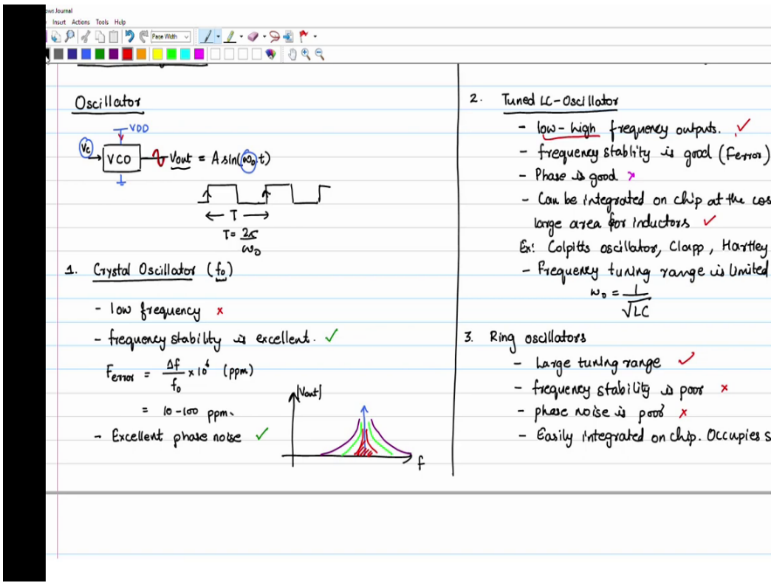
Phase-Locked Loops

Dr. Saurabh Saxena
Department of Electrical Engineering
Indian Institute of Technology Madras

Lecture – 36
Introduction to Oscillators

(Refer Slide Time: 00:15)



Hello everyone. Welcome to this session on PLL building blocks. So, after looking at the loop gain, phase margin and noise analysis in PLLs at system level, what we will do is we will now move on to the block level design for PLLs. The first one which is the most important block in PLL is the oscillator. So, let us look at what is this block, how does it work and how we are going to design it.

So, we have repeatedly used a block named VCO, voltage controlled oscillator, where we control V_c and we get V_{out} whose frequency we can control. So, just to add details to this block, this block connects to the supply VDD and ground, and you do not feed any other signal other than the V_c signal and V_c controls only the frequency. It does not control the amplitude of the oscillator.

So, you have this V_{out}. So, you can think about it, this particular block operates standalone, it takes power from the VDD and converts that to one particular frequency, whatever frequency you have, so, V_{out} can be given as follows:

$$V_{out} = A\sin(\omega_0 t)$$

V_{out} can be a sine wave or it can be a square wave of the same time period. It depends on the kind of oscillator which you have.

So, this is the time period of the oscillator and this time period $T = \frac{2\pi}{\omega_0}$. So, the oscillator as a standalone block generates frequencies depending on the control voltage. Somehow this control voltage will relate to ω_0, but there is no other input to the oscillator, there is nothing like sinusoidal input because of which you are generating ω_{out}, no, it does not happen like that. So, we need to design this block actually and there are multiple options before you for this block which can generate sinusoidal output.

So, let us look at the options which are available to substitute your VCO. The first one is crystal oscillator. This is the part which we saw in the very beginning of this course. So, crystal oscillator operates on the piezoelectric phenomenon and the characteristics of this crystal oscillator are as follows. It is generally a low frequency output which you get. The frequencies are low, it may be ranging from a few kHz to a few 100 MHz at the max.

So, the output is low frequency. The frequency stability is excellent. So, what is frequency stability? I will just tell you. Frequency stability is excellent here, so when you have a crystal oscillator whose output is at frequency f_0, f_0 is a function of device sizes within the crystal oscillator and this frequency f_0 may vary depending on the temperature, depending on the voltages which we are feeding to the crystal oscillator and other process parameters.

So, that variation, in case for the frequency, you are not applying any force, any sinusoidal inputs to the system. This is the oscillator standalone, it has its frequency output at f_0 but f_0 can vary. So, whatever the variation you have in the output frequency by the nominal frequency or you can say the average frequency into 10^6 is measured as parts per million. This is the frequency error.

So, the frequency error in crystal oscillator is very low and that is why we say that the frequency stability is excellent. This is typically in the range of 10 to 100 ppm. It depends on what kind of crystal you are choosing. Then the other important metric for the oscillator is your phase noise.

So, crystal oscillator comes with excellent phase noise. What is this phase noise? Well, it means that if you use the oscillator standalone as a clock, the jitter is going to be very low or the phase noise is very less. So, if you look at the spectrum of the crystal oscillator, you may get a tone and then very low phase noise, it is like the perturbations near the output frequency are going to be quite low, that is what you are going to have whereas in other oscillators, you may get a lot of phase noise. So, this is the crystal oscillator output spectrum. The only bad part about the crystal oscillator is they are quite bulky.

So, crystal oscillators are bulky. You cannot integrate these crystal oscillators with your integrated circuits because they are too big. So, you cannot integrate on your IC and having multiple crystal oscillators even on the PCB board will be too much because they occupy a good amount of area. So, this is one of the drawbacks of the crystal oscillators.

The good part is the frequency stability and the phase noise. So, in case you need low frequency output, you need a low frequency clock and you are not integrating on chip, you are integrating on board, you need only one part of one oscillator, you may be good enough to use the crystal oscillator in place of using any PLL, but that is the limit.

As we move ahead and start realizing the oscillator using tuned oscillators which are tuned LC oscillators, this is something which we saw when we told that the electrical model of the crystal oscillator can be realized using RLC components. From there we found that the frequency was limited by L and C, inductor and capacitor values. So, we can go and choose the inductor and capacitor values as we want, and we can realize that.

So here, the frequency is not the limit. Low frequency will have its own problems because inductors may be very large, but you can get in general low to high frequency outputs. Frequency is not an issue provided you are ready to sacrifice the area for the inductors but that area is not as large as the area for the crystal oscillator unless you go to a very low frequency.

The other thing is that the frequency stability is not excellent I would say but frequency stability is good. So, it is going to vary, no doubt about it. So, F_{error} will actually be large. It can vary from 100 ppm or depending on the values, it can vary up to 1000 ppm in general.

The next one is phase noise. The phase noise is good but not as good as the crystal oscillator. So, phase noise may increase, just for an example, I may have a larger phase noise like this in comparison to your crystal oscillators. Then, when we come to the point of integration, they can be integrated on chip at the cost of large area for inductors. So, as you go to the lower technology nodes where the chip price per unit area is quite large, integrating these inductors may not be an option always.

So, initially the inductors were kept outside the IC because they were huge. As the silicon got cheaper, we have started integrating the inductors on chip also. So, it completely depends on the value of the inductor which you are having whether you will integrate on chip or put it on board.

So, the examples of tuned LC oscillators are Colpitts oscillator, Clapp oscillator, Hartley oscillator. Another one bad part about this I would say is that the frequency tuning is limited, frequency tuning range is limited. In case of the crystal oscillators, frequency tuning range actually does not exist. So, once you design a crystal oscillator, the frequency whatever the frequency you have, that is what you get, frequency tuning range is roughly absent. So, you have to design it carefully for the desired frequency.

Here the frequency tuning range is limited, and the reason is that the frequency at which it is oscillating is roughly equal to or is equal to $\frac{1}{LC}$. So, how much capacitor you can vary will depend on that and there will be limitations on that also, you cannot vary inductor that easily for an LC oscillator.

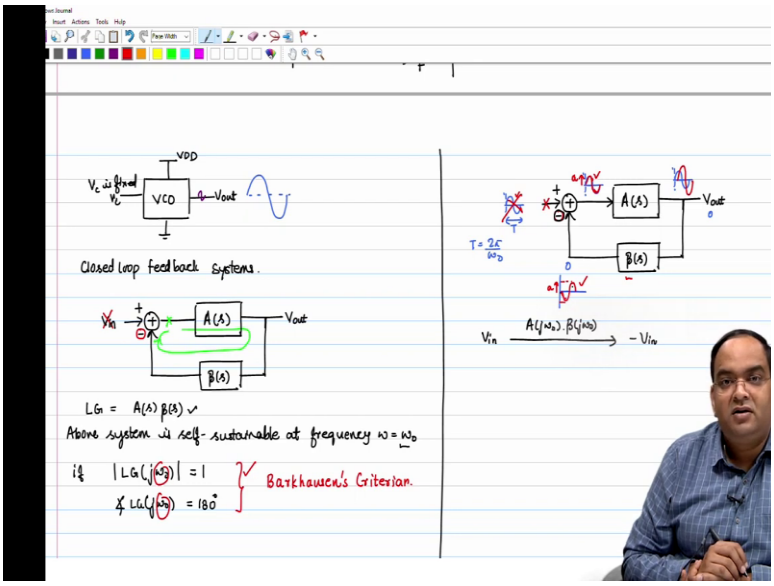
As you see in comparison to the crystal oscillators, the good part about this is that it can be integrated on chip, and you get a wide range of frequencies during the design process. So, this low to high frequency output is during the design process. Once you have designed the oscillator, you cannot tune that frequency that is the frequency tuning is very limited. So, these are the good parts about the tuned LC oscillators. The bad part with respect to the crystal oscillators is that these two things are not that great. Now, the other one because we are looking for the things where in our PLL we can vary the output frequency, we want good stability also in those things. So, there exist another kind of oscillators and these are ring oscillators.

So, in ring oscillators, you can have a large tuning range during the design process and once it is designed by controlling few things, you can have large output tuning range. So, this is like a good part of it. The frequency stability is poor which is like you cannot use the ring oscillator standalone without keeping it in the phase-locked loop because the frequency will vary depending on process, voltage and temperature.

So, frequency stability is poor, phase noise is also poor, a lot of phase noise, so if you are looking at this, standalone ring oscillators may have a lot of phase noise. You can easily integrate it on chip, integration on chip is not at all a problem. They occupy a very small area and can be easily integrated on chip. So, if you want multiple ring oscillators on chip, you can easily integrate them.

So, if we think about it, large tuning range is a good part, frequency stability is poor, this is not good. So, you cannot use standalone, phase noise is poor, again you have to use in the PLL loop, easily integrated on chip and occupies small area, that is good part about the ring oscillator. So, these are three different kinds of oscillators. We will talk about the last two and because in many places we need oscillators with a wide tuning range, so, we will start with the ring oscillators to begin with.

(Refer Slide Time: 15:50)



So, let us come to this block which is VCO. So, I will just draw this and just give a thought to it. What should it do? So, you have VDD and ground, forget that you are controlling V_c, just assume that V_c is equal to some fixed voltage. So, V_c is fixed, for example, and you are getting V_{out} here and in practice what you want to see is this V_{out} should be sine wave and it is self-sustained.

So, think about a system where you are not feeding any input, all the voltages are dc and it is producing sustained sinusoidal or square waves at the output which means that the system we are using should be self-sustainable which in turn means that even if there is some disturbance at the output of V_{out}, the system takes care of it and it still retains the frequency which it is designed for.

So, to understand those systems which are self-sustainable even in the presence of disturbances, we have to understand closed loop feedback systems. So, it is like it is a closed loop system, I use the word closed loop because if you make any disturbance, it takes care of it.

So, in general, let us draw a block diagram of a typical closed loop feedback system. You have a forward path with gain $A(s)$ and you have a feedback path with gain $\beta(s)$. This is the system and if it is in negative feedback, then I can put a minus sign here. Typically, in a closed loop system, this is V_{in} and this is V_{out}.

So, this system will become self-sustainable without any input under given conditions. So, what are those conditions? So, first we look at the loop gain of this system. Loop gain is the gain which you get as you break the loop and go around the loop. So, here, the loop gain is given by,

$$LG = A(s)\beta(s)$$

We have to keep in our mind that there is a negative sign in the feedback, this is important.

So now, given this loop gain of the system, the above system becomes self-sustainable at, because we are writing in Laplace domain, so, we will talk about frequencies, at frequency $\omega = \omega_0$, if

$$|LG(j\omega_0)| = 1$$

$$\measuredangle LG(j\omega_0) = 180°$$

If this happens, then you will have sustained oscillations at the output of this system at ω_0 without any external input to the system. So, you can think that this is like a small signal model of a closed loop system and that is what is going to replace your oscillator.

So, let us understand why these two conditions are important. By the way, these two conditions for sustained oscillations are defined under Barkhausen's criterion, this is the condition for sustained oscillations. So, let us take this example and try to understand that. So, just to begin with, think about it that in the beginning, you feed in a sine wave like this, only this sine wave, with respect to time, it has time period T and $T = \frac{2\pi}{\omega_0}$. This is what you feed in, we will see that how we can think about that this kind of frequency will come on its own, but this is the sine wave you feed.

So, initially all the voltages were 0. So, V_{out} was 0, feedback was 0, everything was 0, and I feed this input. When I feed this input here, so, the input comes here, it will get amplified by the gain $A(t)$ and it will come here. $A(t)$ is going to add some kind of gain to it. So, in place of just having the same blue waveform, I am actually providing some phase shift and some amplification, that is what I am going to have at one particular frequency. So, at one particular frequency, there is going to be an amplification and phase shift.

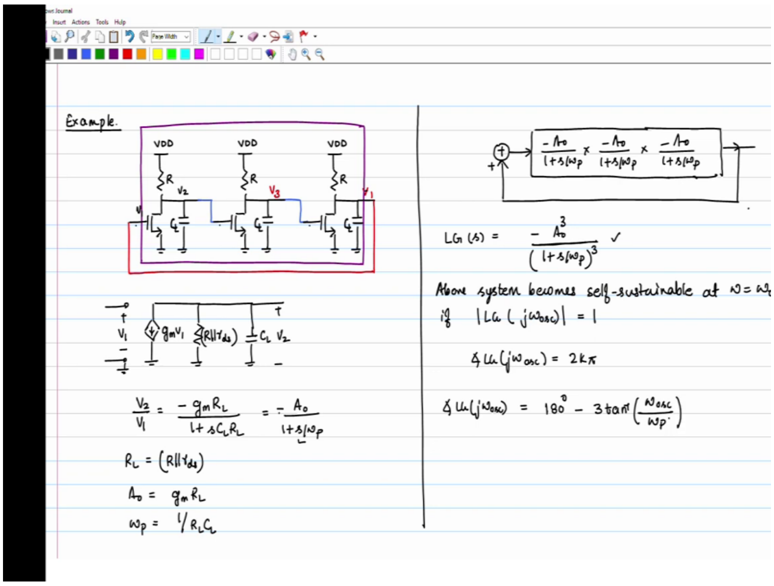
So, that will happen at output $A(s)$. Then what happens? You have $\beta(s)$ also and $\beta(s)$ is going to do a similar thing, it will actually amplify and add phase shift to it. So, the waveform here is shifted in phase and amplified. I deliberately do it like this, the reason is the following. So, let me do it with the same amplitude, so here, what I am trying to make sure is that if this amplitude is a, then this amplitude is also a. Here, it is 0 and then you see positive cycle first, here you are seeing negative cycle first.

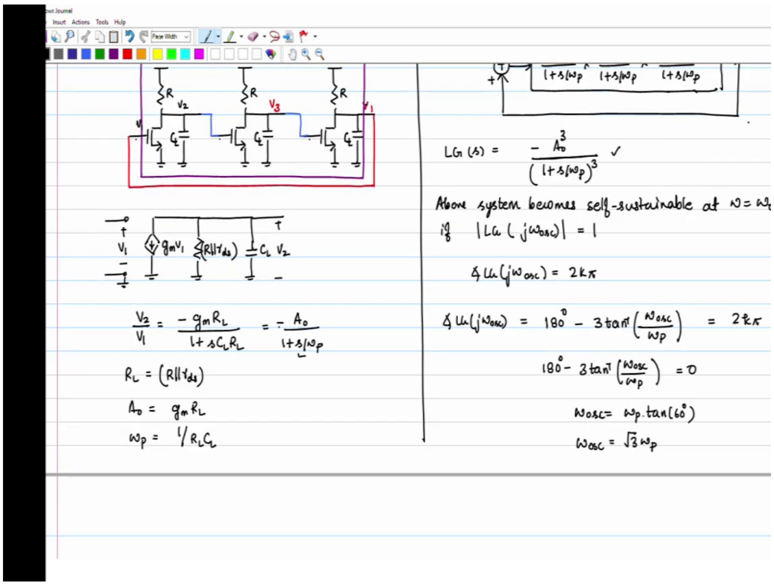
So, effectively, what I am trying to show you here is that this signal is just an inverted signal which you fed at the input which in turn is just telling you the same thing that if I feed this sine wave V_{in}, then from V_{in} when I multiply $A(j\omega_0)\beta(j\omega_0)$, effectively what I get is $-V_{in}$. That is what I am doing. If I get that $-V_{in}$ and you have a negative sign sitting here, so, what happens is this negative will invert this signal and you are going to get the same signal here.

So, now you think that you provided the sine wave to begin with as per our understanding and then the system itself is also feeding the sine wave. If the system itself is also feeding the sine wave of the same value, then you do not need to feed anything at the input and this sine wave will be sustained in the system on its own. You can ask a question that if I do not feed anything, where will the sine wave arise in the first place?

Well, if this system is a model of some circuit block, there is going to be noise in the system. Noise exists at all the frequencies. At one particular frequency which is ω_0, you will satisfy this criterion for sustained oscillations and once you satisfy the criterion for one particular frequency, those oscillations will be sustained, all other input frequencies will die down. So, this is how our oscillator will be designed so that it becomes self-sustainable.

(Refer Slide Time: 26:43)





So, using this particular kind of negative feedback analysis, we will now go ahead and analyze one particular example. So, the example which I am going to choose is based on this loop gain and the angle of loop gain and Barkhausen criterion. So, let us do that, a simple example of a ring oscillator. Why do we call it a ring? You will see.

So, I have a simple common source amplifier designed with the help of a MOSFET, R and C components. So, this is R, and this is C. If I call this as V_1, the gain from V_1 to V_2 for the amplifier at a given operating point can be found out. So, let us say, you have this common source amplifier and it is biased with dc voltages.

So, we can find the gain from the input to the output using the small signal model for this amplifier assuming that it is biased in saturation region, I can very well say, this is V_1, this is

g_mV_1 and you have resistor R in parallel with the r_{ds} of the transistor and whatever capacitor we have, that capacitor I am going to have at the output. Assume that the capacitor is effectively C_L. So, this is the model.

So, we have,

$$\frac{V_2}{V_1} = -\frac{g_mR_L}{1 + sC_LR_L}$$

What is R_L? R_L is given by,

$$R_L = (R||r_{ds})$$

So, this is the transfer function which you have, and this can be written as follows:

$$\frac{V_2}{V_1} = \frac{-A_0}{1 + {}^{s}/{\omega_p}}$$

where,

$$A_0 = g_mR_L$$

$$\omega_p = {}^{1}/{R_LC_L}$$

So, I have this single stage. Here, the gain which you are seeing is you are having -180° and one particular pole.

If I go ahead and cascade these blocks, for example, like this. So, now I am going to connect all of them and form a ring like this and these are V_1, V_2 and V_3. So, from V_1 to V_2, V_2 to V_3, and back to V_1, it forms one single system like this. This is the block which we have. For this particular block, if you think about it, we are connecting the output of the last stage back to the first stage.

So, I am going to model this using our small signal model. So, the gain from the very first stage to the last stage is given by,

$$\frac{-A_0}{1 + {}^{s}/{\omega_p}} \times \frac{-A_0}{1 + {}^{s}/{\omega_p}} \times \frac{-A_0}{1 + {}^{s}/{\omega_p}}$$

The output finally is fed back here like this and it is directly connected, there is no negative sign connected from the output of the last stage to the first stage. So, it is still in a positive sign, so we have to take this into account and there is no input to the system.

Now, for this particular system, let me just draw it little nicely like this. We could have picked up at any particular point, but this is what we are looking at. So, here the loop gain of this system is given by,

$$LG(s) = \frac{-A_0^3}{\left(1 + {}^{s}/_{\omega_p}\right)^3}$$

As per the Barkhausen criterion, the above system becomes self-sustainable at $\omega = \omega_{osc}$, let me write it, if

$$|LG(j\omega_{osc})| = 1$$

$$\measuredangle LG(j\omega_{osc}) = 2k\pi$$

Now, you would ask that why did I write $2k\pi$ here whereas it was 180°. Well, when it was 180° here, that time you had a minus sign here. So, 180° and this minus sign was giving you 360°. Here, I do not have the minus sign, it is connected straight. So, I am writing $2k\pi$ which is either 0° or 360° or -360° or so on. So, that is what we have.

So, in order to do that, I will see whether I can have the frequency of oscillation or not. So, first I will write the $\measuredangle LG(j\omega_{osc})$, if this frequency exists. So, you look at the loop gain. So, for the loop gain, you get,

$$\measuredangle LG(j\omega_{osc}) = 180° - 3\tan^{-1}\left(\frac{\omega_{osc}}{\omega_p}\right)$$

Also, as per Barkhausen criterion for oscillations, if and only if,

$$\measuredangle LG(j\omega_{osc}) = 180° - 3\tan^{-1}\left(\frac{\omega_{osc}}{\omega_p}\right) = 2k\pi$$

Then this is going to be k goes from 0 to ∞, it can take any value, then you will see that this is going to have oscillation.

So, I will pick up $k = 0$ which implies the following:

$$180° - 3\tan^{-1}\left(\frac{\omega_{osc}}{\omega_p}\right) = 0$$

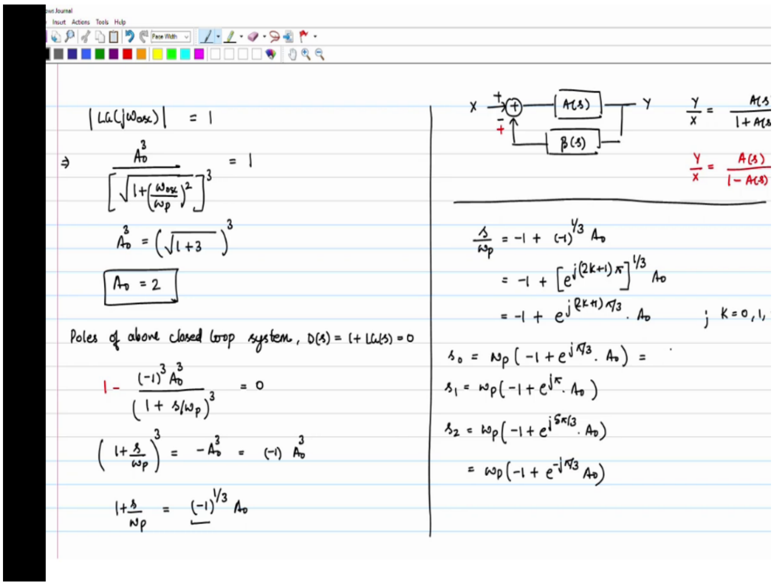
So, we have,

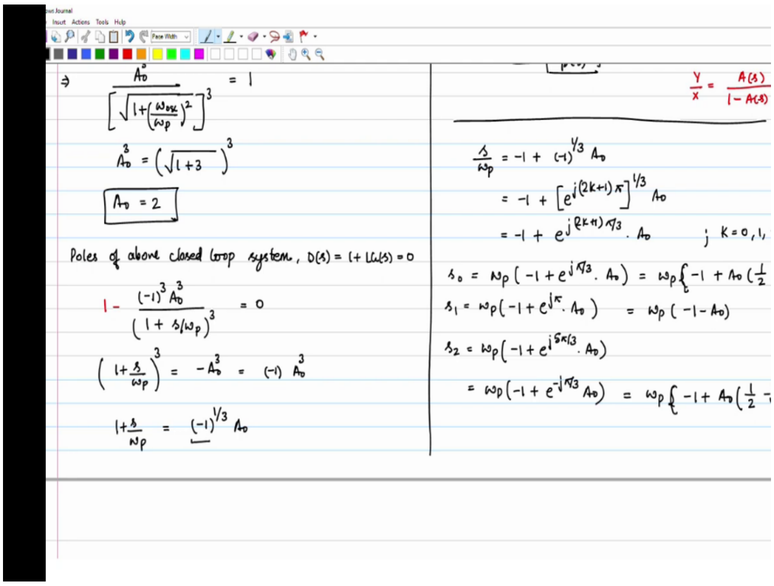
$$\omega_{osc} = \omega_p \tan(60°)$$

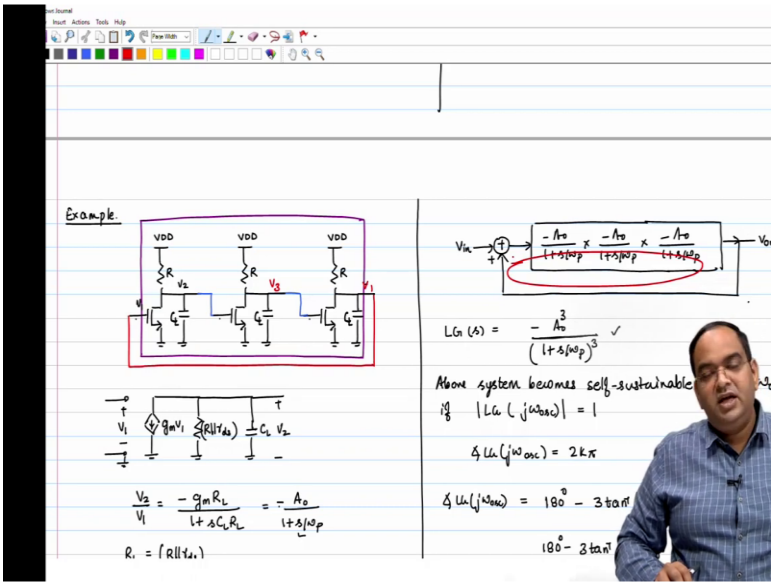
$$\omega_{osc} = \sqrt{3}\,\omega_p$$

So, at the frequency $\sqrt{3}\,\omega_p$, it happens that the phase of the signal is $2k\pi$.

(Refer Slide Time: 35:26)







Now, this is not the only condition. The other condition is that the loop gain should also be equal to 1. So, I have to find this. So, we have,

$$|LG(j\omega_{osc})| = 1$$

$$\frac{A_0^3}{\left[\sqrt{1+\left(\frac{\omega_{osc}}{\omega_p}\right)^2}\right]^3} = 1$$

$$A_0^3 = \left(\sqrt{1+3}\right)^3$$

$$A_0 = 2$$

So, if $A_0 = 2$ and then you have $\omega_{osc} = \sqrt{3}\,\omega_p$, then you can get sustained oscillations for this particular example, and all the oscillators which are working on this negative feedback system are working on this principle only. Here, you have seen a common source amplifier, you can have any other amplifier, you can cascade and if you have the phase and the magnitude like this, you will see sustained oscillations.

Now, it is not only this open loop system or this loop gain system which you have understood that you will have the sustained oscillations. It is also important to look at the closed loop systems for this example. So, given this example, what do you have here? This is, let us say, V_{in} and V_{out}. So, what we are going to look at is that if you have such kind of system, where do the poles of this closed loop system lie? The poles of this closed loop because when we make any closed loop system, it is stable if and only if all the poles are in the left half s-plane. If you have poles on the right half s-plane, the system is going to be unstable.

So, what happens in the case of oscillator whether the poles are in the left half plane or right half plane or jω axis, we will confirm that. So here, if you look at the closed loop system, we need to find the poles or the roots of, let me write it, poles of above closed loop system will be given by,

$$D(s) = 1 + LG(s) = 0$$

These will be the poles.

So, to find the poles here, so, look at this particular example here. So, what is the loop gain here? Loop gain is this and you are closing the loop in this particular manner. So, we have,

$$1 - \frac{(-1)^3 A_0^3}{\left(1 + {}^{s}/_{\omega_p}\right)^3} = 0$$

So here, we find the poles of this closed loop. One important thing here is that in this particular case, you are actually closing the loop with a positive sign.

So, to give you an idea, let us just look at this system in general. When you have $A(s)$ and here you use β or this is positive and this is negative. In this particular case, what you see is the following, $A(s)$ and $\beta(s)$ in this case. So, this is X and this is Y, so, we have,

$$\frac{Y}{X} = \frac{A(s)}{1 + A(s)\beta(s)}$$

If it happens, you can rewrite those equations, if in place of this negative sign, you have a positive sign here, then we have,

$$\frac{Y}{X} = \frac{A(s)}{1 - A(s)\beta(s)}$$

So, looking at our oscillator, the way we are combining, it is going to be $1 - \frac{(-1)^3 A_0^3}{\left(1+{}^{s}/_{\omega_p}\right)^3} = 0$.

We need to find the poles for this. So, what we will get here is,

$$\left(1 + {}^{s}/_{\omega_p}\right)^3 = -A_0^3 = (-1)A_0^3$$

So, for the closed loop poles, you have to take the cube root on both the sides. So, we get,

$$1 + {}^{s}/_{\omega_p} = (-1)^{1/3} A_0$$

Now, taking $(-1)^{1/3}$ can be done with the help of complex numbers. So, I can write,

$${}^{s}/_{\omega_p} = -1 + (-1)^{1/3} A_0$$

$${}^{s}/_{\omega_p} = -1 + \left[e^{j(2k+1)\pi}\right]^{1/3} A_0$$

$${}^{s}/_{\omega_p} = -1 + e^{j(2k+1)\pi/3} A_0$$

Here, for the roots, $k = 0$, 1 and 2.

So, what roots are we going to have? Let me just write it. I will substitute $k = 0$. So, we have,

$$s_0 = \omega_p\left(-1 + e^{j\pi/3} A_0\right)$$

$$s_1 = \omega_p\left(-1 + e^{j\pi} A_0\right)$$

$$s_2 = \omega_p\left(-1 + e^{j5\pi/3} A_0\right)$$

$$s_2 = \omega_p\left(-1 + e^{-j\pi/3} A_0\right)$$

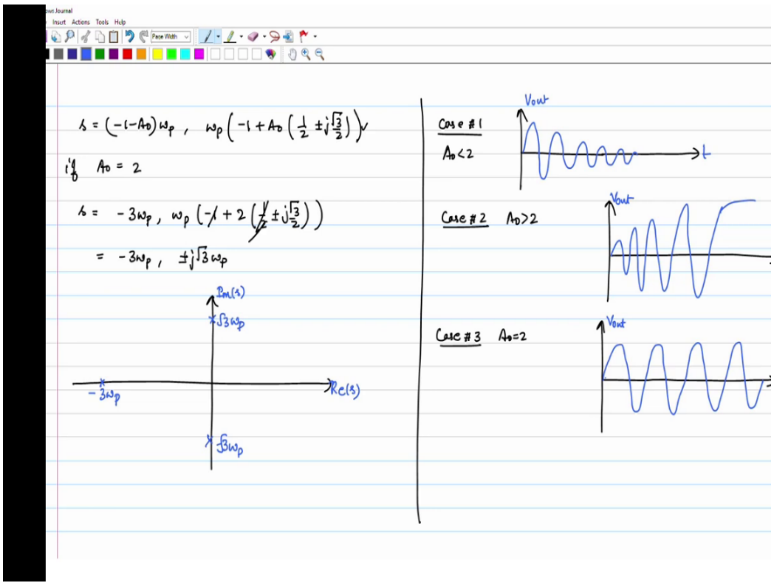
So, I will just expand this, so, we have,

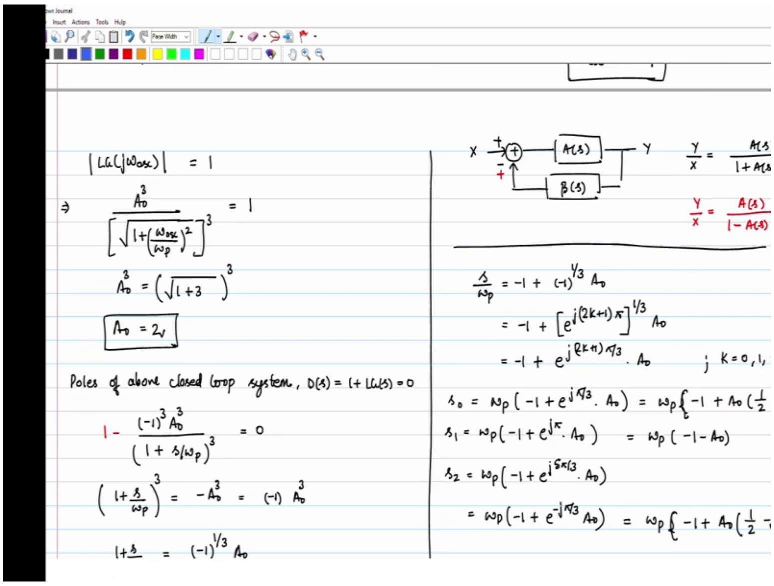
$$s_2 = \omega_p\left\{-1 + A_0\left(\frac{1}{2} - j\frac{\sqrt{3}}{2}\right)\right\}$$

$$s_1 = \omega_p(-1 - A_0)$$

$$s_0 = \omega_p\left\{-1 + A_0\left(\frac{1}{2} - j\frac{\sqrt{3}}{2}\right)\right\}$$

(Refer Slide Time: 44:44)





So, the 3 poles which you are seeing in the closed loop which you will get, the 1st pole is equal to $s = (-1 - A_0)\omega_p$, so, this pole is going to be in the left half plane. The other poles are $\omega_p \left\{-1 + A_0\left(\frac{1}{2} \pm j\frac{\sqrt{3}}{2}\right)\right\}$.

So, for our sustained oscillations in this case, we found a couple of things that $A_0 = 2$. If $A_0 = 2$, then the roots of this closed loop system are given by,

$$s = -3\omega_p, \omega_p\left\{-1 + 2\left(\frac{1}{2} \pm j\frac{\sqrt{3}}{2}\right)\right\}$$

$$s = -3\omega_p, \pm j\sqrt{3}\omega_p$$

So, if you look at it, the locations of the poles are like you have one at $-3\omega_p$ and the other two poles are on jω axis. This is Re(s), this is Im(s). So now, you can understand that if I choose $A_0 = 2$, then at frequency $\sqrt{3}\omega_p$, what happens is you have poles on the jω axis and which will give you sustained oscillations.

So, what we analyze with the help of the loop gain and the phase, the same thing is revealed by the closed loop analysis by the closed loop poles. If you have closed loop poles along the jω axis, then you will have sustained oscillations. Now, if I take this particular closed loop system and I disturb this system which we have, and I just observe in transient domain. In case 1, $A_0 < 2$, what you are going to see there may be oscillations but those oscillations will die down at V_{out}.

If you take case 2, where $A_0 > 2$, what you are going to see for the closed loop system, you can very well get it from here, if $A_0 > 2$, then in that particular case, you are going to have closed loop poles in the right half plane, the oscillations will build up and then with the real limit of the supply voltages, it may saturate either to VDD or ground.

Then in case 3 when $A_0 = 2$, in that particular case, you will get sustained oscillations, whatever amplitude we are not talking about that, but the oscillations are going to be sustained. In all these cases, I am plotting V_{out} versus t. So, you can say it is important at some time that you have $A_0 > 2$ so that the oscillations can start but in sustained case there should be only $A_0 = 2$ to have sustained oscillations.

This analysis which we have done here using a closed loop system and the loop gain forms the basis of all kinds of oscillators which we can analyze using their small signal model, no matter which circuit comes up having 3 in cascade, 5 in cascade, the way we have done in the ring, 5, 7, whatever it is, we can calculate the frequency of oscillation and the gain required in this manner. Thank you.