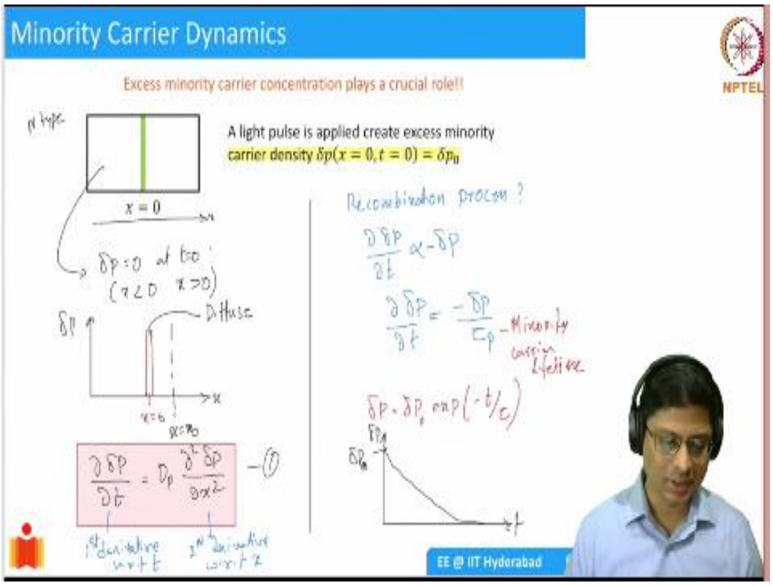
Introduction to Semiconductor Devices
Dr Naresh Kumar Emani
Department of Electrical Engineering
Indian Institute of Technology – Hyderabad

Lecture – 3.5
Minority Carrier Dynamics

This document is intended to accompany the lecture videos of the course "Introduction to Semiconductor Devices" offered by Dr. Naresh Emani on the NPTEL platform. It has been our effort to remove ambiguities and make the document readable. However, there may be some inadvertent errors. The reader is advised to refer to the original lecture video if he/she needs any clarification.

(Refer Slide Time: 00:11)



We want to determine how these quantities change as a function of position as a function of time. To do that, we need to solve some equations. We try to build that equation. To do that, let us just consider a situation where you have basically a semiconductor. Let us call it an n type semiconductor. We could easily take a p type semiconductor. There is no harm. There is no difference at all so, n type semiconductor.

And it has a doping, some doping. Right now, I do not need to worry about what is the doping and I will shine light only at you know, I will apply light pulse. So, only at x = 0 some position, you know this is position x. At position x = 0, I apply light pulse and then I create excess carrier density which is equal to δp_0. Somehow, just shine light only focus it only on certain section of the semiconductor. So, now what happens?

You are created the other places you have. δp_0 might confuse you, but essentially I am saying the δp_0 is at time $t = 0$. Its not the changes in the p_0. p_0 is the equilibrium carrier density. But δp_0 means that δp at time $t = 0$. So, now what happens? At other locations in the semiconductor at time $t = 0$. At time $t = 0$ here, I mean away from the semiconductor, away from the $x = 0$ point.

$\delta p = 0$ because it is at equilibrium. There is nothing that is disturbing that. At $t = 0$, for I will put it in is only for $x < 0$ and $x > 0$. At $x = 0$, we are having exactly δp_0. At every other place, there are no excess minority carriers. So, there is a concentration gradient and we saw that there is a diffusion process that happens whenever you have such a situation. So, what will happen is the initial if I take the, if I plot out let me plot this is δp versus x.

So, initially a time $t = 0$, I have my carrier density, minority carrier density, p type right n type semiconductor. So, δp at $x= 0$. Let us say this is the number of holes that are introduced. Now, as time changes what is going to happen? These things are going to diffuse. So, what I need to do is essentially monitor. For example, I can monitor a location maybe here you know, at some $x = x_0$ I will try to monitor what is the number of holes okay.

So, because there is a gradient the holes are going to diffuse. It is going to diffuse and it will cross the surface. This $x = x_0$ at that particular surface. So, I will try to monitor what is the rate at which the minority carriers are causing that surface and that I will denote by a partial derivative of δp with respect to time. So, excess minority carrier at $x = x_0$ is going to be changing with respect to time, and that I will represent by this.

And it turns out that this is a diffusion problem. This is a very classic problem that is introduced probably in plus 2 for some of you. If not just you know, look up Wikipedia and it is a very straightforward problem. So, what this tells you is that, whenever you have this diffusion like processes, the excess minority carrier concentration is going to be dependent on the diffusion coefficient times the second derivative of δp with respect to space.

$$\frac{\partial \delta p}{\partial t} = D_p \frac{\partial^2 \delta p}{\partial x^2}$$

So, this is called as a diffusion equation. So, what this is telling you? You remember this is first derivative with respect to time and this is second derivative with respect to x. And the coefficient that is linking is the diffusion coefficient. This is the same diffusion coefficient which is on the diffusion process. We wrote the current density as you know product of diffusion coefficient times the gradiant.

If you want to look at the time change also, then you could do this. I mean, they are all interrelated. So, this is the original equation from that you can derive the current densities and so on. So, this is your diffusion process that is happening. So, if you just shine the excess minority carriers, and just if you create the excess minority carriers, they will simply diffuse.

In addition, we have also been talking about the recombination process. Let us say, what happens to recombination? How does that happen? We want to monitor how the minority carrier density changes with respect to time. So, let us write it as partial derivative, same partial derivative with respect to time. So, over here, in let us see, if I call this as equation 1, equation 1, I am only talking about a diffusion process.

But definitely we know that there is going to be a recombination process. That also has to be accounted for. And this is partial derivative with respect to time of delta p excess minority carrier density. And whenever you consider recombination process, we know that it is going to be proportional to the excess minority carrier density δp.

$$\frac{\partial \delta p}{\partial t} \propto \delta p$$

Think about it, how will be the rate of change of minority carrier density?

Will the minority carriers increase with time or decrease with time. Whenever you are recombining electron and holes your minority carries density is reducing. So, your proportionality constant should have a you know, I mean, this equation should have a - sign. To denote that you have a decrease in the rate at which the minority carriers are changing with respect to carrier concentration.

And if you write out the actual equation, I mean, the proportionality constant, you could actually solve it. The proportionality constant turns out to be the partial derivative with respect to time,

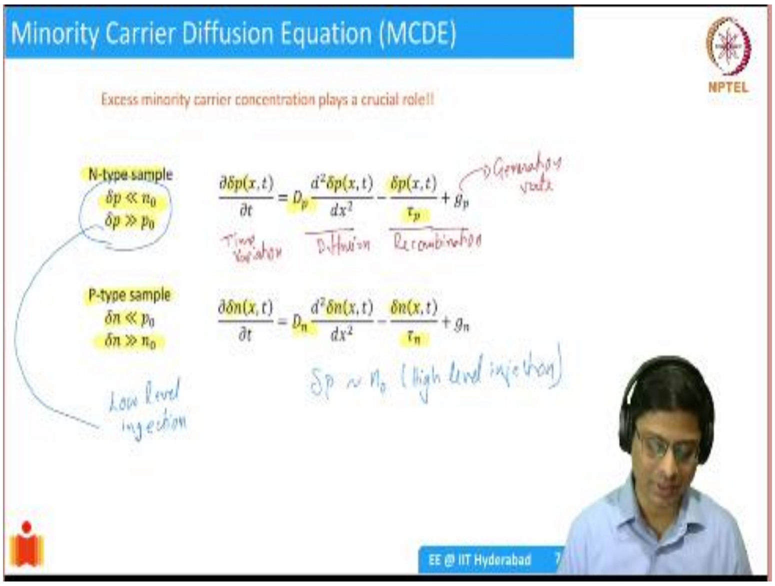
$$\frac{\partial \delta p}{\partial t} = \frac{\delta p}{\tau_p}$$

So, what I am trying to say is this proportionality constant, which I call is minority carrier lifetime. And if you solve this, you will see that actually,

$$\delta p = \delta p_0 e^{\left(\frac{-t}{\tau}\right)}$$

That is why I wrote in one of the previous slides that this is basically an exponential decay. So, you have time here and this is going to be δp, essentially, we want to have an expression. So, this is δp_0. So, this is how minority carriers will change. So, these are the main processes.

(Refer Slide Time: 08:26)



So, what we will do is we will try to sum them up into a equation, which we call as minority carrier diffusion equation. This is something that you will see you know in many places. So, what we did is we are just recapping, we are saying that excess magnetic carriers concentration plays a very crucial role, and the diffusion equation that controls that can be written in this form.

So we are combining both the diffusion process and the recombination process. This is basically time variation at a particular position, how does the magnetic carrier density change in time. That is going to be dependent on the diffusion processes and recombination process. And for complete sake, of course, I did not really describe it. But it also depends on the generation rate.

We are only talking about how the deficient ones are created, but then you also have if you are talking about time dependence, you also have to include the generation rate. And that is what we call us excess minority carrier diffusion equation MCDE. This is for n type semiconductor.

If you have an n type semiconductor you have δp which is a minority carrier, p is a minority carrier and we will write it for minority carriers.

If you have a p type semiconductor, you will write it for n. And that is why this is going to be τ_n, this is going to be τ_p here. Remember all of the coefficient everything are corresponding to only the minority carriers. I should also point out this is a simplified version. If you want to actually solve it in devices, you also have to take into account what happens with the field.

Because you are diffusion process is going to be affected by the field. That is an additional variable, which I have not really taken into account. Well, I mean, if you want to do that, it becomes a little bit more. It is complicated, and we do not gain much out of it right now. So, for the physics necessary for understanding the operation of the devices is this. I mean, if you understand the spatial and diffusion and the time dependence that is enough.

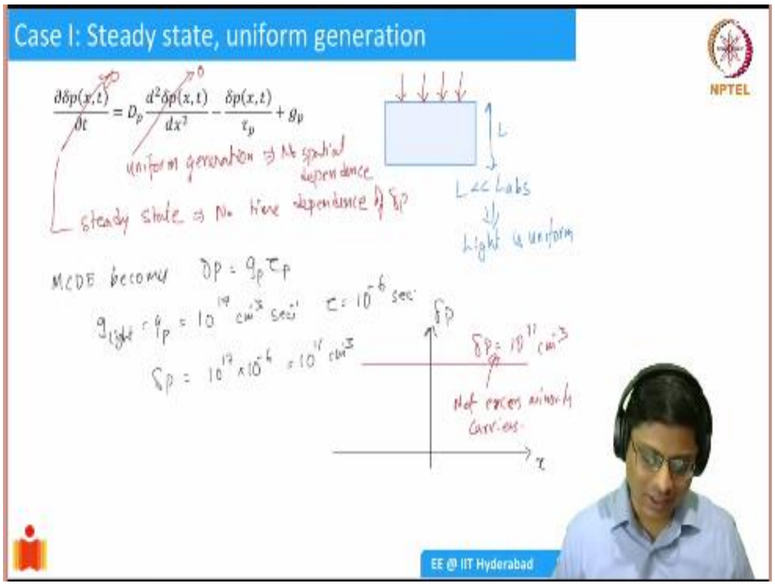
That is why we put in that the field dependence part of it okay. But it is still there, I mean, in the real devices, because we are definitely going to have introduction of minority carriers and then there will be some fields also applied. So, the diffusion process is going to be accelerated sometimes. We have to see that as well. Anyway I also want to point out one specific case this equation is valid essentially for what we call us low light injection.

Low level injection, by which I mean that if I am introducing this you know, the stimulus that is introducing this excess minority carriers is such that the minority carrier density itself is quite small compared to the majority carrier density. The majority carriers are n_0. They are significant in number. We are not reaching a level where you know, you are reaching the minority carriers are reaching that level.

Suppose if you consider a situation where you know, δp is of the order of n_0, the majority carrier density, then this is called as high level injection. Then something has changed. I mean, because we have to have a few more processes that have to be accounted for and all that. Anyway low level injection means that minority carrier density is much smaller than the majority carrier density, but still it is significant compared to the equilibrium minority carrier density.

The δp>p_0. Similarly, for holes, you have the same relation. So, this is what we will consider. If not, then we have to do some corrections. Anyway, so, this is a background, you know what we need to solve? Let us take a few examples to try to solve them. With that, we will know how to interpret the results.

(Refer Slide Time: 12:34)



So now let us consider one case, where you have a semiconductor and shining I mean I am illuminating the semiconductor with the light. Let us assume that this is the thickness of semiconductor. I am drawing it in 2d, but equally applicable to 3d. And because I am doing it in 2d, because I will only need to look at you know, that character in one direction here.

So, my, let us assume that L is much much smaller than the absorption length L absorption. L is much much smaller than L absorption. So, because I am shining light there is going to be absorption length. Light is going to be absorbed at a certain you know rate as of which is given to the absorption coefficient of the semiconductor. And then if your device length is much, much smaller, then you can assume that this implies light is uniform.

Uniform across a semiconductor. There is no spatial dependence. That is why we are assuming that the length of the semiconductor is so. Basically at each point in this device, you have the same amount of light that is incident, so same amount of electron hole pairs. So, essentially, this implies that uniform generation and if uniform generation is happening, uniform generation implies no spatial dependence.

Just different way of seeing the same thing. So, the deficient term is going to go to 0. And I am also saying that this is a steady state problem, by which I mean that I have turned on light and I am waiting for some time till everything stabilises. It is not the same as equilibrium. Because once you introduce electron hole pairs by using light, there is an excess minority carrier density. And then there is some recombination.

So light is driving the creation of excess minority carriers and recombination is actually reducing. So, there is a certain equilibrium you know, there is certain steady value of electron hole pairs each. And that is why we will say that you know, we are saying that it is that steady state. So, essentially there is no change with respect to time. So, basically what I will say is this one steady state implies no time dependence of δp.

No, I am taking an n type semiconductor. So, δp is time dependent. So, that is what it is. So, that is it, we have everything that we need to solve this particular situation. So, what does MCDE reduce to? Well, MCDE becomes, the minority carrier diffusion equation MCDE becomes well, taking the other side, so basically δp when there is no time, there is no time dependence. There is no space dependence.

I can simply write as $\delta p = g_p \tau_p$. Let us say I am having light, generation rate of light generated electron hole pairs g_L, write it as lights you know. This is equal to g_p. The rate at which holes are generated is going to be let us say 10^{17} centimetre cube per second. This is the rate at which I am generating. So, what is the minority carrier density?

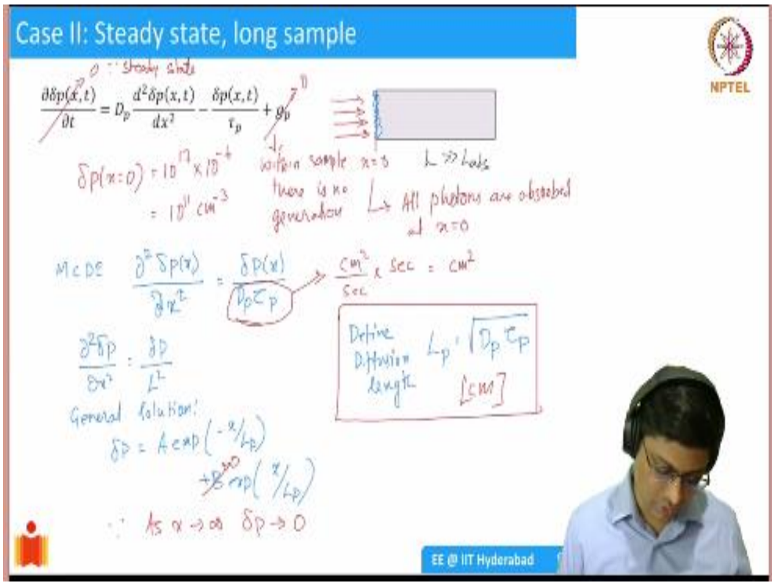
δ_p is going to be what you might think that it is 10^{17} but no, there is a recombination process which is going to result in a steady state. So, the amount of excess minority carrier density is going to be, I have to also tell you what is τ? So let us say $\tau = 10^{-6}$ seconds. This is a kind of large number I would say largest number, typically the lifetime is going to be smaller.

So then $\delta p = 10^{17} \times 10^{-6} = 10^{11} cm^{-3}$. So, you have this much of minority carrier density in the semiconductor. So, you could actually, how does that depend on position? Well, if you take this as a semiconductor, if I plot δp versus x, what is it going to be? Well, it is going to be a uniform generation.

So there is no spatial dependence. $\delta p = 10^{11} cm^{-3}$. I want you to appreciate this that even though the rate of generation is 10^{17} in my case, in the steady state, the net excess minority carrier density is going to be 10^{11}. This essentially the net excess minority carriers. Remember, this is happening because we have continuously the light on.

If you switch off the light, immediately, you know, it will start decaying and then you will go to 0. You should have the physical picture. Equation is simply telling you the physical picture, if you are understanding the physical description of the problem, then equations are very easy to apply, not the other way around. It is difficult to decode the physics from the equation, but the other way around is very simple. So this is the first case where you know, fairly simple.

(Refer Slide Time: 19:08)



Now let me take the second case, where I will say that my length of the semiconductor is large compared to the absorption length. So, what I will do is, now I will take a semiconductor which is long I wanted a rectangle. Let me do it one more time. So, now my length is going to be L is going to be greater than L absorption, much, much greater. So, what I am trying to say is, when you shine light, for example, let me say I will shine light.

Now this side is very easy. All my photons are going to be absorbed only at x = 0. This left side is x = 0. All photons are absorbed at x = 0. What I am trying to say is the length is much, much longer, let us say, you know, the photon has an absorption length of, let us say, one micrometre or something like that. If you are considering 10 microns, or you know, one millimetre sample, then essentially you could take the absorption is happening only at x = 0.

That is why the long sample, and it is still steady state. So, what happens? You turned on the light and then see, you want to watch what happens. Well, one important point is we do not have to worry about we are simplifying the problem. We are assuming that the light is not penetrating into the sample. So, there is no generation. So, zero within the sample right? Within sample, there is no generation.

But still, depending on the extent of you know, generation rate at δp at x = 0, it is going to be some number. It is still going to be if you have a rate of 10^{17}, let us say. That is what we took last time, right? We took 10^{17}. So, let us take the same numbers $10^{17} \times \tau_p$, $\tau_p = 10^{-6}$. So, we will have $\delta p(x = 0) = 10^{17} \times 10^{-6} = 10^{11}\ cm^{-3}$.

We are shining light. It is getting absorbed all of it, and it is creating some sort of a you know, at x = 0, we will have this much of excess minority carriers. So, now what happens within the sample? So this is the rate at which the photons are coming and then that is generating an excess steady state number of 10^{11}. So, previously, we had 10^{12}, no 10^{11}.

So what happens now? So you are generating these, you know, huge number of minority carriers. In this case, holes are being generated here. What happens? Of course, they are going to diffuse. So, we want to solve the equation in the semiconductor. So, to solve in the semiconductor, at steady state, we turn on and we just wait for some time, we do not do anything.

We wait for some time. When we do that, this is basically steady state. So, this is going to go to 0, this expression is going to go to 0 because steady state. I should write it in a different way. This goes to 0 because steady state. So, now our MCDE becomes what? The second derivative with respect to position, so I will write it as δp. Now this can depend on position.

δp is going to depend on position the time dependence is gone. But there is still position dependence. I mean, I can write it as partial derivative or even the full derivative, it does not really matter. And I will take the recombination term on the right side. So, basically, I can write it as δp(x) divided by τ_p.

$$D_p \frac{\partial^2 \delta p(x)}{\partial x^2} = \frac{\delta p(x)}{\tau_p}$$

This is my MCDE, now, simplified MCDE.

So now what I will do is I will actually move this guy, D_p, I will put in here the denominator.

$$\frac{\partial^2 \delta p(x)}{\partial x^2} = \frac{\delta p(x)}{D_p \tau_p}$$

So, what happens is, what are the units think? Think about the units of the denominator here. By dimensionality you should already see that it should have a position square but let us just see. So, this is the D_p is going to be the diffusion coefficient. So, centimetres square per second times second, τ_p seconds. So, you should have a centimetres square unit. So, now what I will do is I will define this.

I will define quantity which I will call as, diffusion length L_p.

$$L_p = \sqrt{D_p \tau_p}$$

So, this is my, this diffusion length is one important part of my problems. So, please remember so this is going to have a unit of centimetre. Always remember, you know, in semiconductor device physics, the convention is that we use centimetres for lengths, we do not use metres.

This is a strange convention, but no other textbook use it. And It is been historical reasons, you know, it is much more common. So, this is going to be having centimetres units. So, now what happens to this equation? This equation now becomes.

$$\frac{\partial^2 \delta p}{\partial x^2} = \frac{\delta p}{L^2}$$

So, I can solve this. Well, when I say I can solve this, I will not solve it, but I will tell you the general solution.

You should have solved this in some of your differential equation ordinary differential equations course. If you have not done that, just verify that this solution is correct.

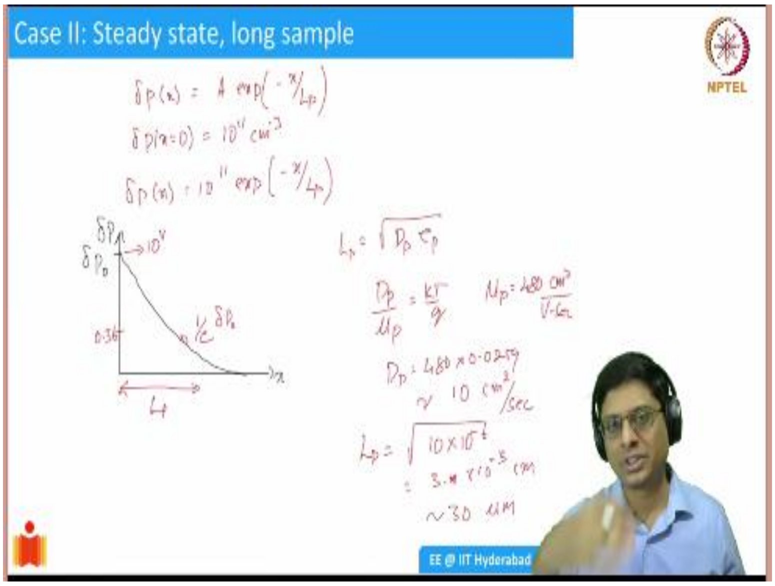
$$\delta p = Ae^{\left(\frac{-x}{L_p}\right)} + Be^{\left(\frac{x}{L_p}\right)}$$

This is a general solution that is allowed for the equation. You can verify it, you know.

Yeah, take a secondary derivative with respect to position. You will see that you know, it is coming out to be this. So, the problem will be which of the coefficients is allowed? A or B? So, now, we are shining light on x = 0. If you allow the second coefficient to be there, if you know this goes to 0, essentially, let me summarize. B goes to 0 because as x tends to infinity, δp has to tend to 0.

Because you are only having light at the left edge. As you go further into semiconductor the carrier concentration has to drop. So, because of that, B has to be 0. Therefore, B is 0. So, only term that is remaining is A term, and we can find out, how to solve for it. So, let us do that.

(Refer Slide Time: 27:39)



Let us say our equation has turned out,

$$\delta p(x) = A \times e^{\left(\frac{-x}{L_p}\right)}$$

So, A is going to be, now we know that δp(x=0) was 10^{11}per centimetre cube. That is what we did. This is 10^{11}, we substitute that. So, essentially, you are going to get δp(x) is going to be,

$$\delta p(x) = 10^{11} \ \times e^{\left(\frac{-x}{L_p}\right)}$$

So, what does this tell us?

How do the semiconductors you know how does the distribution of minority carriers look like? So, you take this. Plot it as a position of x this is δp, we are going to have an exponential decay. Fantastic, I mean one of the simplest things that we can think of. But δp_0, this is equal to 10^{11} here and as you go, we define the diffusion length.

Now we can define the diffusion length as the position as x at which the minority carrier, excess minority carrier concentration falls to 1 over e, when it becomes 33%. So, that is when you know, this is .36. So, when basically $\frac{1}{e}\delta p_0$. So, if you have this, then assume that is a diffusion length. Essentially your minority carriers are decaying over a certain distance. And what is this number?

How much is this length? Let us try to estimate. We said L p was,

$$L_p = \sqrt{D_p \tau_p}$$

Well, I mean D_p, what was diffusion coefficient? So remember, diffusion coefficient was,

$$\frac{D_p}{\mu_p} = \frac{KT}{q}$$

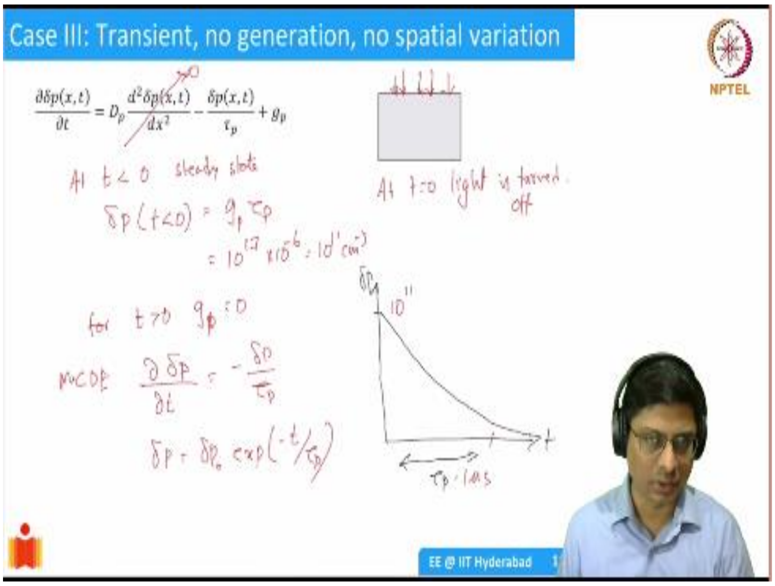
Einstein relation. And μ_p, μ_p was 480 centimetre square per Volt- second. This is the mobility of holes. You multiply it by .0256 or .0259, you should get it.

And you know, so $D_P = 480 \times 0.0259 \sim 10 \frac{cm^2}{sec}$. Just you can verify this, I do not want to calculate this. So, now with this what will happen? Your $L_P = \sqrt{10 \times 10^{-6}} = 3.x \times 10^{-3} cm \sim 30 cm$ approximately. Remember, there is always centimetres you have to put additional thing.

So this is actually a quite large number. As I said, you know, the minority carrier lifetime is going to be smaller, 10^{-9} something. It depends on the semiconductor, you know, if you have a indirect semiconductor, this might be long, direct semiconductor it will be small. So, it is sample dependent, it is not necessary right now for us.

So what this is telling us is, if you introduce minority carriers, over a, you know, 10 microns, 10 to 20 - 30 microns range, they will all decay. You are not going to see the effect of these minority carriers deep in the semiconductor. It is going to be close to the surface. And we will see that in pn junctions, the minority carrier diffusion is going to be close to the junction itself. So, that is why we introduced this.

(Refer Slide Time: 32:01)



So Well, yeah, I am running out of time, only 15 minutes, let me quickly wrap up. So, the third case I wanted to discuss was a transient case, you know. Let us say there is no spatial variation. We have talked about it in a different way, there is no spatial variation. So, this is going to go to 0. I will just quickly do this. I am not going to spend as much time and then there is a no generation.

Well, what it means is, let us say I will take a sample I will shine it with photons uniformly so that there is no spatial variation. And then at time t = 0, I will switch it off. So, at t = 0, light is turned off. So, we already said that, you know, it is about exponential decay. And the coefficient, the time constant is τ_p. But I just want to show you that.

So before, you know, basically what happens at p < 0? Basically, light is on. It is going to be in the steady state. You can solve the problem, same time, we have done it a couple of times already. So, there is steady state for time less than 0. And therefore, $\delta p(t<0)$ is going to be simply generation rate, g_p times τ_p.

$$\delta p(t < 0) = g_p \tau_p$$

Let us assume the same generation rates.

$\delta p(t < 0) = 10^{17} \times 10^{-6} = 10^{11} cm^{-3}$ at time less than t = 0.

Now, for t > 0, g light is going to 0.

$$i.e, t > 0 \;\; g_l = 0$$

I mean, it is basically not creating any electron hole pairs. So, g_p is going to be 0, okay? When I say g_p , g_l , it is all you know, when I say g_l , it is light generated carriers is going to produce electrons and holes.

And the rate at which electrons have been produced or holes are being produced is going to be g_p, same thing. So, what is this at time $g_p = 0$. So, essentially, diffusion equation MCDE becomes partial derivative of excess minority carrier density with respect to time. That should be equal to,

$$\frac{\partial \delta_p}{\partial t} = -\frac{\delta_p}{\tau_p}$$

Other terms are going to 0. So, you can solve this.

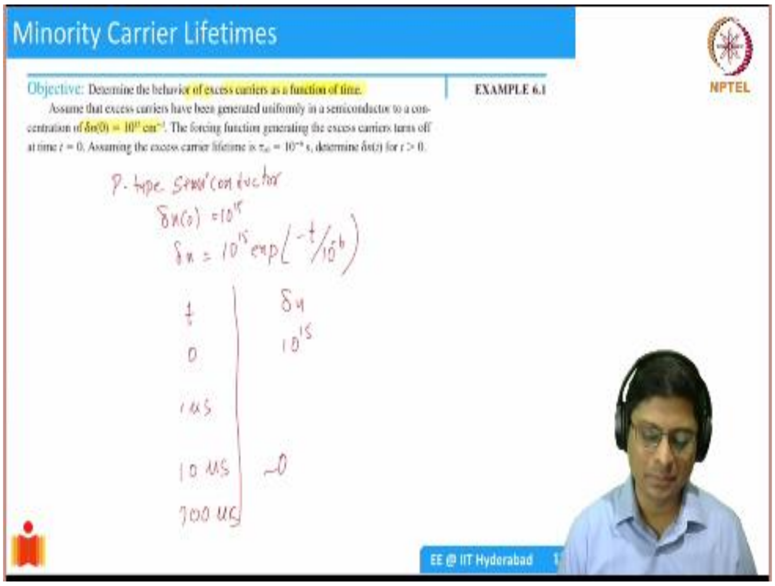
And this solution turns out to be,

$$\delta p = \delta p_0 e^{\left(\frac{-t}{\tau_p}\right)}$$

So you can solve these things, if you are interested. Just take some moment and solve it. It is not very difficult equations. So, I have this as a function of time, and this is a function of δp. So, my axis is δp. So, you are going to have exponential decay.

And that is what we have shown earlier actually, this one τ_p. So, this is in our case example 10^{11} . This is some time you know, we got one microsecond or something like 1 microsecond, τ_p=1 microsecond. Anyway, you will see that you know what happens. So, this is a already we have solved this case. So, I just quickly went over this.

(Refer Slide Time: 35:47)



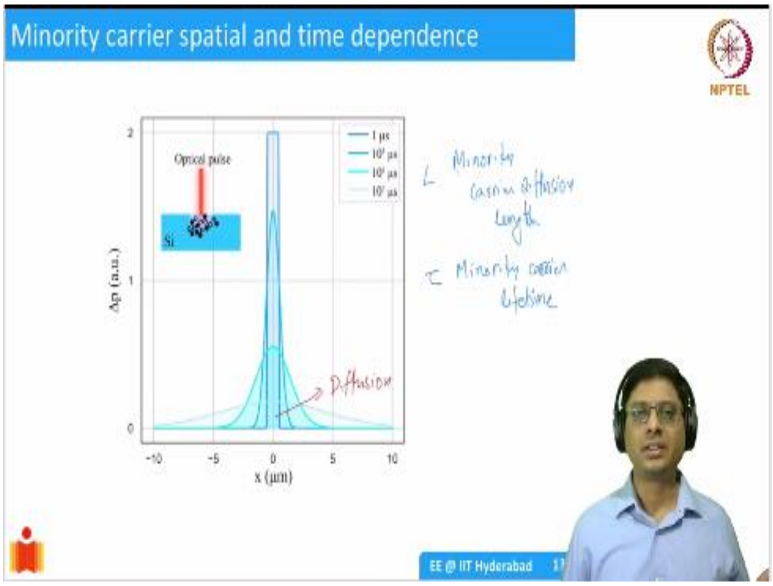
So, you can take some examples, this is basically excess carriers as a function of time. The same problem just we did. I want you to work out. So, we saw that the equation is going to be you know, one small difference here. This is a δn is given to you. That means we are talking about a p type semiconductor, remember. This is p type semiconductor, because we are only talking about minority carriers, the p type semiconductor.

So, the minority carriers are basically going to be δn. $\delta n(0)=10^{15}$. And you are switching of the light. So, essentially, it is going to be $\delta n = 10^{15} e^{(\frac{-t}{10^{-6}})}$. This is a time. So, please find out what it is? I will tell you the simple cases t = 0 and δn. So, of course at t=0, I will do this for you 10^{15}.

What happens at t = 1 microsecond? What happens at t = 10 microseconds? Let us put in some numbers and calculate. And you will see that quickly in 10 microseconds nothing will be left I mean, you will reach basically the n0 whatever it is. This excess minority carriers is going to become 0, almost 0. Check it out. Check it out what happens? Negligible compared to the original starting point.

So, as you know even 100 microseconds maybe you should try to check it out, what happens? As you go to longer and longer times δp is going to become 0. So, this is basically how you calculate you know, minority carrier lifetimes and diffusion lenghts. These are very, very important.

(Refer Slide Time: 37:34)



And in the end, I just want to show you, if you have a combination of both, you know, so far we have done problems with spatial dependence or time dependence. But if you combine both, you could solve it. I just know it becomes mathematical a little bit more involved. But the idea is that you start with Lets say I introduce that same problem as it was just started with. I shine I take a piece of semiconductor, I shine light at a location $x = 0$.

And I switch off the pulse. What happens? So, originally you have this position dependence exactly, you created that you know, that many electron hole pairs, but as time progresses, this is basically as a function of time you see here, as you know, 1 millisecond, 10 milliseconds and 100 milliseconds. As you go and longer and longer in time, you see that the profile is becoming spread out. So, there is diffusion process.

This is basically because of diffusion. And also, there is going to be some recombination happening, which will essentially reduce the total area under the curve. If you calculate the area under the curve, that will tell you the total number of carriers. And you will see that eventually that becomes 0. As you go to longer and longer times, essentially indicating that all the electron hole pairs are getting annihilated.

So, I mean, this is basically, it is going to make sense. Qualitatively it should make sense to you, there are numbers which you know, I think is not really necessary for this course. In an advanced course, maybe we could talk about it. So, already 58 minutes, I would like to stop here now. So, yeah, so today, we talked about one very important concept of non equilibrium dynamics.

So we see that, you know, in all our semiconductor devices, we are going to introduce some electron hole pairs, right, we are going to distribute from equilibrium. Equilibrium is that it exists without any external stimulus. Once you distribute with an external stimulus with voltage or current, optical light or something like that, so it is going to change the minority carrier density.

And that is going to be captured by the diffusion length and the diffusion time, minority carrier lifetime. These are very, very important parameters. Please make sure that you are comfortable with what do you mean by diffusion length L. I will write it. So, please make sure that you are comfortable minority carrier diffusion length and minority carrier lifetime. This we called as τ and this we called as L.

If you are talking of p type semiconductor you will talk of τ and L_n and vice versa n type semiconductor.