

**Introduction to Semiconductor Devices**  
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**Lecture – 3.7**  
**Quasi Fermi Level and Minority Carrier Diffusion Length**

This document is intended to accompany the lecture videos of the course “Introduction to Semiconductor Devices” offered by Dr. Naresh Emani on the NPTEL platform. It has been our effort to remove ambiguities and make the document readable. However, there may be some inadvertent errors. The reader is advised to refer to the original lecture video if he/she needs any clarification.

**(Refer Slide Time: 00:11)**

The slide illustrates the minority carrier diffusion length in a band diagram. It shows energy levels  $E_C$  and  $E_V$  with a Fermi level  $E_F$  and a quasi-Fermi level  $E_{Fn}$ . The carrier concentration profile is given by  $\delta p(x) = \delta p(0) \exp(-x/L_p)$ . Handwritten notes include: "As  $x \rightarrow \infty$ , sample approaches equilibrium. Hence,  $E_{Fn}$  &  $E_{Fp}$  should move towards  $E_F$ ".

We talked about the minority carrier diffusion length. And we said that if you have a semiconductor with a doping density  $N_d$  and you shine light, you are creating electron hole pairs at  $x = 0$ . And then because the light is not really penetrating into the semiconductor, we said that this length was greater than the  $L$  absorption, absorption length in the light that particular wavelength of light.

Essentially, you are going to have exponential decay in the carrier density and we said, we can define a minority carrier deficient  $L_p$ . So, this is what we have seen already. Now, we want to represent that in a band diagram. What will we do? So, to represent this, you know, this sort of a decay in the carrier density, excess carrier density, how do we represent that?

So, well, I mean, you can guess it already. You might have guess but anyway, let me go through the motions. So, basically, I will say my  $E_F$  is going to be the same. I did not change the problem point  $E_V$ . This is going to be  $E_F$ . Now, let us say this is your  $x$  variation. So, this is basically  $x = 0$ . At  $x = 0$ , let us assume that you know the carrier density is the same  $10^{17}$  just for convenience.

Basically,  $\delta P(0)$ . Let us assume that it is  $10^{17}$ . So, if your  $\delta P$  was that, of course,  $\delta n$  also was that much. So, basically, where will your Fermi level see at  $x = 0$ ? There will be at you know, this point and this point. QFL's 0.42 eV away from  $E_i$ . We have seen this in the last couple of slides. Now, let us assume the carrier density, you know what should happen as  $x$  tends to infinity.

So, rather you know, we just at, you know, when we are just reusing the concepts as  $\delta n$  tends to 0 and  $\delta p$  tends to 0,  $E_{Fn}$  tends to  $E_F$  and  $E_{Fp}$  tends to  $E_F$ . That's what we have seen. So, if you go far away, as  $x$  tends to infinity, let us say, the Quasi Fermi level should merge with  $E_F$ . I will put as you know,  $x$  tends to infinity. Let us take just one more case at  $\delta n$ , at some  $x$ , where  $\delta n = 10^{10}$ ,  $\delta p = 10^{10}$ . Because my  $\delta p$  is reduced the exponentially, some  $x$  you will have this situation.

What will happen? At this point,  $E_{Fn}$  is going to be equal to  $E_F$  but  $E_{Fp} = E_i$  because  $10^{10}$  is intrinsic carrier density. So, the Quasi Fermi level for hole has to be at  $E_i$ . So, at this point, where you know, this was happening  $10^{10}$  and this is almost here. This is at  $\delta p = 10^{10}$ ,  $E_{Fp} = E_i$ . Please verify these things. So, what we are seeing is the same thing you know, in the previous case, we saw that as its approach equilibrium, the Quasi Fermi levels should move towards  $E_F$ .

So, what should happen qualitatively is that? This is a Quasi Fermi level. As  $x$  increases, the Quasi Fermi level should move towards  $E_F$ . Similarly, the electron Quasi Fermi level also should come towards the same point. So, this is qualitative. Qualitative analysis tells us that as  $x$  tends to infinity, sample approaches equilibrium. Hence,  $E_{Fn}$  and  $E_{Fp}$  should move towards  $E_F$ . This is the conclusion we have made in this from the analysis.

So, now, let us try to do qualitative calculation. I mean, you could actually do that. So, this is still hand waving right and we just put few numbers that we want to have.

(Refer Slide Time: 05:09)

The slide illustrates the minority carrier diffusion length in a band diagram. It shows energy levels  $E_c$ ,  $E_i$ , and  $E_v$  with Fermi levels  $E_{Fn}$  and  $E_{Fp}$ . The carrier density is given as  $N_d = 10^{15} \text{ cm}^{-3}$ . The minority carrier concentration is shown as  $\delta p(x) = \delta p(0) \exp(-x/L_p)$ . A handwritten note states: "Exponential decay can be represented by a straight line on the band diagram." The slide also includes a video inset of a speaker and the NPTEL logo.

We can actually introduce exponential decay into the expressions. So, what is the definition of  $E_{Fp}$ ?

$$E_i - E_{Fp} = KT \ln\left(\frac{p}{n_i}\right)$$

So, that is going to be equal.

$$E_i - E_{Fp} = KT \ln\left(\frac{p_0 + \delta p(0) e^{\left(\frac{-x}{L_p}\right)}}{n_i}\right)$$

Let me assume that you know the excess hole density is much higher than the equilibrium density. This is a low level injection and we saw that it was actually quite possible.

We said, it is be possible. Assume  $\delta p$  is greater than delta, sorry,  $p_0$ . We were significantly perturbing the minority carriers density. If that happens, the expression becomes,

$$E_i - E_{Fp} = KT \ln\left(\frac{\delta p(0)}{n_i} e^{\left(\frac{-x}{L_p}\right)}\right)$$

This is the expression, so, log a into b is log a plus b that is going to,

$$E_i - E_{Fp} = KT \ln\left(\frac{\delta p(0)}{n_i}\right) - \frac{KTx}{L_p}$$

So, you see, what is happening is the distance, this was  $E_i - E_{Fp}$ .  $E_i - E_{Fp}$  is like a you know,  $y = mx$  form,  $mx + C$ . So, in the way we are writing  $C$  plus  $mx$  sort of it. So, this  $\frac{KT}{L_p} = m$ , and

$C$  is going to be equal to what  $10^{17}$ . So,  $C = 60 \times 7 = .42 \text{ eV}$ .  $0.42 \text{ eV}$  minus some slope, you

know  $L_P$  we do not know what it is and into  $x$ . So, you can actually try to put an  $x = L_P$  and see what happens.

Try to calculate it by the actual exponential and try to see by this approximate not approximate, yeah approximation, because we said,  $\delta p$  is much, much greater than  $p_0$ . But at  $x = L_P$ , this will be true. Try to check it out or same order of magnitude. So, you have to see that what happens. Please do that. Please try to put in the numbers and check it out. So, that you are confident.


So, what we are now saying is: this exponential decay here, can be represented by a straight line on the band diagram. So, this is the conclusion of the analysis here So, what I will do is: I will pick up my  $E_i$ , you have done it in the last slide. So, essentially this is going to be your  $E_{Fp}$ . This is  $E_{Fp}$ . You see, the Quasi Fermi level for the electron is not really moving that much, because it is a majority carriers and we saw that the total density does not move so much.

The minimum, it can get to is a equilibrium majority carrier density which is  $10^{15}$  in this case. It only changes by 2 orders of magnitude whereas, the hole density changes by 10 power you know,  $10^{17}$  to upto  $10^5$ , so 12 orders of magnitude. So, you see that there is a large change in the minority carriers density  $E_F$  that is why  $E_F$  is changing by significant amount. So, this is a concept of Quasi Fermi levels.

And we will actually analyse and we apply next week of every where we will analyse a p-n junction will actually show you or rather when we talk of p-n junctions in forward bias, we will talk about, maybe the week after. This is going to be useful. So, please practice some of these things that I mentioned.

**(Refer Slide Time: 10:07)**

**Quasi Fermi level and currents in nonequilibrium**



$$J_p = q\mu_p p E - qD_p \nabla p$$

$\downarrow$  drift       $\downarrow$  diffusion

$$p = n_i \exp\left(\frac{E_i - E_{Fp}}{kT}\right)$$

$$\nabla p = \frac{dp}{dx} = n_i \exp\left(\frac{E_i - E_{Fp}}{kT}\right) \cdot \frac{1}{kT} \left(\frac{dE_i}{dx} - \frac{dE_{Fp}}{dx}\right) \quad \left\{ \frac{1}{kT} \frac{dE_i}{dx} \right.$$


$$= p \frac{q}{kT} E - p \frac{1}{kT} \frac{dE_{Fp}}{dx}$$

$$J_p = q\mu_p p E - qD_p \left( p \frac{q}{kT} E - p \frac{1}{kT} \frac{dE_{Fp}}{dx} \right)$$

$$= q\mu_p p E - q\cancel{p} D_p \frac{q}{kT} E + p \frac{q}{kT} D_p \frac{dE_{Fp}}{dx} = p \mu_p q E_{Fp}$$

$J_p = p \mu_p q E_{Fp}$   
 $J_n = n \mu_n q E_{Fn}$

$\rightarrow$  Gradients of QFLs give the current density.



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So, there is also another advantage of using Quasi Fermi levels that is, you know, we saw already that the current in the semiconductor is consisting of the drift and the diffusion terms. These are the 2 terms that are contributing to the current. We have seen that in the last 2 lectures, 2\_3 lectures, you have seen this. So, you see that you know, Quasi Fermi levels, now,  $p$  is basically the term defined in terms of the Quasi Fermi levels so and you have the  $(D_p/dx)$  in the expression. So, let us try to do that you know.

So, I wrote it as grad  $p$ , essentially grad  $p$  in one dimension is  $(D_p/dx)$  and that is going to be; I will take a derivative of the carrier density. So, basically that is going to be,

$$\nabla p = \frac{dp}{dx} = n_i e^{\left(\frac{E_i - E_{Fp}}{kT}\right)} \frac{1}{kT} \left(\frac{dE_i}{dx} - \frac{dE_{Fp}}{dx}\right)$$

Remember, we said, electric field is going to be,

$$E = \frac{1}{q} \frac{dE_i}{dx}$$

So, we can substitute that into this expression. So, this is going to be;

$$\nabla p = \frac{dp}{dx} = p \frac{q}{kT} E - p \frac{1}{kT} \frac{dE_{Fp}}{dx}$$

this is a simplification. So, now, let us put it back into the current expression. So, this will take you to the derivative of the carrier density. Let us put it back into the current expression.

So,  $J_p$  is going to,

$$J_p = q\mu_p p E - qD_p \left( p \frac{q}{kT} E - p \frac{1}{kT} \frac{dE_{Fp}}{dx} \right)$$

So, I can expand it out. And If you apply Einstein relationship, because Einstein relationship,

$$D_p \frac{q}{KT} = \mu$$

So, basically, this is going to be,

$$J_p = qp\mu_p E - qpD_p \frac{q}{KT} E + p \frac{q}{KT} D_p \nabla E_{Fp}$$

So, again, Einstein relationship will give you this to be,

$$J_p = p\mu_p \nabla E_{Fp}$$

So, what is happening is: if you are using Quasi Fermi levels, we are able to represent the current densities  $J_p$  as simply  $p\mu_p$  gradient of Quasi Fermi levels.

$$J_p = p\mu_p \nabla E_{Fp}$$

$J_n$ , when I did not show it, but  $J_n$  is going to be,

$$J_n = n\mu_n \nabla E_{Fn}$$

You will see quickly in the next weeks that when you have diffusion, drift, it becomes very difficult for us to keep track of what are the contributions to currents.

So, simply, we can make our life easy by showing the current as gradients of QFLs give the current density that is it. So, that is why Quasi Fermi levels are very useful. So, yeah, I mean there are multiple applications, and we will use them in subsequent lectures. So, this essentially, you know, finishes the basic fundamentals that I want to cover.