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Lecture - 10 Deriving the common v-source amplifier - part 1

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Good morning, and welcome to Analog Electronic Circuits, this is lecture 5. So, in the last class we were looking at what one needed for a non-linear device to be able to get incremental gain. And after a long discussion we concluded that the incremental Y matrix must be such that? What should be the incremental Y matrix?

The first row must be 0, Y_{11} and Y_{12} must be 0, Y_{22} must be 0 and this must be as large as possible. And from this incremental matrix we went and tried to construct what the characteristics of the device itself must be, in order that it will have an incremental matrix like this, Okay. And we concluded that if that input port current $I_1 = 0$, right, ideally all that it needs to do to ensure Y_{11} and Y_{12} are 0 is that they must be a constant. That input current I_1 must be a constant, a good value for that constant 0 and it so happens that many devices actually have $I_1 = 0$.

So, if that is the case then we are in a situation where we can plot the output characteristics and we said that because the device itself has no sources of energy inside. It must follow that the output characteristics cannot remain parallel to the x-axis, for all values of V_2 and they must eventually dive into the third quadrant. So, basically you will get therefore, in principle you will basically get a family of curves one for each value of for each value of V_1 , Okay.

And one of the devices that exhibits this kind of behavior is the n channel MOSFET. Enhancement mode MOSFET, Okay. And as far as we are concerned, we are only interested in the I-V characteristics of this box and so, this is the drain this is the gate and this is the source.

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And we said that if $V_{GS} \leq V_T$ then the device is off. If the $V_{GS} \geq V_T$ and $V_{DS} \geq V_{GS} - V_T$, then the current the drain which is the equivalent of I_2 . The drain current is independent of the drain source voltage and depends only on the gate source voltage and is given by,

$$I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \left(V_{GS} - V_T \right)^2$$

Okay.

And consequently, the g_m therefore, can be written in multiple ways the trans conductance which is the proxy for Y_{21} is nothing but,

$$g_m = Y_{21} = \frac{\partial I_D}{\partial V_{GS}} = \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_T) = \frac{2 I_D}{V_{GS} - V_T}$$

This is yet another form, Okay, right. This V_{GS} - V_T as you can see plays a prime role in determining the current through the MOSFET. And this is often what is called the gate overdrive voltage, and why does it make sense to call it the gate overdrive?

Well, any voltage applied across the gate is often called the drive and it is not nearly enough to drive the gate, it must be driven above the threshold voltage. So, the voltage that actually determines the current is not V_{GS} , but V_{GS} - V_T and therefore, V_{GS} - V_T is called the gate over drive voltage.

So, in the triode region where $V_{GS} \ge V_T$, but $V_{DS} \le V_{GS} - V_T$ or $V_{DS} < V_{GS} - V_T$. We find that two things happen first, the trans conductance. What comment can we make about the trans conductance?

It is smaller than that we get in saturation, right? And what is the trans conductance? It is

$$g_m = \mu_n C_{ox} \frac{W}{L} V_{DS}$$

And the output resistance or the output conductance of the MOSFET is g_{ds} ,

$$g_{ds} = \mu_n C_{ox} \frac{W}{L} \left(V_{GS} - V_T - V_{DS} \right)$$

Does it make sense? Okay. So, for a special case you might think that this is not really a useful region of operation. But it turns out that in many cases it turns out that let us say you are operating the device around an operating point of $V_{DS} = 0$, right? or V_{DS} is very very small. So, for V_{DS} very very small meaning much much smaller than $V_{GS} - V_T$. So, what comment can you make in the operating region, where are we operating on that in that picture?

Student: Triode.

We are operating somewhere here, deep in triode then what can we neglect here?

g_{ds} is approximately? What can we neglect?

Student: V_{DS}.

 V_{DS} , so, basically this is

$$g_{ds} = \mu_n C_{ox} \frac{W}{L} \left(V_{GS} - V_T \right)$$

Okay, and what is g_{ds} by the way? By definition what is g_{ds} ? It is the change in current due to a change in the? Student: Drain source V_{DS} .

Drain source voltage, Okay, and so, what is this telling us?

In English what does that what does this equation mean?

Student: Not depending on $V_{DS.}$

Yeah, of course it is not depending on V_{DS} .

So, for a fixed V_{GS} what comment can we make about $\frac{\partial I_D}{\partial V_{DS}}$?

Student: Constant.

It is constant. So, what does that mean?

Student: Resistance.

It is basically a resistance, right? And the value the conductance is determined by.

Student: V_{GS} - V_T

 V_{GS} - V_T So, basically in deep triode this basically is nothing but a voltage-controlled resistor and what is the value of the resistance? It is

$$\frac{1}{\mu_n C_{ox} \frac{W}{L} \left(V_{GS} - V_T \right)}.$$

So, you can think of it as a voltage-controlled resistor, alright. This also brings up a very important property of a MOSFET which we will not cover in this course. But is very useful nevertheless, you can see that when the gate source voltage is less than V_T then how does the device behave between the drain and the source?

Student: Open circuit.

It is an open circuit, right, now when V_{GS} is very large.

Student: It is a resistance.

It is a resistance, right; The value of the resistance can be made as small as possible by making W/L very very large. So, this now you know enables the MOSFET to operate as a switch, right? As a voltage-controlled switch, Okay, which is extremely useful in a whole lot of applications. This is just for your reference, we will not be seeing much of that in this course, alright. So, with this in place our amplifier now looks like this. Remember that this is what we had the device is now has a new symbol, Okay.

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And rather than keep drawing batteries all over the place, you know an easier thing to do is to basically say well. This is R_L this is V_B I will call this not to get confused with capital V_B and this is V_A , this is v_i , this is R_S , alright, does it make sense folks? Alright. Now, the question is what do you call? you can see that one of the you know I would say irritants with this circuit on the, right is the fact that we have two batteries, right, which do you think will be higher?

Student: V_B.

I mean I do not know if you are able to see this. So, let us say assume that you have you are looking at the circuit for the very first time is clearly a non-linear circuit. So, the first thing to do is to find the?

Student: operating point.

Operating point. So, to do find the operating point what will you do?

So, whenever you see a non-linear circuit, the first thing to do is to find the operating point. So, to find the operating point what will you do you?

You set all small signals to zero, correct? if we do that what happens? What do we get?

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We will basically end up with a circuit like this. So, this is V_A , this is some R_S , this is V_B , this is R_L ., Okay. So, how will we find the operating point? What is V_{GS} ?

Student: V_A.

V_A, why?

The gate current is zero. So, the voltage across the gate is simply V_A . So, from the equation of the MOSFET, let us assume that the voltage of the drain is large enough that the transistor is operating in saturation. So, I_D is,

$$I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \left(V_{GS} - V_T \right)^2$$

What is the drain potential?

Student: $V_B - I_D R_L$.

Yeah, so, the drain potential V_D is nothing but, this is the drain, it is nothing but $V_D = V_B - I_D R_L$, and if you want to ensure that the device is indeed in saturation, what comment can we make about the drain potential?

Student: Should be greater than.

Should be greater than or equal to?

Student: V_A - V_T . Does that make sense? Okay. So, now, can you tell me in practice which do you think will be larger, V_A or V_B ? Ideally what comment can we make about V_B ? Should it be very large compared to V_A or should it be very small compared to V_A .

Student: Large.

Large, why?

You want to make sure that the transistor is you know as deep in saturation as.

Student: Possible.

Possible. So, a for all practical purposes therefore, V_B is going to be hopefully much much larger than V_A , correct? So, now, staring at the circuit one can say well you know there is obviously, an inefficiency in the circuit and what is that?

Student: R_L voltage drop.

I mean if you do not have R_L, then you have no load no.

Student: Two voltage source.

I mean we know that V_B is much larger than V_A . So, there is no need to have two voltage sources, one can actually have derive the smaller voltage source from the.

Student: Larger.

Larger one, how do you derive the smaller voltage source from the larger one?

You put a voltage divide, right? One way of doing it is to put a voltage divider.



So, this is typically called V_{DD} , Okay, DD stands for drain and then so, all the drain currents eventually find their way there. So, and so, we will assume that we derive V_A . Let us call this R_1 and R_2 , Okay. So, as far as simply biasing the transistor is concerned, do we need R_S or we do not need?

Student: Do not need.

It if we just needed to bias the transistor and saturation, this R_s is actually coming from the source it is strictly not necessary. And it may not be as we will see going forward that may not be the only location where we can put the source. But as far as biasing the transistor is concerned the R_s is not really necessary. So, this is basically one of the simplest networks you can think of that will make sure that the transistor is biased in.

Student: Saturation.

Saturation and is ready for action, alright, Okay. Now as far as the transistor is concerned therefore, the incremental Y parameters are you know because it is operating in saturation. The incremental model incremental model for the transistor looks like this. This is the gate, this is the source, this is the drain, this is nothing but Gm times the incremental V_{GS}, alright? Okay. Now, you know we need to add the source, Okay. So, the quiescent voltage at this node is $V_{DD} R_2 / (R_1 + R_2)$, and the quiescent voltage at that node is $V_{DD} - I_D R_L$, alright. Now, the transistor is all ready for action. So, what should we make sure that the incremental voltage? At the gate if you want gain what should be the incremental voltage at the gate?

Student: v_i.

We want to make sure that the incremental voltage here is.

Student: v_i.

 v_i , alright?, Okay, and if the incremental voltage there is v_i , what comment can we make about the incremental voltage of the drain? How will we do it? we will need to draw the.

Student: Incremental model.

The incremental model. So, what do we do? We go look at the circuit and replace every element by its incremental equivalent. So, as far as the incremental equivalent is concerned, therefore, you know what comment can you make about V_{DD} ?

Student: Short circuit.

Becomes a short circuit. So, this is R_1 , this is R_2 , this is the transistor, this goes to ground, this goes to R_L correct. And we should somehow make sure that this node, we should apply our incremental input here, Okay. I mean when I draw the circuit like you can connect a MOSFET this up this way and you can actually make the circuit work, right?

What is this a shorthand for? What does it actually mean, what is the implication? I mean can I take a physical MOSFET connect two resistors you know from between ground. R_1 connects to ground on one side and R_2 also connects to ground on one side, can I will the circuit work?

Student: No.

No, right, why?

So, when somebody draws a circuit like this on the left-hand side, the implication is that you have to mentally replace that MOSFET by its?

Student: Model.

Incremental model, which is this character here, Okay. So, what this means is that as far as the incremental network is concerned, R_1 and R_2 are in.

Student: Parallel.

Parallel. We will somehow apply the gate voltage v_i there and this is the controlled source, this is R_L , this is v_o , and this is $g_m v_i$. So, what comment can we make about v_o ?

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Student: - $g_m R_L v_i$.

- $g_m R_L v_i$. So, what in this picture, therefore, the total voltage at the drain therefore, is the?

Student: Non-linear.

In a non-linear network, first thing you do is find the operating point then you replace every non-linear element with its incremental model. Then you analyze the incremental network what kind of network is that?

Student: Linear network.

It is a?

Student: Linear network.

Linear network, Right. Once you analyze the linear network you will get all this all the voltages and currents at you know every node and every branch, they all refer to what kind of quantities?

Student: Incremental.

They all refer to incremental quantities. So, that is what we have done here. The incremental voltage of the drain is basically given by - $g_m R_L v_i$, what is the total voltage at the drain?

The total voltage at the drain is basically the quiescent value plus the increment. The quiescent value is V_{DD} minus?

Student: I_D R_L.

 $I_D R_L$. The increment is $-g_m R_L v_i$. So, the total voltage at the drain is $V_{DD} - I_D R_L - g_m R_L v_i$, Okay, alright. So, now, the question is how do we get an incremental voltage at? I mean in other words we want here is our quiescent circuit, here is our circuit which sets up the operating point, for incremental signals we want to make it look like this, correct? So, we want this behavior for incremental signals, correct? For the time being let us assume that the source is ideal, it does not have any output resistance.

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So, how can we make sure that this voltage here also has an incremental component v_i ? The simplest thing I can think of is say, Okay what happens if I simply connect v_i here?

Student: There will be some change in operating point.

Why?

Student: It was like I_D.

I mean what is the incremental voltage of the gate now?

Student: It is the addition of V_{DD} or $V_{DD} R_2 / (R_1 + R_2)$.

Please take a look at this carefully and tell me what is the incremental voltage at the gate? Student: v_i .

v_i, that is what we wanted, now the question is can I simply do this?

Student: No sir.

No, why not?

Student: Because DC.

Yes, Ralph?

Student: Sir, v_i fluctuates more, then this can be also affect the quiescent point.

So what is the quiescent operating point now?

Whenever you add the incremental voltage you must make sure that the quiescent operating point of the non-linear devices is not changed, because you are going to linearize the device about the quiescent operating point, the addition of the incremental source should not change the operating point, correct? So, now, if we did this, what comment can you make about the quiescent operating point. So, to find the operating point what would we do?

Student: short circuit.

We make v_i equal to 0. So, this becomes the?

Student: That becomes the short circuit.

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This becomes the short circuit, Okay. So, what comment can you make about the gate voltage?

Student: Zero gate voltage zero.

Gate voltage is zero. So, then what comment can we make about the transistor's operation.?

Student: Off cut off.

Off, right? $V_T > 0$, Okay, and by the way that is what it means to say an enhancement mode device where $V_T > 0$. So, if we simply stick the small signal voltage at the gate, for sure the gate incremental voltage at the gate is definitely v_i . Unfortunately the quiescent voltage is forget not fluctuating, the quiescent operating point is completely ruined, because the gate where the transistor is cut off. Why is the operating point changing?

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This was v_i , Okay, under quiescent circumstances this node on the left x the quiescent voltage there is? Let us say I do not connect x to let us call this the gate voltage and the gate G, what is the quiescent voltage on the left?

Student: V_{DD} - v_i .

On the left.

Student: v_i actually zero.

It is.

Student: Zero.

Zero, right? Remember for quiescent operating point v_i is 0. And so, therefore, on the left side it is 0, on the, right side it is?

Student: R₂/R₁.

This quiescent voltage is $R_2/(R_1 + R_2) V_{DD}$.

So, what comment can we make about so, obviously, when you short x to G what happens?

Why does the what happens when you short node x to node G?

Student: Violation of the.

Nothing, no violation of anything. What happens?

Student: Current.

The current is being drawn from the node G; and therefore, its voltage will?

Student: Drop.

Will drop, correct? Because the source is ideal it will drop to 0. If the source is not ideal what will happen? Let us say the source had a source resistance of R_s , what will happen?

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Student: Current will flow.

Some current will flow through R_s the voltage will drop.

Student: To minimum.

But it will not drop to 0, Okay. It will drop to R_s parallel R_2/R_s parallel ($R_2 + R_1$) V_{DD} , at any rate the quiescent operating point is going to?

Student: Drop.

Change, right? Which is not intended because you know we bias the transistor at desired operating point. And the act of putting the incremental voltage source at the gate has gone and changed that operating point. Is this clear? alright. So, the root cause as you can see is that the potential of X and the potential of G are?

Student: are not the same.

Are not the same, if they were the same then shortening the two would make.

No difference to the operating point is that clear? So, in principle what should we do between X and D, I mean the potential of X is 0.

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The potential of G is $V_{DD} R_2 / (R_1 + R_2)$. There is a difference between the two, Okay. So, if we short the two, current is going to be drawn, if you want prevent to prevent current being drawn from the gate node, the potential on of X must be?

Student: Same as G.

The same as the potential at G. So, in principle what will we do?

In principle we want to equalize both the potentials, right, if we want to equalize two things we either pull you know the G down or pull, X up.

Student: X up.

Pulling G down is no solution, because that is going to?

Student: Kills the operation.

Kill the transistor. So, the only solution is to pull.

Student: X up.

X up. So, in principle what will we do? We will put a battery of value?

Student: Same.

 $V_{DD} R_2 / (R_1 + R_2)$. This is in principle, right. If we did this then the quiescent operating point is not going to change, correct? And we can go about our incremental analysis, how will the incremental network look like now? on the left side what must we do?

Look carefully, the source has changed now. How will the incremental network look like?

Student: Same as before.

Same as before. So, there is now a source resistance which I do not see in the incremental equivalent. Where will I put that?

Student: put in the v_i.

Yeah. So, because we have an R_s, this will be the incremental equivalent. And therefore, what will be the gain? what will be the incremental gain? What will be the incremental output?

Student: put the v_i will change the voltage R_s and R_1 .

Okay.

So what will be the output voltage now, therefore?

$$v_{o} = \frac{-g_{m}R_{L}v_{i}(R_{1}/R_{2})}{R_{s} + (R_{1}/R_{2})}$$

Okay. So, you know as an aside what comment can we make about $R_1 // R_2$, how should we choose $R_1 // R_2$. If you want to make sure that the gain is not affected very much.

Yeah, so, must choose $R_1 // R_2$ to be much much larger than R_S , Okay, then this will approximately be - $g_m R_L v_i$, Okay, alright. So, stare at this picture. Therefore, let us come back to this picture here, Okay, the whole idea we put the potential divider on the gate side was to?

Student: eliminate a battery.

To eliminate a battery, Okay, now it seems that we have come back to a battery, Okay. So, how do we what do you think I mean and what do you think we can do to solve this problem, what is the current being drawn from the battery?

What is the quiescent current being drawn from the battery?

How much current how much DC current is flowing through the battery?

Student: Blue battery.

The red battery.

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Student: Currently 0.
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0, why?

Student: Because the two potentials are the same.

The two potentials are the same, alright, Okay, alright. So, there is no current being drawn from the battery, Okay. And remember we also talked about how to realize a you know a battery with a capacitor what did we say?

Student: Infinite capacitor.

An infinite capacitor charged to a certain voltage is equivalent to a battery, charged to the same voltage. And in this particular case even if that infinite capacitor is replaced by a finite one, right. Notice that there is no current being drawn from the there is no DC current being drawn through the going through the battery correct. So, therefore, in principle even if I replace that capacitor by a?

Student: Finite capacitor.

Finite capacitor as far as the operating point I mean you know the voltage across that capacitor will not Change.

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So, for the time being let us assume that we replace this by an infinite capacitor, alright. So, what will be the voltage across the infinite capacitor?

Voltage across this infinite capacitor?

What is the you know what is the purpose of that infinite capacitor what is it planning what is it doing?

It is replacing a battery of voltage.

Student: $V_{DD} R_2 / (R_1 + R_2)$.

So, what is the voltage drop across that infinite capacitor therefore.

What is the potential on the, right?

 $V_{DD} R_2 / (R_1 + R_2)$. What is the potential on the left?

What is the potential on plate X?

Student: 0.

What is the quiescent voltage at node X?

Student: 0.

0. So, what is the voltage drop across that capacitor?

This voltage is going to be $V_{DD} R_2 / (R_1 + R_2)$. So, all that the capacitor is doing is behaving like a battery with a voltage = $V_{DD} R_2 / (R_1 + R_2)$. So, what happens in the incremental equivalent?

The incremental equivalent just simply becomes a?

Student: Short circuit.

Short circuit, Okay, alright. So, we will come back of course, in reality it is not possible to have an infinite capacitor all the capacitors will be?

Student: Finite.

Finite. So, we cannot make an infinitely long large capacitor. So, we will say we will make we will replace it with a sufficiently?

Student: Large capacitor.

Large capacitor and then the obvious question that comes up is?

Student: How large?

You know how large is large, and what does large mean? Okay, alright. And you know another kind of little bit of a problem with the circuit here is that the load what comment can we make about the current through the load is it a DC current. Is there a is there a DC current flowing through the load?

Student: Yes.

Yes, Okay. So, it turns out that in practice many times it is not permitted to have DC current through the load. An example of such a load is a loudspeaker, right. Because there is some you know it is a coil you know around a magnet and so on, it does not makes you know you will saturate the magnet if you have DC current flowing through the coil. And therefore, in

many cases what happens is that you are also constrained to make sure that the load does not have any DC current flowing through it, Okay, and it also turns out that oftentimes you have yet another constraint and that is the load is one terminal of the load is?

Student: Grounded.

Grounded, you do not have access to that terminal. So, in other words you have a load like this. Let us call this R_{L1} , Okay. And you also cannot have DC current flowing through R_{L1} . So, what comment how do we therefore, connect R_{L1} to the output of the amplifier.

One way of doing it is to simply short those two what will happen now?

Student: DC current.

DC current will flow through it, why is DC current flowing?

Because before you connected the quiescent voltage on the, right is different from the quiescent voltage on the left. So, how will you prevent current from flowing?

You I mean in principle you will put a?

Student: Battery voltage.

Battery of voltage of V_{DD} - $I_D R_L$, right? And that battery is going to be like this. So, this is V_{DD} - $I_D R_L$ and how do you realize the battery?

You replace it with an infinite capacitor.

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Now what comment can you make about the has the quiescent operating point of the transistor changed? The quiescent operating point of the transistor has?

Student: Not changed.

Not changed and the only thing that has what has changed then?

What comment can we make about the incremental gain now?

There is also an additional term. R_{L1}. So, what comment can we make about the gain now.

$$v_{o} = \frac{-g_{m}(R_{L}/R_{L1})(R_{1}/R_{2})v_{i}}{R_{s} + (R_{1}/R_{2})}$$

So, is this gain larger than before or smaller than before?

Student: Smaller.

Its smaller than before. And that is the penalty to be paid. See whenever you have you know you have constraints, correct? You have to pay something to be able to overcome those constraints, alright. So, here you wanted the load to be grounded, you wanted no DC current to flow through the gate through the load. I mean that basically means that you are asking for something. If you ask for something you must be prepared to pay something and that pay something is equivalent to decrease the gain.

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Now we have seen that an infinite capacitor for incremental signals. An infinite capacitor charged to a voltage V_B is equivalent to a battery of voltage V_B , and therefore, for incremental signals this looks like a?

Student: short circuit.

Short circuit, Okay. So, whatever you can do with voltage you can do with current whatever you can do with the capacitor you can do with a?

What is the if I say voltage is to current as capacitor is to?

A capacitance stores charge, right, Okay. Inductor stores energy in the form of flux, correct? So, a capacitor can be left open, an inductor must always be left.

Student: Shorted.

Shorted, right. So, an inductor which is carrying a current I am assuming that there is some path here, an inductor which is carrying a current I is equivalent to a therefore?

Student: Current.

A current source of value I which in the incremental. This is an open circuit. So, what do you notice between an infinite capacitor is open in the quiescent in the circuit that finds the quiescent operating point, the capacitor is open whereas, in the incremental network it is a

short circuit, an inductor an infinite inductor on the other hand is a short circuit in the quiescent network but open in the incremental network, right? So, these are extremely you know I mean these are useful things to know, right? So, one way we saw that when we added this R_{L1} which was grounded and we did not want any DC current to flow through it we had to couple it with an infinite capacitor and thereby lose gain, Okay.

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Why are we losing gain by the way? The incremental current coming from the transistor. If you did not want to lose gain you have to ensure that all that incremental current flows through which of the resistors through R_{L1} , right, but now part of it is flowing through?

Student: R_L.

 R_L , right, Okay. So, if you want to fix this problem what do you think you can do now that you know about inductors.

You replace.

Student: R_L.

 R_L with?

Student: Inductor.

An infinite inductor or you can put R_L as he suggest you can put the infinite inductor in series with R_L . So, basically if you put an infinite inductor here, then in the incremental network the inductor will become an open circuit and all the current will pass through?

Student: R_{L1}.

 R_{L1} , right. Of course, you know the practicality of using an infinite inductor is a different matter, but at this point this is an in principle argument, Okay. Nobody does this usually, because inductors can tend to become physically very big, Okay. And by the way this is not the only way of coupling the input source to the gate and it can somebody tell me another way of inserting the I mean making sure that the gate voltage is v_i incrementally.

Student: Transformers.

In the same network if we did not want to use transformers. We will stop now we will discuss tomorrow.