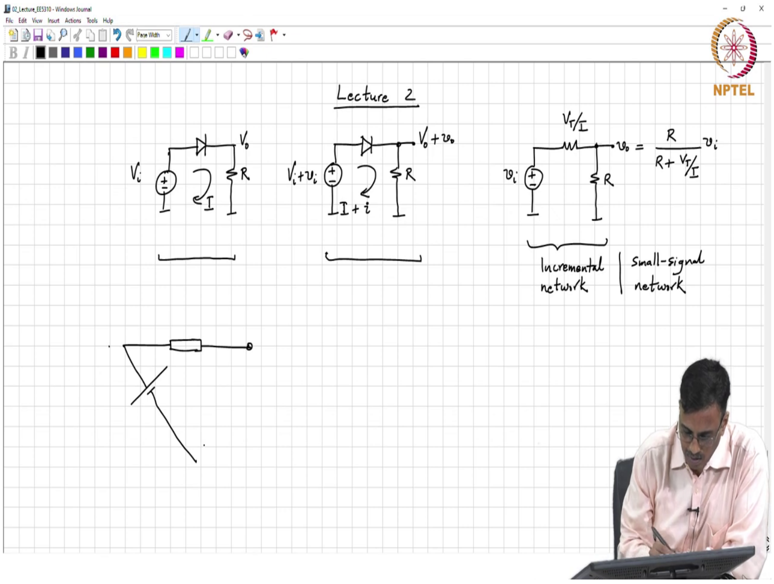
Analog Electronic Circuits
Prof. Shanthi Pavan
Department of Electronics and Electrical Communication Engineering
Indian Institute of Technology, Madras

Lecture - 03
Small Signal Analysis (contd)

(Refer Slide Time: 00:18)



Ok. Good morning, everybody, and welcome to Analog Electronic Circuits. This is lecture 2. In the last class, we were looking at a simple non-linear network, and we concluded that fundamentally finding the branch currents and the branch voltages, and thereby the node voltages in a network which is non-linear is a difficult job, right.

Because you get a system of equations which is non-linear in the network unknowns, namely the branch voltages and the branch currents. And these are also coupled. To solve them you most often have to resort to numerical techniques unless you are happen to be particularly lucky.

Now, the however, we said that once you find the branch voltages and branch currents with a given excitation, right. If you change the excitation by a small amount then the changes in the branch currents and the changes in the branch voltages can be related to each other with the linear equation as we saw in the simple-minded diode example yesterday, right.

And this notion of small signal is was related both to the nature of the nonlinearity, right the function as well as well as the quiescent operating point, ok. Now, so, we saw that for a simple a diode network and what we concluded was that if this was V_1 or V_i and this was our network, this was some R and this was V_o then and if this was I, then when we changed the excitation to $V_i + v_i$.

Then, the resulting output voltage would be $V_o + v_o$. And likewise, the current would be I + i. And by writing out Kirchhoff's Voltage Law around the loop and subtracting the two voltages and so on, we got a network relating only the incremental quantities and assuming the signal v_i is indeed a small signal. We found a linear network relating the change in the branch currents and branch voltages to the change in the excitation, ok.

And this is, since this represents only incremental quantities, this is called the incremental network, also called the small signal network. And from the small signal network being linear is very easy to evaluate, right. And in this case, the v_o is by inspection seen to be simply,

$$v_0 = \frac{R}{R + \frac{V_T}{I}} \cdot v_i$$

Is this clear? And so, what have we done? If you stare at this picture, we have seen that we have replaced every non-linear element by its incremental equivalent, the two-terminal diode. Though non-linear around the operating point, the change in the current through the diode can be related to the change in the voltage across the diode by a linear relationship. And therefore, the as far as changes are concerned, you know di/dv of the diode is a conductance of value I/V_T.

The I is the is related to the quiescent operating point. And the particular nature of this small signal relationship comes about because of the exponential nature of the diode, right. If the diode had some other characteristic, then this formula for small signal resistance as a function of operating point would be different, ok.

Now, this is for a simple network. And remember what we did. We wrote the equations for the original network. We wrote the KVL equation for the network where the excitation has changed. And then how did we get the incremental network?

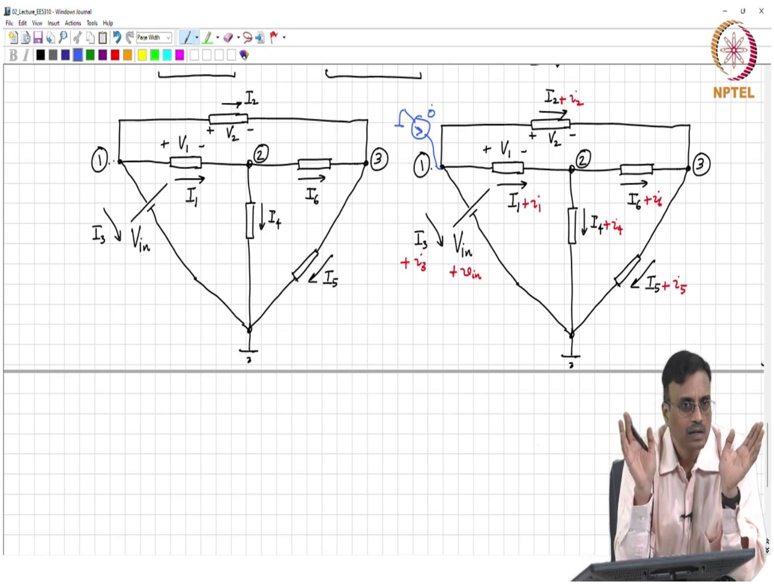
We wrote these equations, we wrote those that equation, and then we subtracted the two equations, right. Then from that after making the small signal approximation, we got an equation which was linear. From that we got the network again, the small signal network, right. It is kind of you know eating like this, right.

You have a network, you write the equations, then you know you have slightly different network, you write the equations again, you subtract the equations, you get another equation. From the equation you go back to the network. It does not seem to make sense to write equations, go from network to equations, back to network.

It seems reasonable that there must be a way of going directly from; I mean given the two networks we should be able to draw the incremental network right away without having the need to write the KCL, KVL equations you know for each network subtract and all that, right.

Rather than subtract sets of equations, it is likely that we will be able to subtract the networks itself. We will see what that means, going forward, so, let us take a generic example. And so, all these elements could either be linear or non-linear. I am just taking you know some picture like this, ok. So, this is let us say V_1 or V_{in}. And let us number these nodes 1, 2, 3, alright. And let us number at least some branch. Let us call this say I_1 and I_2, I_3, I_4, I_5, ok and I 6, ok. So, we numbered the branches.

(Refer Slide Time: 07:27)



So, the voltages across the branches are going to be V_1, V_2 etcetera, ok. I am not labelling all the branches. The idea is that if a current I_5 is flowing through a branch, then the voltage across that branch is V_5, ok, alright. With this background in mind. And these elements could be linear or non-linear.

Now, what am I doing? Well, I am going to basically increase the excitation by a small amount. Consequently, what comment can we make about all the branch currents? They will also change by some amount. So, this current which was I_2 will now become the i_2, this i_4, this i_6, this is i_5, that will make sense, ok.

Now, quick question if I add likewise the node voltages V_1, V_2, V_3 will also change by small amounts. Now, quick question. If I add to this node a current which is 0, what comment can we make about the branch currents and the branch voltages now?

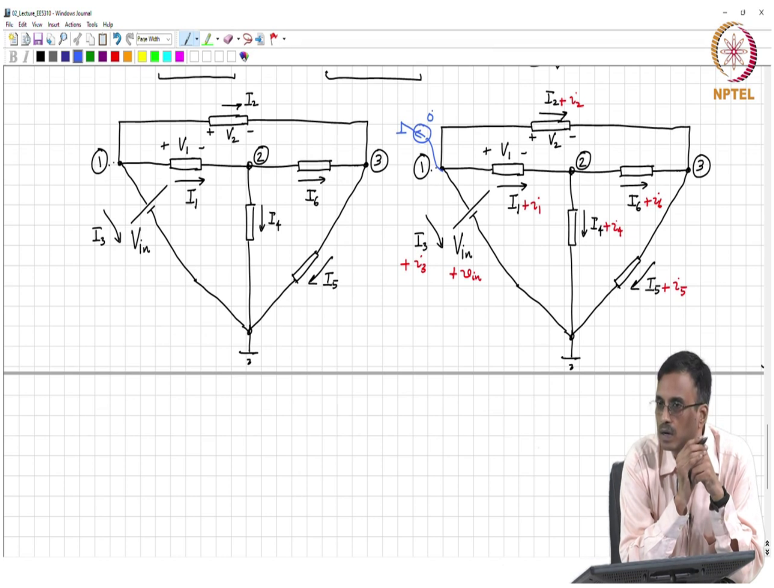
Student: No change.

There is.

Student: No change.

There is no change, correct, alright.

(Refer Slide Time: 11:19)



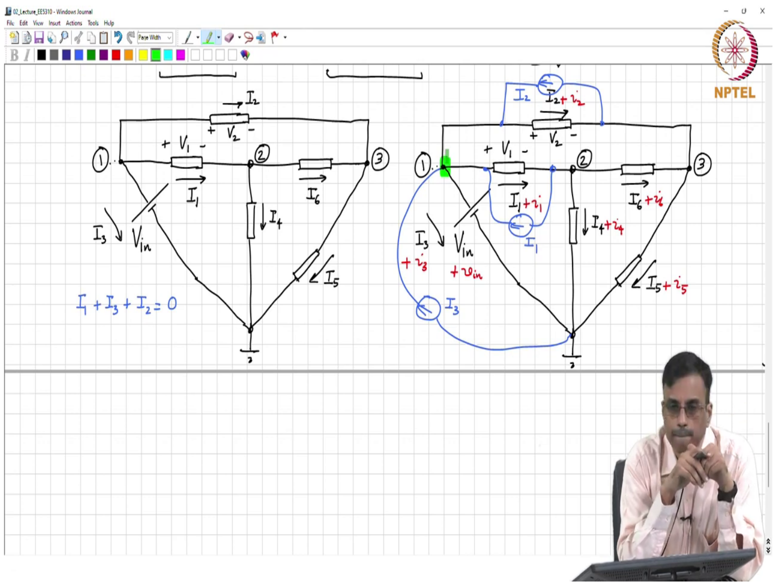
So, now, if I pull current out which is 0 in the opposite direction, what comment can you make?

Student: No change.

No change, very good, ok.

Now, I would like to draw your attention to an observation.

(Refer Slide Time: 11:37)



What comment can we make about $I_1+I_3+I_2$?

Student: 0.

That must be 0 because of.

They must satisfy.

Student: Kirchhoff's Current Law.

Kirchhoff's Current Law, ok.

Now, watch carefully as I do this. I am going to add a current source like this in the opposite direction. I am going to add a current source like this in the opposite direction and likewise, I am going to add a current source like this in the opposite direction. And I am going to do that for?

Student: For all the branches.

For all the branches, ok. Now, what comment can you make about the node voltages and branch currents?

What comment can you make about the current I mean about this node voltage?

Student: voltage will reduce.

Why?

Student: voltage.

Well, there must be only 3 answers, either the voltage will reduce or the voltage will increase or the voltage will remain the same.

Student: Remain the same.

Remain the same, why?

Student: 0 point.

Why do you say the voltage will increase?

Student: node the current is being pushed.

No. How is the current being pushed into node 1, net current being pushed into node 1?

Student: apart from that network.

No, no, what is the extra current that I have pushed into node 1?

Student: some sources I_3, I_1 and I_2.

Yeah. So, I have added 3, what is the net current that therefore, going into node 1?

Student: $I_1+I_3+I_2$.

And what is $I_1+I_3+I_2$?

Student: zero.

It is 0. So, so there must be.

Student: No change.

No change, simply because at every node I am injecting a.

Student: 0 current.

0 current, right. And how would I get that 0 current? I mean how are these I_1 and I_2 and I_3 currents derived, are they arbitrary? Are they arbitrary or what?

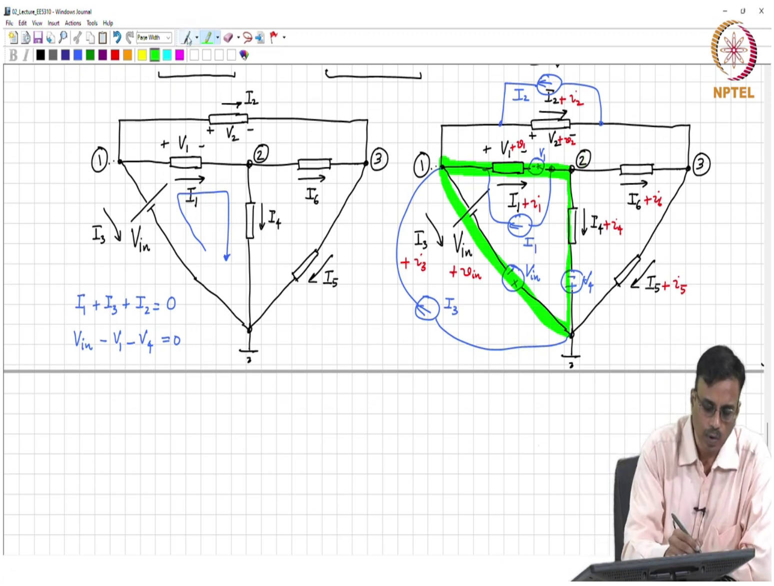
Student: They are the they are the currents.

They are the they are the currents in the original network which all satisfy Kirchhoff's.

Student: Current law.

Current law. Does this make sense folks? Ok, alright.

(Refer Slide Time: 14:52)

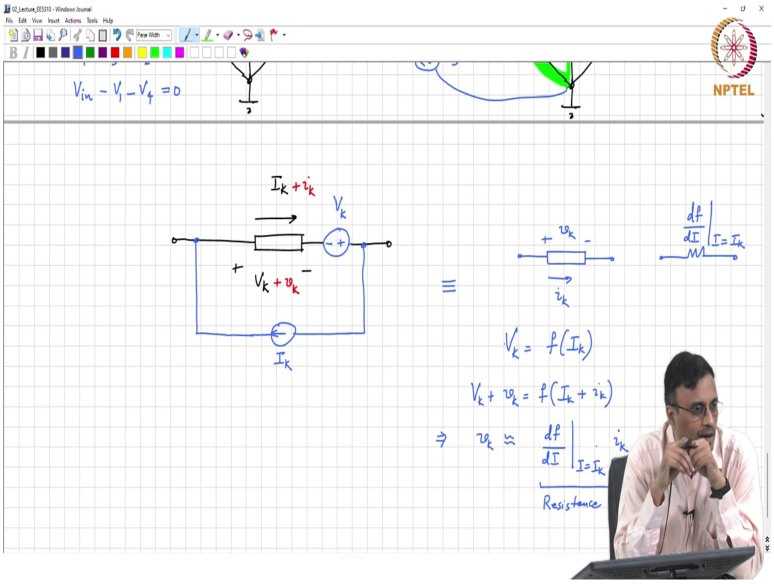


Now, by the same token what I am going to do is to introduce in each branch a voltage which is in the opposite direction as the original, ok. V_{in}, this is V_4 and V_i, alright. So, the voltage here was $V_1 + v_1$, this was $V_2 + v_2$ and so on. So, what comment can you make about the net voltage added in the loop? By the way what is looking at the original network? What comment can we make about $V_{in} - V_1 - V_4$? It is.

Student: 0.

0, because that forms a KVL loop, ok. So, what is the net voltage I have added in this loop? The net voltage I have added is 0 and therefore, there must be no change in the loop currents, correct.

(Refer Slide Time: 16:59)



Similarly, if we do that for every branch, so basically what we are doing is replacing every branch. Therefore, if there was an original branch like this with some $I_k + i_k$ and $V_k + v_k$. What have we done? In parallel with every branch, we have.

Student: We have added a current.

We have added a current, what?

We have added a current I_k going in the opposite direction, alright. And in series with every branch, we have added a voltage source there. What is the value of the voltage source?

Student: V_k.

V_k in the opposite direction, correct. And we just discovered that if we replace every branch with this contraption then KCL and KVL will be satisfied at every node. Does it make sense? Right, ok.

I hope you are able to see the construction. I mean essentially what are we doing?

Basically, we are writing out KCL, KVL equations subtracting the two. Rather than subtract, writing the equations subtract the two, we can do this at the network level, correct, ok. So, now, if you do this, what comment can you make? You can think of this as a branch. What is the voltage across this branch?

Student: v_k.

v_k. And what is the currents of the branch?

Student: i_k.

i_k, alright. Now, does this depend on small signal operation or this is true for arbitrary changes in excitation?

Student: Arbitrary.

Arbitrary, why?

Student: approximate.

Pardon.

Student: approximate.

We have not made any approximation whatsoever. We have just used KCL and KVL and then you know just made some connections and it seems like. So, this is valid for all values of excitation. Does it make sense?

Ok. Now, when we make a small signal approximation, what are we going to do what are we going to do?

If the excitation was a small signal one, what comment can we make now, can we make a further simplification? Yes. What is that simplification?

Student: We can find the relation between V_k and I_k.

Yeah. So, if the signal is a small signal, then an additional thing that you can do is to relate the change in the voltage across this branch to the change in current through this branch by a.

Student: Linear equation.

By a.

Student: Linear equation.

Linear equation. Does it make sense? Ok. And how do we do this? Well, if the V_k is some function of the I_k for instance, then the $V_k + v_k$ is a function of $I_k + i_k$. And therefore, this basically means that v_k is approximately equal to.

$$V_k + v_k = f\left(I_k + i_k\right)$$

$$v_k = \frac{df}{dI}\Big|_{i=I_k} + \ higher\ order\ terms$$

Which we already saw right. Basically will define what it means for the signal to be small signal. But in general if we assume that those higher order Taylor series terms are negligible compared to the first order term, then what are the dimensions of this character?

This has got dimensions of resistance and therefore, this can be replaced by a resistor of value.

Student: df/dI.

df/dI evaluated at $I=I_k$. Does it make sense? We did this in the special case of a diode yesterday, right. But this is now true for I mean its I mean as perhaps not surprisingly, it is true for an arbitrary non-linear network where you have all sorts of a non-linear components.

Finding the operating point is going to be a difficult affair and has to be done numerically. But once you find the operating point if you change the excitation by a small amount, the changes in the branch currents and the changes in the branch voltages can be related to the change in the excitation by.

Student: Linear network.

A linear network, ok. Now, the linear network may take time to evaluate, if the network is big, we know that the linear network will be big, but we know how to evaluate it. It may take time, but we can do it, right. Whereas, with the original operating point calculation, it may simply not be able to do so analytically. You have to go for numerical techniques. Does it make sense? Ok.

So, the bottom line is that what is the advantage, basically from this whole exercise here? Why have we done this exercise?

Student: From the main network.

Pardon.

Student: From the main network.

From the main network of course, you can derive the incremental network. But you know let me kind of a step back a little bit and motivate you know why we are doing this small signal analysis in the first place. We as I already told you we spent a lot of time learning about, about?

You know linear networks and ways to analyze them, correct. And then but now we find that all are networks are non-linear, that basically means all that studying is apparently a complete waste, right, ok.

So, I mean there are two ways to go about it. One is to say I mean our previous teachers have pray you know played a cruel joke, right. Because you know they have taught us what they know, but you know unfortunately it looks like none of it is applicable in practice because nothing in the world is linear, correct, ok.

Then, the other approach is to say, yeah maybe that is so, but let us try to make the best of it, correct. So, in general, you know analyzing a non-linear network is impossible with the tools we have.

But let us try and figure out if we can at least come up with a scenario where the tools we have learnt will be useful, and that is we say now that, ok, well let us give up on the fact that finding the operating point you know analytically we give up on it, right. We go to a computer.

However, having found the operating point, we are able to relate changes through a linear network. That means, as far as incremental signals are concerned, we can come back, we can go back to all our linear analysis tools, right. Fourier transforms, Laplace transforms, except with the small caveat that they only apply for?

Student: Small signals.

Small signals, right, ok. So, this is like you know let us say your you have an English exam and you know an essay will be on cow, ok. So, all night you open a guide and then you know mug up essay on cow, ok. So, next day you go to the exam and then suddenly he says you know the essay or you write the essay on the aeroplane. So, what do you do? One thing is to say, well I do not know anything and then that is the honest thing to do, correct?

But if you are well your at least studied all this, you might as well use it. So, what do you do? You say in the essay you say well there was a plane I got onto the plane and then the plane took off and then I looked out of the window and there was a meadow and there was a cow. And then I write the essay on the cow, right? So, this is like that.

Well, you kind of you know you either say you know we pack up and leave, right or we make the best of. Basically, if you think about your undergraduate courses, right at least half your courses are all dedicated to I mean whether it is DSP or whether signals in systems or whether it is you know linear circuits or whether it is circuit theory. It is all whether you know it or not, you spent all your time learning about tips and tricks for analysing systems that are linear, right? So, you do not want all that to go to waste. That is the bottom line. Is this clear?