Introduction to Semiconductor Devices
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Lecture - 4.5
PN Junction Electrostatics – Examples

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Hello everyone, welcome back. So, we will continue from where we left off in the last lecture.

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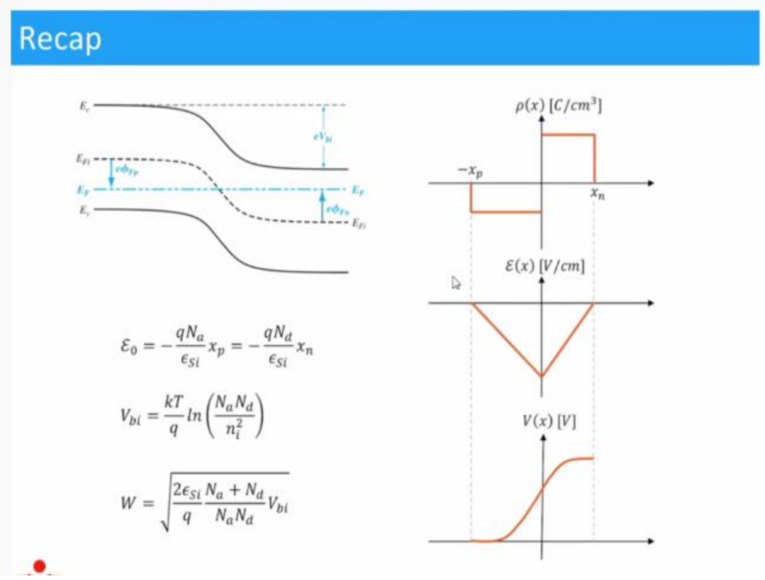

So, in the last lecture, we discussed the PN junction in some depth. We derived expressions for the electric field and the peak electric field at the center of the depletion region. And then we also derived expressions for the built-in potential and the width of the depletion region. So, I mentioned that the width of the depletion region plays a very important role because a lot of the junction physics is essentially that. So, today we will try to move forward.

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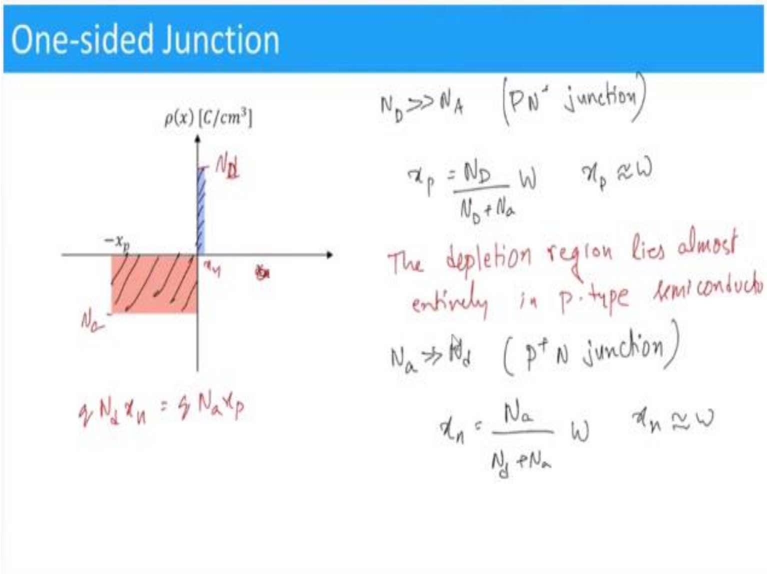

The agenda is to discuss a few special cases. So, I will pick up problems where you have various doping profiles and we will try to analyze what happens. We will only be applying concepts that we have already learned. We are not going to learn any new concepts right now in this lecture. So, the first situation I want to discuss is a one-sided junction. I think I have mentioned this in passing in the previous lecture but just wanted to do it here one more time just for sake of completeness.

So, let us say, we have a semiconductor with the doping in such a way that the N_d, the N type doping is much larger than N_a. What would you expect? So, this is what is known as PN^+ junction. So, basically N type doping is high. So, what do you expect in that situation? So, in the previous lecture, we have seen that x_p, the depletion width in the P region of the semiconductor was equal to that in the n type doping region, which is $x_p = \frac{N_d}{N_d + N_a} W$. So, whenever N_d is large, the above expression essentially becomes 1 and x_p becomes approximately equal to the W, width of the depletion region. Thus, the depletion region lies almost entirely in P type semiconductor for PN^+ diode.

The same thing is schematically represented in here. With a larger doping, the depletion width is small and when the doping is smaller, the depletion width is large. This relation, $qN_d x_n = qN_a x_p$, will always hold true. The total charge should always be conserved. In a junction, the area under the N_d and N_a regions is always equal.

So, you could also analyze what happens when N_a is much greater than N_d. This scenario is called as P^+N junction and is again another type of one-sided junction because the depletion region is going to lie in only one type of semiconductor. The depletion region is going to be on the side whichever has a lower doping. So, essentially I mean you could understand intuitively also that you know when you have a large doping then it will lead to excess carriers which can diffuse into the other region and then create a space charge. So, in this situation, $x_n = \frac{N_a}{N_a+N_d} W$ and x_n will be almost equal to W. These are what are known as one sided junctions. And they happen often in our semiconductor devices. For example, we will talk about them when we talk about MOSFETs.

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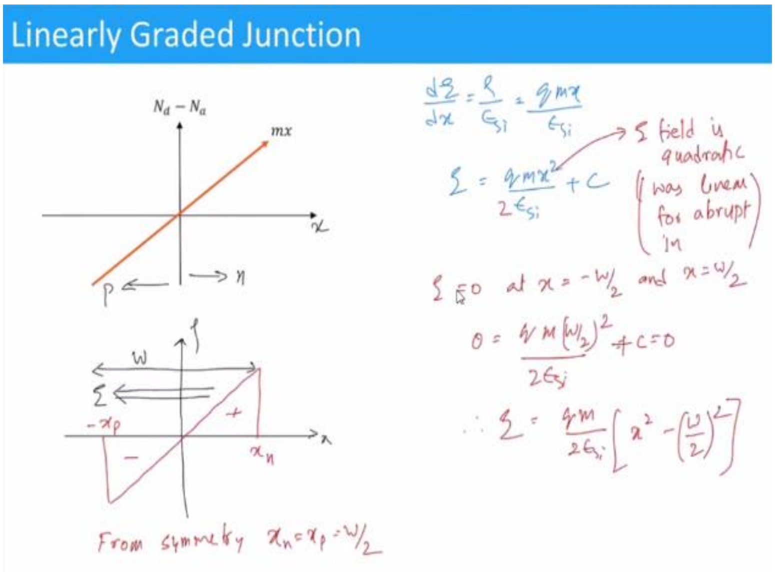

So, the next type of doping profile, I wanted to introduce was linearly graded junction. You remember in the first lecture on PN junctions I have mentioned that there are various types of junction profiles possible because the diffusion is never going to be abrupt, that is, not going to be realistic. So, we are going to have some profiles depending on the slope of the N_d versus x.

So, depending on how the slope is you can have different sort of responses. So, as depicted here, for linearly graded junction the doping is linearly graded. So, what will happen? How do we analyze this? So, how is the space charge region in this case? What will happen? Please think about that. So, in the previous case it was straightforward uniform doping and then there was diffusion.

What would happen in this case? Still you would have the same. You know this is N type region on the left side here. This is N type and this is P type. So, you still have excess carriers. But, at the junction, you have relatively few free charges. But, you still have a lot of excess carriers on the other side in the bulk of the semiconductor. So, they are going to cause some diffusion. So, how will the space charge profile look like?

Since there is going to be some diffusion you can draw the space charge versus x. So, how do you think the profile should be? You know this lecture is going to be somewhat interesting with interesting cases. So, if you are unsure please pause the video, think about it, come up with an answer and then you can verify. In the lecture I will just give you the solution.

But I would really encourage you to pause the video and then think about it before going ahead. As there is going to be diffusion, some amount of free charges will move to the other side, uncovering space charge. And because the doping profile is now linearly graded even the space charge has to be linear. This is a certain point, please make sure that you are convinced by that.

So, your space charge region is going to be like this. You are going to uncover a finite amount of space charge and, which will cause an electric field just like in the abrupt junction case. The only subtlety here is that your space charge is going to have a shape of linear profile. That is the only subtlety. And the other interesting thing is, let us say, if you call this as x_n and this as $-x_p$, the edge of the depletion regions.

What can we say about the edge of the depletion region? Is there any relation between x_n and x_p? Well, it is a simple linear doping with symmetry about the $x = 0$ point. So, $x_n = x_p$, and if you call this entire region as depletion width, $x_n = x_p = W/2$, the depletion region is symmetrically spread. It is not one sided or it is not non-uniform as you saw in the previous case. It is just coming because of the symmetry of the doping profile. So, we got the space charge. So, to analyze any of these junctions, this is a sequence or the recipe that we adopt. First, we look at the doping profile, we look at the diffusion, and then identify where the space charge is and what is the sign of the charge.

Once we know all these, we calculate electric field and then calculate the built-in potential. So, now, again, what will be the direction of the electric field? Well, you have a positive charge in the region $x > 0$ and negative charge for $x < 0$. The electric field is going to be in the $-x$ direction.

In the abrupt junction case, we see that electric field is of negative value and it is triangular in profile. What will happen here? We get the electric field from the Gauss law. Gauss law turns out to be $\frac{dE}{dx} = \frac{\rho}{\epsilon_{Si}} = \frac{qmx}{\epsilon_{Si}}$ with ρ being a linear function. So, this is going to be your electric field differential equation, which will dictate your electric field. And so, you can integrate this and quickly you get $E = \frac{qmx^2}{\epsilon_{Si}} + C$. So, note, this is an important difference from the abrupt junction. Here, E field is quadratic. It was linear for abrupt junction because there you had a uniform doping. But now it is quadratic as doping is linear. Fine, but, what is the constant C? What is the recipe? So, this is an interesting problem for you to clear up your concepts you know how we derived the expressions. So, please do not get bogged down by the mathematics.

Think about it from a physical point of view. Then you would clearly answer this. So, can the electric field be non-zero in the quasi-neutral regions? Because outside this depletion region it is all quasi-neutral. So, there is no charge in that region and thus electric field is zero. And you could also think about it like a quasi-metal.

You have lots of free charges. So, they will respond in such a way that electric field will be 0. So, what will be the electric field now? So, since I have to make it 0 at $x = 0$, so, $E = 0$ at $x = -W/2$ and at $x = W/2$. Substituting $x = W/2$, you get $0 = \frac{qm({W}/{2})^2}{\epsilon_{Si}} + C = 0$. Therefore, your electric field is going to be $E = \frac{qm}{2\epsilon_{Si}}\left[x^2 - \left(\frac{W}{2}\right)^2\right]$. So, it is a quadratic form and looks like this.

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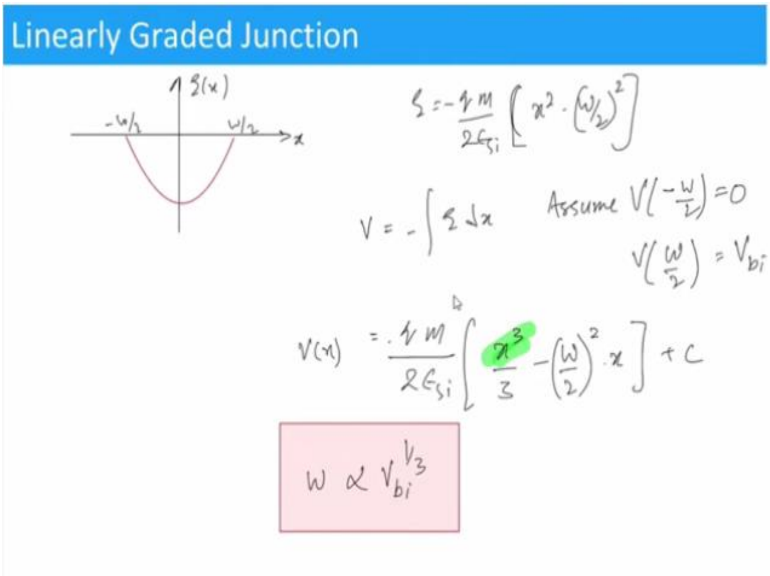

So, what happens to the potential built-in potential? It is of the form $V(x) = \frac{qm}{2\epsilon_{Si}}\left[\frac{x^3}{3} - \left(\frac{W}{2}\right)^2 x\right] + C$ with depletion width, $W \propto V_{bi}^{\frac{1}{3}}$.

So, in the previous case, the width of the depletion region was proportional to the square root of voltage. But if you have a linearly graded junction it becomes cube root and so on. So, I mean there are more complicated profiles that researchers study because they want to look at exact analysis. But the recipe used is same.

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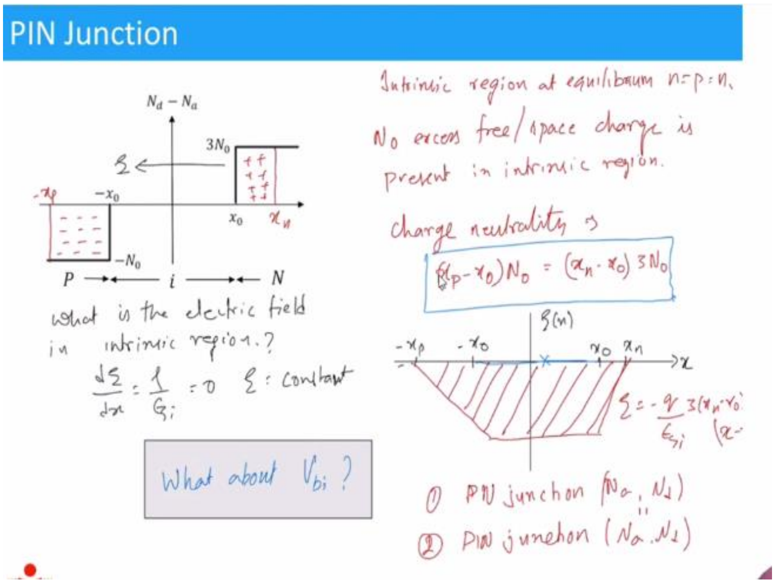

And then I wanted to present one more case which is a very important device as far as opto electronic applications are concerned. It can be used in solar cells and photo detectors. And it is a very interesting device.

So, instead of a simple PN junction you have a PIN junction. So, what it means is, you have a P type region, you have an N type region, both are semi-infinite, extending to large x. And in between them you have an intrinsic region which is sandwiched.

Let P region has some doping density, $-N_0$ and N region has a doping density as $3N_0$. Analyzing the condition of the intrinsic region, there is no excess charge here because we have not doped it. So, there cannot be any space charge. And there is no free charge or excess free carriers because both N and P are equal. So, essentially we have no excess free or space charge present in intrinsic region.

But lot of holes present at below $-x_0$ and lot of electrons present above x_0 and so, there will be some diffusion. Now, you have electric field due to opposite charges present. Of course, you are going to have electric field here all across this. What is a functional form of electric field in the intrinsic feature? The governing equation for electric field is always the Gauss law, $\frac{dE}{dx} = \frac{\rho}{\epsilon_{Si}}$ and since $\rho = 0$ in the intrinsic region, E field is constant.

So, will this constant be 0? Or, will it be something? You have to think about. We will come back to that. This is in the intrinsic region. So, what is the E field in the extrinsic regions N and P? Well, we can solve it. We have solved it multiple times now already. It is going to be a linear function because the doping is uniform. The only difference here is, remember, we still have to satisfy charge neutrality because there was no charge originally.

The charge neutrality implies your P type doping is only N_0. So, the relation, $(x_p - x_0)N_0 = (x_n - x_0)3N_0$ should be maintained. We can quickly write the electric fields and derive expressions for it.
I will show you the form of electric field, *i.e.*, $E(x)\ vs\ x$. So, you have a few critical points here. The electric field for the region greater than x_n and below x_p have to be zero, and because of Gauss law the electric field has to be of linear form in P and the N region. In between them,

will it come down to 0 here? Well, electric field has to be continuous. There is no reason for it to have this no discontinuity because there is no a charge.

If there is a boundary charge, then there can be discontinuity in the electric field otherwise no. So, because of that electric field here will be this one, constant. So, this is your electric field profile in a PIN junction. It has a lot of implications especially in photo detectors. So, you can compute the expression for electric field on N-side of the junction. It will be $E(x) = -\frac{q}{\epsilon_{Si}} 3(x_n - x_0) N_0 (x - x_n)$. Similarly, it can be computed for the P side. So, this is how electric field behaves. I will revisit this when we discuss photo detectors in the last eleventh week of the course in the towards the end of the electronic devices. The built-in potential will be the area under the curve.

So, I will give you a brief demo of a tool on nanoHUB today. And then you can try to use the tool to compute. We will give you a problem, say consider two devices: (1) A simple PN junction with some N_a and N_d doping given, and (2) A PIN junction with same N_a and N_d. So, with this situation I will ask you to run the tool and see, what happens to the built-in potential, V_{bi} in both cases? We will just help you run the code and answer the questions in the assignment.

And before I close I want to show you one more interesting problem.

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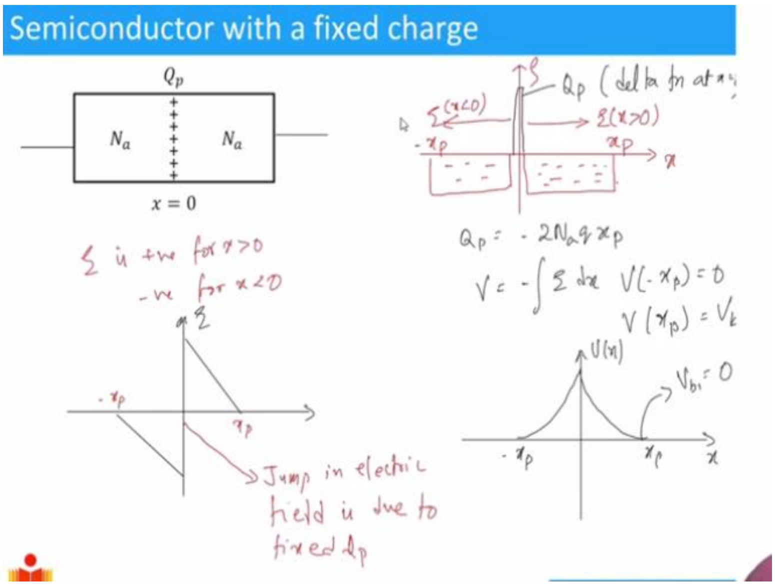

So, please do not get worried if you do not if you are not able to solve it immediately or you know it will take you some time to solve it. This is I would say a challenging problem. And if you are able to analyze this correctly then you would I would assume that you understand the junction electrostatics very well.

I do not expect it out of a UG student. But I am just including it here so that it is a good sanity check for your understanding. I have seen that UG students were able to solve this problem in the past. But, out of a class of 60 or so, there might be 2 students who will solve it correctly fully correctly. So, many people would get to different levels. So, do not feel too worried if you are not able to get it. But, take your time and analyze this.

So, the problem is like this. You have a piece of semiconductor and then somehow there is a fixed charge in that. In this case, let us say it is a P type semiconductor and then somehow there is a fixed amount of charge Q_p which is inserted into the semiconductor.

It can happen during a growth process in MBE or something. It can happen or maybe when it is exposed to some radiation and then there is some damage to the crystal. And then there is some fixed charge at a point. This can happen experimentally also. So, what happens when there is some fixed charge at a point in the given semiconductor

So, you have a positive charge is being introduced into a P type semiconductor, which is symmetric about $x = 0$ having a lot of holes which are the free to move. What would happen? The positive charge is going to repel the holes.

And you are going to have a space charge which looks like this. So, you have the fixed charge in the center which you know is already there. It is like a delta function. This is going to be Q_p. It is like a delta function at $x = 0$. And you have the holes which are repelled on either side of the junction. So, because of that you have a space charge region here and a space charge region here. It is all it is going to be symmetric between $x = -x_p$ and $x = x_p$. And this is going to be negative on both sides because holes are being repelled on both sides. Well, the charge neutrality still should be valid. So, $Q_p = -2N_a q x_p$.

See the important thing here is that electric field E is positive for $x > 0$. So, E field direction is going to be in the $+x$ direction. And, for $x < 0$, E field is going to be in the $-x$ direction and there is going to be some sort of a jump at $x = 0$. The jump in electric field is due to fixed Q_p charge. This is in accordance with the boundary conditions as per the Maxwell's equations.

And whenever there is a jump in electric field, there has to be a charge surface charge there. In this case, since we are solving in 2 dimensions, there is going to be a line charge. So, this that charge causes the discontinuity in the electric field. It was not there in any of the previous scenarios that we considered. Only in this scenario, we have that because of just the fixed charge.

So, then what happens to the potential? The potential is simply going to be integral of x.
$V = -\int E(x)\,dx$ with $V(-x_p) = 0$ and $V(x_p) = V_{bi}$. So, now as you integrate it you would see that you have to take a boundary condition. Please integrate it. You will see that since it is an electric field is linear the potential is going to be quadratic. And the built-in potential is going to be something like this.

So, this problem might be a little challenging. I do not expect any regular undergraduate student to really solve it. But if you are somebody that is interested in understanding the device physics in depth, this would be useful thing.

So, with that now, if you are able to solve it fully, I am sure that you have a very good hang of electrostatics of PN junctions. So, with that I would like to stop today. I will just also add a brief demo of the nanoHUB tutorial that I mentioned, nanoHUB tool that I mentioned. And then in the next week, we will start analyzing what happens with applied voltages. So far, we have not applied voltages to the junction.

So, we will start applying voltages and see what happens to the current. Thank you so much for your attention. I will see you in the next week.