must be met? Ok. Let us start with an easier question. If  $g_m$  tends to infinity, what will be the input resistance? How is the negative feedback loop working? First of all, you recognize that this is a negative feedback loop ok. So, how is the negative feedback loop working? What is it compared to R ok. So, the output voltage does not change irrespective of the load resistance you know provided the  $g_m$  is infinite. And therefore, the output resistance is 0, you understand ok alright. So, basically if  $g_m$  tends to infinity of course, the input resistance and the output resistance are both 0. And, what is the trans con trans impedance? What is the trans impedance?

Student: R.

So, and if you want to make the trans impedance you know largely independent of the  $g_m$  of the transistor, what all constraints must be satisfied?

Student:  $g_m$  greater than  $1/R_s$ .

So,  $g_m R_S$  and  $g_m R_L$  must be greater than 1.

So, now if we add an  $r_o$ , if the transistor had an output resistance  $r_o$ ; what comment can we make?

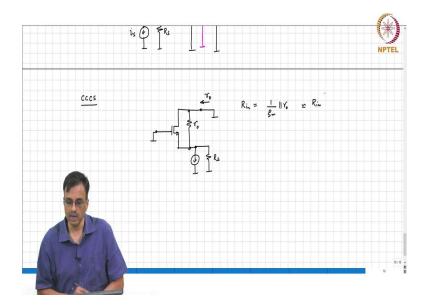
Student: It will be the  $R_L$ .

It will be in parallel with R<sub>L</sub>. So, is it benign or is it, you know, a serious problem?

Student: No.

Yeah, for all practical purposes we remember that  $g_m$  you know if  $g_m$   $R_L$  is much much larger than 1. And, in any case  $g_m$   $r_o$  is much much larger than 1 because the transistor is operating in saturation. So, to first order output resistance of the transistor has a minor effect or almost no effect ok.

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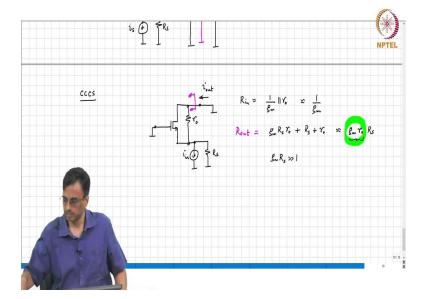
Now, the last controlled source is the current control current source and again it's mostly a rehash of what we have seen earlier. So, this is the common gate amplifier, this is  $R_s$ , this is the output current  $i_o$  and now the transistor has got an output resistance  $r_o$ . So, what comment can you make about the input resistance? What is it before without  $r_o$ ? What is the input resistance without  $r_o$ ?

Student:  $1/g_m$ .

Now it is going to be  $1/g_m$  in parallel with  $r_o$ . So, what comment can you make if this is a serious problem or if it's not? Which will dominate  $1/g_m$  is going to be much smaller than  $r_o$ .

So, basically the input resistance is for all practical purposes going to remain unchanged, ok.

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What about the output resistance, sorry this is capital  $R_o$  and this is  $i_o$ . So, what comment can we make about the output resistance  $R_{out}$ ? We have done this you know at least; it will be blue in the face is just simply  $g_m$ .

Here symbols are different, that is all, what is the output resistance seeing now?

Student:  $g_m R_S r_o + R_S + r$ .

And, again as we basically. So, if  $g_m R_S$  is much much larger than 1, then basically the output resistance is  $g_m R_S g_m r_o R_S$ . So, basically this is a bad current source with an output resistance of  $R_S$  by using the current controlled current source, you can make its output resistance look much better and it enhances the output resistance by a factor of  $g_m r_o$ . Does it make sense? Ok, alright. So, I will leave it as an exercise to figure  $i_{out}/i_{in}$ , without doing the math can somebody tell me what the approximate output  $i_{out}$  will be or do we anticipate a big change because of that small  $r_o$ . That is all I am asking you whether we anticipate a big change due to the? It will not be a big change, because input resistance is roughly the same. So, the input current going in is also going to be the same as it was without that small  $r_o$  and so, whatever current goes into the source must come out through the drain right. So, basically  $i_{out}$  will be the same approximately  $g_m R_S/(g_m R_S + 1)$ . So, with that we have completed the analysis of the effect of finite output resistance on the performance of all the four basic series alright.

So, we will continue in the next class.