Introduction to Semiconductor Devices
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Lecture - 7.4
Exact Solution vs Delta-Depletion Approximation

Welcome back. So, in the last week, we understood how to analyse the electrostatics of a MOS capacitor. So, we have derived expressions for electric field and voltage in a MOS capacitor. We have drawn the profiles. And towards the end, we introduced what is the surface potential and then the relationship between the surface potential and the gate voltage.

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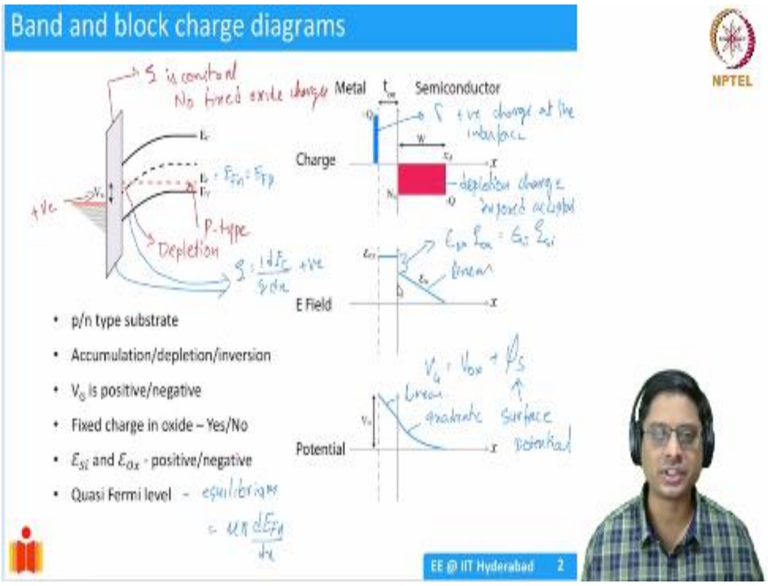

We will quickly recap some of these things today. So, as I have been emphasising multiple times, band diagrams are a very, very useful tool to understand what is happening in the semiconductor device and also you know, in the last lectures, we have introduced the block diagrams, block charge diagrams, for example, how does the charge vary as a function of position in MOSCAP and how does electric field vary and how this potential vary? So, these are also very useful.

So, together with you know band diagrams, they convey a lot of information. For example, we have looked at all this before. Let me try to quickly recap. So, if I look at the semiconductor band diagram here, what can I conclude? what are the few inferences? For example, we can look at the bulk Fermi level and then decide what is the doping at Fermi level closer to E_V and conclude that my substrate is a p type substrate.

And if I look at the Fermi level relative position of the Fermi level closer to the surface, then I can decide whether the MOS capacitor is in accumulation, depletion or inversion. So, for example, here you see that the Fermi level has now moved above E_i. So, it is definitely you know, it does not look like, it has reached inversion, but it is definitely depletion. So, if the Fermi level moves even higher such that the surface potential becomes $2\phi_F$ then that will be inversion, all set of inversion.

We will discuss that in greater depth today. And what is the applied gate voltage? Well, the applied gate voltage in this case is positive. The reason is whenever you have a positive gate voltage related to the substrate here, the Fermi level move down. The Fermi level in the semiconductor is here and related to that this has moved down by this much distance. So, that is a positive gate voltage.

And you could also ask: is there any fixed charge in the oxide? In this case, what happens when you have a fixed charge? When we try to solve the electric field in the oxide, we have assumed that you know we have considered dE/dx is equal to 0 because dE/dx is equal to ρ and ρ is 0. There is no fixed charge. So, whenever you have a constant electric field, this is a linear gradient, bands are bending in a linear fashion.

So, electric field E is constant, and this implies there are no fixed charges, fixed outside charges. Suppose if there was an oxide charge, then dE/dx will be some let us say, some position or some function of x, then it will not be linear. It might be quadratic or cubic or whatever if there are some charges present in the oxide. So, E, what is the electric field in silicon? What is the electric field in oxide?

We are well we know that how to do this. If for example, we can take the gradients or in oxide, you can take the gradients, so basically,

$$E = \frac{1}{q}\frac{dE_C}{dx}$$

So, in this both cases, gradients are going the same direction. So, electric field is positive. And is the system in equilibrium or non-equilibrium? Well, I mean, we have not explicitly showed any Quasi Fermi levels.

So, it looks like it is not; it is an equilibrium region, it is not non-equilibrium. It is a equilibrium. So, whenever you have E_F is equal to $E_{Fn} = E_{Fp}$, where your E_{Fn}, E_{Fp} is a Quasi Fermi levels and both are coinciding E_F, then it is basically a equilibrium situation. So, this will tell you that its equilibrium situation. So, is there any current flow in the MOS capacitor? That is an interesting question to ask.

So, what happens? How will the current depend on the Quasi Fermi level? We saw that it was basically $\mu_n \frac{dE_{Fn}}{dx}$. So, if you look at the gradient in the Quasi Fermi level, you can find out but right now, there is no gradient because there is no Quasi Fermi level at all in the semiconductor. So, there is no current that is you know possible here, in the way that it is shown right now.

So, all of this information is contained in just a band diagram and that is why it is so important to analyse the band diagrams carefully. So, if you have not already done it, I would urge you to draw these diagrams by yourself, understand each and every line you have drawn a band diagram represents something physically. So, if you keep doing that you will take it as an exercise and repeat it.

Then you will understand the physics very well and this block charges diagrams also we have seen them in the past in the last class. For the depletion case in this you know, when you want to apply, you have to apply positive voltage for this you know, this is a p type substrate. So, when you apply a positive voltage, you push away the holes and uncover that expose the acceptor ions.

So, this is the depletion charge consisting of exposed acceptors. And other side, this is a delta function, delta positive charge because it is metal. Metal cannot have charge in the bulk. So, it has to be at the interface. What interface? Interface between metal and oxide. So, this is what we can figure out from the charge diagram and of course, it is a uniform charge density that is why you have this height equal to a N_A that is what is given by doping.

And then based on this, we have calculated how the electric field should be. This is straightforward. It is positive, of course and then it is linear here and constant. And there is this jump here which is very, very significant that comes because $\epsilon_{ox}E_{ox} = \epsilon_{si}E_{si}$. This is Gauss law.

So, that jump has to be maintained so that jump you know, we will actually run a simulation on Nanohub and show you that this exactly turns out to be and of course, your potential you know, you can always like the applied gate voltage V_G should be equal to the potential drop across oxide plus we call it surface potential, let me call it phi S surface potential.

$$V_G = V_{ox} + \phi_s$$

In the last class, I use psi S but I will call it phi S from now onwards. So, this is quadratic, and this is linear. So, why each of these things turned out to be that you should be comfortable. So, this is what we did last time. And then you should do this for accumulation, for inversion, for PMOS, all of that.

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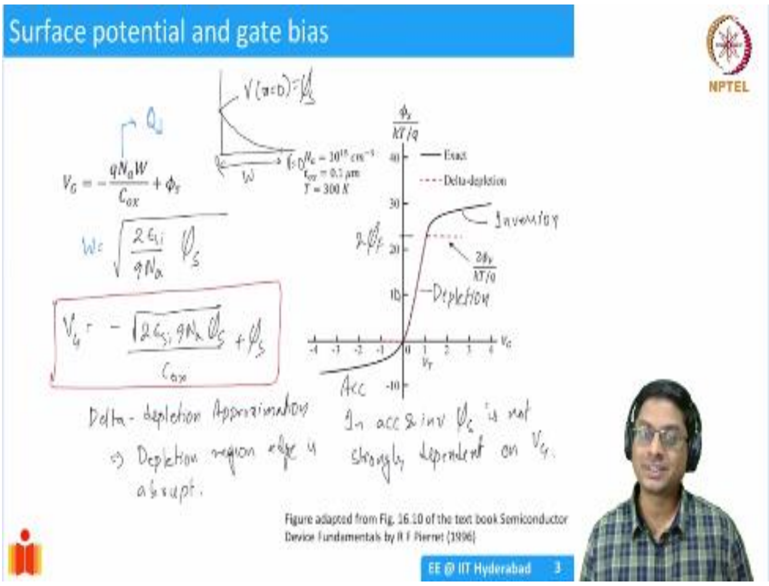

So, towards the end of the last lecture, we introduced what is known as surface potential. And we wanted to relate it to gate bias and we derived an expression you know, gate voltage is basically in terms of the depletion charge and surface potential.

$$V_G = \frac{-qN_AW}{C_{ox}} + \psi_s$$

It is a function of surface potential. Essentially, what we are having is let us say, if you have let me draw the potential diagrams only in the semiconductor while draw it, let us say in the bulk you have, 0 potential and this is the width of the depletion region. The bulk V is equal to 0 or one potential is 0. So, you are actually, there is a quadratic relationship here.

And the voltage or in a potential at x = 0 in this case, the interface basically this is if I call it 0 here, this is your ϕ_s surface potential. That is the amount of potential that is required to be applied at the interface between silicon and silicon dioxide such that you processed the same depletion. The depletion width will be whatever maybe. So, how do we calculate the width of the depletion region given a surface potential?

$$W = \sqrt{\frac{2\epsilon_{si}}{qN_A}\phi_s}$$

$$V_G = -\frac{\sqrt{2\epsilon_{si}qN_A\phi_s}}{C_{ox}} + \phi_s$$

This essentially is telling you that given any surface potential, you can find out what is the V_G or vice versa. There is a fixed relation between both of them. And if you plot it, we will see that you will get some expression, some curve which is looking like this, the red here. So, we should remember that this expression when we derived, we use the depletion approximation.

So, we said the depletion region is flat, it is abruptly changing. So, essentially, you have this you know, space charges. If I have, for example here, if I go back, so this is your depletion approximation where you have a block charge like this, but we know that in reality, it is going to be something like this spread out. It is going to be spread out like this, but we are not considering that.

So, if you consider that sort of a depletion charge profile, then you will get an exact solution. So, it turns out that you know, we can do a full numerical analysis of this and then determine what is the relation between surface potential and get bias. And if you do the exact calculations, we get a relation which looks like this. We will discuss why you know this is important, but if you did the simple depletion you know, delta depletion you know, we call this delta depletion approximation.

Delta depletion approximation essentially means you are depletion is abrupt. Depletion region edge is abrupt. It is not a smeared out depletion charge. If you take this approximation, we find that in the potential, you know, surface potentials in the depletion, for example, when surface potential is 0, considered that that is flat band, and as you go to surface potential of 2 ϕ_F, this is basically surface potential of 2 times ϕ_F.

When you go there, in between both of them, essentially, this region is depletion. And if you apply a negative surface potential, then it is essentially accumulation. And this can happen when you apply a negative gate voltage. So, as you apply negative gate voltage in MOSCAP, you get the surface potential. Why are we differentiating between the two? What is the difference between surface potential and the gate bias?

Well, we know that the gate bias is going to fall across the oxide and the semiconductor. Whatever falls across the semiconductor, we are calling it surface potential. So, there is going to be a slight difference. It is not a simple proportionality, because we also have you know, this Gauss law which tells us that there is a certain ratio of electric fields that has to be maintained.

And then if you apply gate voltage which will result in a surface potential greater than 2ϕ_F, then that regime will call it an inversion. So, we see that whatever approximation, we make this, you know abrupt charge approximation that works perfectly well in the depletion part. So, in this part of the curve, it is perfectly you know, correct. But if you go inversion or accumulation region, you see that the actual solution is telling us that the surface potential is not really dependent much on the gate voltage.

So, you have a large change in the gate voltage, but still reasonably small change in that surface potential. And similarly, the accumulation so in accumulation and inversion ϕ_s is not strongly dependent on V_G. Why this is so? I mean, we have another way of looking at it, we could think about it in terms of the bands.

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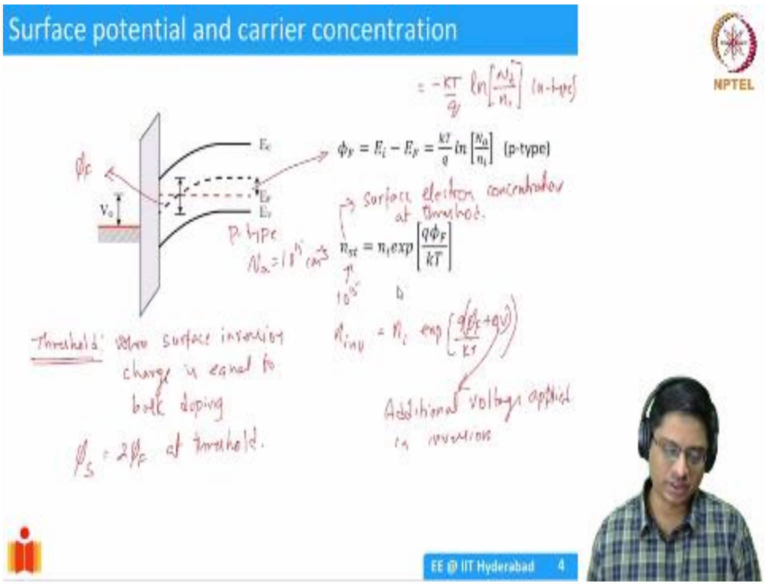

So, for example, if you look at this band diagram, when do we say that it goes into inversion? We already mentioned this, we said that the threshold definition. So, I am essentially looking at the same thing from various perspective, so that you get you know all round understanding of it. So, threshold was defined as when surface inversion charge, let us say, surface inversion charge by which I mean this discharge here at the interface, when surface inversion charge is equal to bulk majority carrier concentration or rather, we can call it bulk doping.

So, if you have let us say, we are using you know in this case p type, this is a p type semiconductor, let us say $N_A = 10^{15}/cm^3$. You have that much of bulk doping. And by applying a gate voltage and switching at the interface, I am getting the same amount of electrons. In the bulk, we have the holes but I get the same number of electrons at the interface that gate voltage call it as threshold voltage.

So, when will that occur? Well, we defined ϕ_F. ϕ_F is essentially the distance Fermi level from E_i.

$$\phi_F = \frac{kT}{q} ln\left(\frac{N_a}{n_i}\right) \; for \; p \; type$$

$$\phi_F = \frac{-kT}{q} ln\left(\frac{N_d}{n_i}\right) \; for \; n \; type$$

So, n type, you have Fermi level higher than E_i, so, that will be negative by this definition $E_i - E_F$.

So, in the bulk, you have a ϕ_F. How many electrons should be there? The number of electrons at threshold, this is basically I will say, surface electron concentration at threshold. So, what should we that? Well, if your substrate has a 10^{15} doping, this has to be 10^{15}. And then you solve what is. It turns out that the same ϕ_F is here.

So, for example, here, this should be ϕ_F when the distance of Fermi level from E_i is ϕ_F that is when we reach threshold, we say, we reach threshold. So, the overall surface potential is going to be ϕ_s is equal to 2 ϕ_F at threshold. So, we are coming to the same point as what we mentioned last time you know, in this here, we said that in the depletion region, it is exactly matching.

And we call the threshold point at which surface potential equal to 2 ϕ_F. The reason we are doing that is at that point when surface potential becomes 2 ϕ_F, then the carrier concentration inversion charge concentration at the surface will be exactly equal to the bulk doping whatever is the bulk doping. So, well and good. Now, what happens if you increase the gate voltage beyond that?

If you increase gate voltage beyond that, we know that as you approach you know, as the Fermi level approaches E_C, it does not change so much. The moment in the Fermi level is going to be slower and slower that is why, when you actually here, if you go to gate voltage is higher than 2 ϕ_F, you will notice that it is becoming more you know, flat response. Similarly, the accumulation also, if you accumulate it, after a point, you do not really increase much, the Fermi level does not move that much.

That is where we get this sort of S kind of a curve, which tells you that we do not need to really focus too much on you know, how what is you know, this rate of change in the inversion or accumulation. Instead, in the delta depletion approximation, we will assume that it is flat. So, basically an accumulation if you apply negative voltage, the surface potential is 0. And if you have a positive voltage beyond 2 ϕ_F, then it is flatting.

We do not really worry about the differences. So, now, we understood what happens to the surface potential. But the actual you know, carrier concentration is an inversion charge. So, how will that change? So, for any inversion charge beyond threshold, let us say n inversion, I will call it, which let us say, you know, what is this electron, you know, what is the charge?

$$n_{inv} = n_i exp\left(\frac{q(\phi_F + \Delta V)}{kT}\right)$$

So, this is basically additional voltage applied in inversion. So, basically, this tells you that if you have a small change in the delta V you know, small change in the gate voltage that is going to reflect in exponentially increased in inversion charge.

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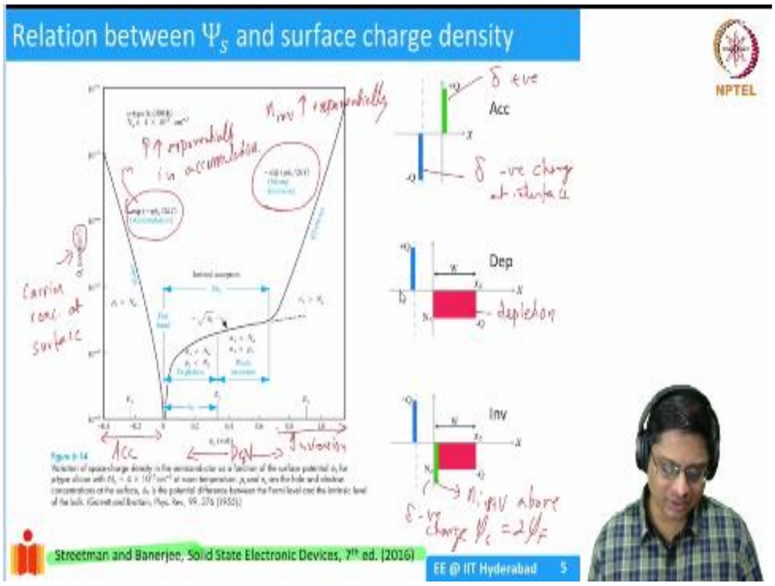

So, this say, I want to summarise I mean, you could summarise in this graph. This is not from the textbook, but another book by name Streetman and Banerjee, I have taken this. This is also a good undergraduate textbook. So, what you see is here on the y axis, we are plotting the carrier concentration. Carrier concentration at surface at just below the gate oxide these units are per centimetre square.

Please look at that. We have to do something to calculate this. So, how does the carrier concentration change as a function of; I mean ideally, we would like to know what the function of gate voltage is but we can also refer to it in terms of surface potential. So, we have seen that when surface potential is negative, we are in the accumulation regime. When the surface potential is greater than 2 ϕ_F, we are in the inversion regime.

Surface potential between 0 and 2 ϕ_F, we call it a depletion. So, what we see is; in the inversion the charges, the charge at the surface changes exponential you know, n inversion increases exponentially. Similarly, hole accumulation, p acc, we call it; p increases exponentially in accumulation acc. So, what does it tell us? We have a range of voltages in which you are not going to see that much surface charge.

But once you reach inversion, it is going to be a huge number of carriers that are generated. So, block diagram, we have representing this would be what we have shown in last class. So, if you have accumulation, you have this delta negative charge at interface and to compensate know to mirror that, there will be a positive charge at the surface delta positive church. Because this is majority carriers, we do not have a depletion region.

We just have the majority carriers accumulating at the interface. So, you have the balanced charge. But once we go into depletion, we need to keep pushing the holes away to uncover the acceptor ion. So, we have to uncover the depletion charge. We have to keep pushing holes away. So, in this case, on the surface, the charge will be varying some sort of you know, quadratic function, we have seen the exact relationship.

In a way, it is square root of ϕ_s. We are not at reached the inversion part. So, then this will be the behaviour. But the moment you hit inversion, it again starts exponential. That is why n inversion, we show this additional n inversion above $\phi_s = 2\ \phi_F$. Once you reach this, this just at the interface, this is the delta negative charge. When I said delta, I mean that it is just at the interface. It is like a delta function.

You can represent it like a delta function. So, this is the way the charges behave. So, I am essentially presenting you the same information in multiple ways. So, that you appreciate the physics what is happening. It is essential that you understand this, because we will talk about C V and then when we talk about C V will use these facts. You might say, you know, this looks okay. But this is not quantitative. It is still qualitative picture.

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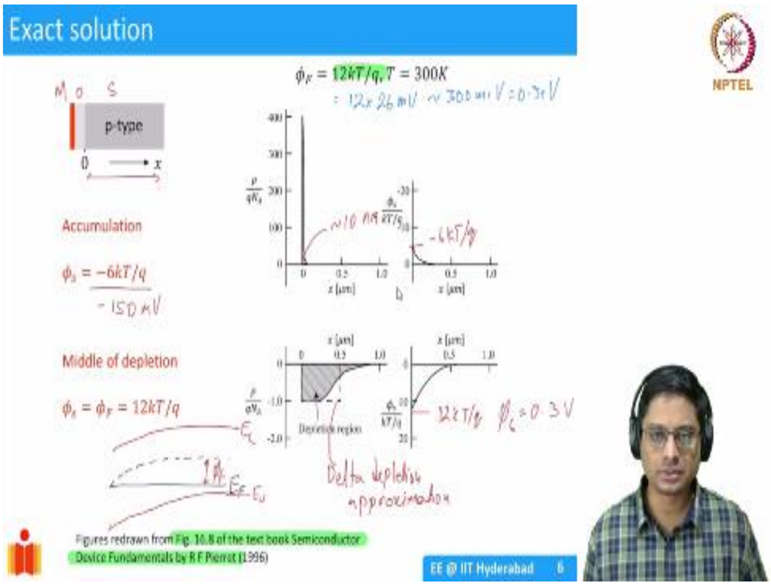

So, we can actually do an exact calculations of this using software tools. So, here what is simulated is considered MOS capacitor in this fashion. There is a metal. There is an oxide. There is a semiconductor, MOS capacitor. It is a p type substrates, it is an NMOS capacitor. And I am going to plot the; I am showing you the plots of various quantities only in the semiconductor region. So, from 0 to x.

This is basically semiconductor, only in that region we are showing in different biasing regions. And before we do that, the ϕ_F for the simulation has been assumed to be 12 kT /Q. By the way, this is taken from this textbook here, R F Pierret, you know, there is a figure 16.8. If you want, you can read up about that. I just use that to draw these figures. So, the substrate doping is basically 12 kT. It is a different way of representative.

$$\phi_F = \frac{12kT}{q} = 0.3eV$$

And to that system, we apply gate voltage such that the surface potential is just – 6 kT.

$$\phi_s = \frac{-6kT}{q} = -150mV$$

Small voltage, you are applying, small negative voltage. So, what will happen to the charge?

Well, whenever we apply a negative surface potential, we are going to say accumulation. So, it might not be apparent you know when you look at it first time, but I would suggest you to just keep analysing it and you will get it. So, why should you have a negative surface potential to get accumulation? Well, accumulation means in p type substrate you have to have holes.

If you have to support a hole, there you have to have a negative voltage at the interface that is a negative surface potential. So, how will the carrier concentration be? So, the first thing, I have here, we showing you that this is the plot of charge density as a function of x and what you see here is that there is accumulation when there is accumulation, the charges is nearly 400 times the doping density.

So, we have a strong accumulation if you are dropping density was 10^{15}, then this is basically 4 *10^{17}. So, a strong accumulation of holes and right at the interface, you remember. So, this distance is very, very small. It is less you know roughly about 10 nanometres or so it should be 10 to 20 nanometres or something like that.

We will see the numbers in the Nanohub. So, we have the sharp accumulation just under the surface and the surface potential of course is this. We have applied you know, gate voltage at the surface potential was – 6 kT so, you know just not the axis basically, its upwards, negative upward slope. This is fine. This is what that is why we call it accumulation. So, we have huge accumulation of charges. What happens in depletion?

So, we have another graph, which is plotted it in the middle of depletion varying essentially surface potential is ϕ_F. When the surface potential is ϕ_F, how will the bands look like? Well, you have this; we have this and we have E_i. So, this is my E_i, E_C, E_V, well I mean, it is not correct. This is my E_V.

So, originally it was close to; E_F was close to E_V. But then when surface potential becomes ϕ_F, this distance is ϕ_F. At surface, it becomes E_i, E_F is close to that E_i. This is the meaning of ϕ_s equal to 12 kT. So, please you know, practice this multiple times how to identify these numbers and surface potential in different biasing. Once you become comfortable, it will be. So, what does it mean that it is in depletion?

Well, the exact solution tells you that the charge density now, $\rho/(qN_A)$, we are normalising the substrate doping density. It is equal to exactly 1 you know, it is – 1 because of the charges minus so, – 1and then decay like this. And what did we approximate? You know, we use this particular line. This is delta depletion approximation. I mean, it is not actually capturing the decay at the edge of the depletion region but it still gives you a lot of physics currently.

And so, that is why we keep referring to this as uniform charges density. So, qN_a, all the time and that is exactly what you will see in the simulation. And of course, surface potential, this time is 12 kT. So, how much is this? 12 kT/q should be equal to surface potential is 12 kT again back to same thing 0.3volts. ϕ_s is equal 0.3 volts. So, this is what happens in depletion?

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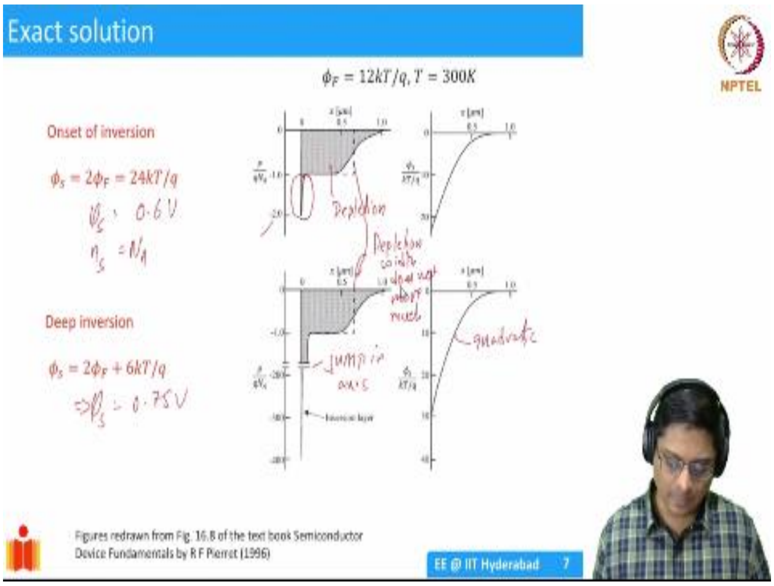

What happens in inversion? Right at the point where surface potential is equal to 2 ϕ_F. In this point, your ϕ_s = 0.6 volts I think, ϕ_F, ϕ_F 300 millivolts. So, this is 0.6 volts. When you have that, look at this, we have depletion region. This is your original depletion region and there is a very sharp spike here and the peak appears to be exactly at – 2. Why is that?

The reason is at ϕ_s equal to you know, 2 ϕ_F, n at the surface, ns call it; electrons at the surface = N_A. That is a definition of threshold. Correct? So, we have N_A contribution from the background, you know, the space charge, and another N_A contribution from the universal charge that is why exactly this peak is – 2. Of course, here, this thing is 24. Once you reach threshold, you see there is a big difference.

You know, when of course there is no applied gate voltage, there is no depletion at all. I did not show you that. So, once you go halfway through depletion, your depletion width in this case was you know, 0.5 microns. 500 nanometres in depletion width and if you go here, there is a slight increase. But about 0.6 or 0.7, it has become now, we can take that as a depletion width, because we have increased the gate voltage, so depletion is going to expand.

But once we do that, you add now another deep inversion, essentially, you add another 150 millivolts, 6kT 150 millivolts. So, this is ϕ_s = 0.75 volts, not a lot of difference. We went from here 0.3, this is phi s = 0.3 volts. From here to 0.6 essentially doubled it. And we saw that the depletion region has changed by maybe about I would say, 30% or so. And there is inversion happening.

But once you go there, even if you, you know change by let us say 150 millivolts, not a very large amount, but you see that the charge density has spiked up. So, this is reaching up to – 400. There is a jump here. This is a jump in the axis. So, jump in. This is the way, you represent whenever you want to represent a large number, but I plot it in a linear scale 400, then you will not see the depletion charge. There is a jump.

So, what you see is if you go into deep depletion or you know, you are going to; after inversion, if you increase, your potential is going to be a huge inversion charge that appears at the interface. And the depletion width does not change so much. You see here, depletion width does not move much. When you compare this to this, there is not much change in the depletion width that is very, very important for us.

Once you reach inversion, the depletion width does not change and we have corresponding surface potential as well. So, this is the exact solutions. So, please take some time and analyse this data. These graphs are very, very useful in understanding of physics. So, this quadratic shape, we have a multiple times of problematic quadratic potential.

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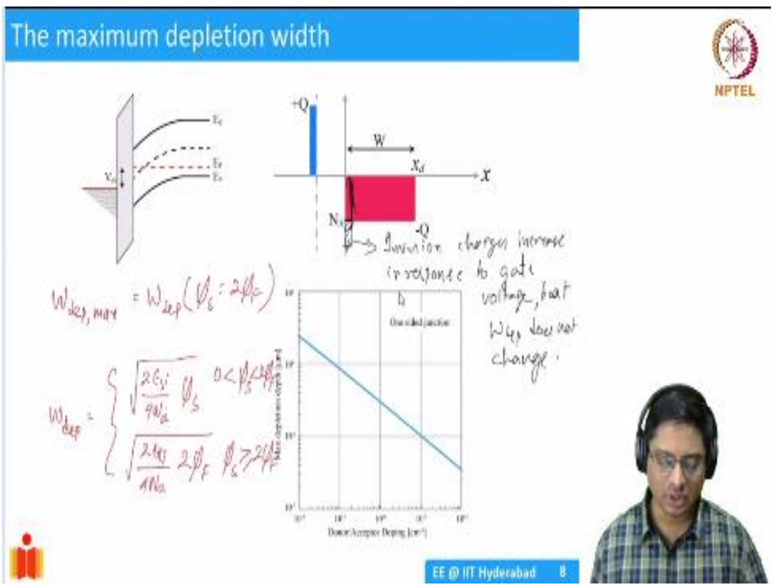

So, before I stop this video, I just want to briefly talk about the depletion width. So, we have seen that as you increase V_G, your depletion width is going to increase but it turns out that we have seen here; once we reach the inversion, the depletion width does not change so much. So, that is why we say that we define a quantity which we call as maximum depletion width which is given by W depletion max is equal to W depletion when phi s = 2 ϕ_F.

Till ϕ_F the surface potential is equal to 2 ϕ_F depletion width keeps changing but once we reach phi s or 2 ϕ_F, then it will not. So, the way to calculate that would be you know, we have seen this expression.

$$W_{dep} = \sqrt{\frac{2\epsilon_{si}}{qN_A}\phi_s} \quad (0 < \phi_s < 2\phi_F)$$

$$= \sqrt{\frac{2\epsilon_{si}}{qN_A}2\phi_F} \quad (\phi_s \geq 2\phi_F)$$

So, we are taking the depletion width not changing. We are reaching the maximum depletion width. And why does it make sense? When it makes sense because after you know, when you say that inversion has reached, you essentially have this inversion charges appearing at the interface. Now, if you further increase the gate voltage, what happens to the inversion charge?

It will quickly respond because this rises exponentially. We saw that in the inversion the charges increase exponentially. So, as you keep increasing gate voltage, they respond and that depletion which does not have to increase. So, inversion charges increase in response to gate voltage but W depletion does not change. And I just calculated the W depletion, the maximum depletion width for various concentrations.

What I want you to take home here is that the depletion width is only about, I would say, you know 0.1 to 1 micron typically or even less sometimes you know, maybe even 0.05 microns, 50 nanometres to about 1 micron. So, that is a depletion width that you can have a typical doping concentration and not more than that. So, we will have a, you know, 500 micron semiconductor wafer, but only the top micron is where most of the physics has happened.

It is like a purely surface phenomenon. So, I will stop the theoretical part of the video right now. In the next video, I will solve a few problems. I will introduce the concept of threshold voltage one more time, I will review it and then I will solve a few problems. I will see you then.