Introduction to Semiconductor Devices
Dr Naresh Kumar Emani
Department of Electrical Engineering
Indian Institute of Technology – Hyderabad

Lecture – 2.5
Temperature Dependence of Fermi Level

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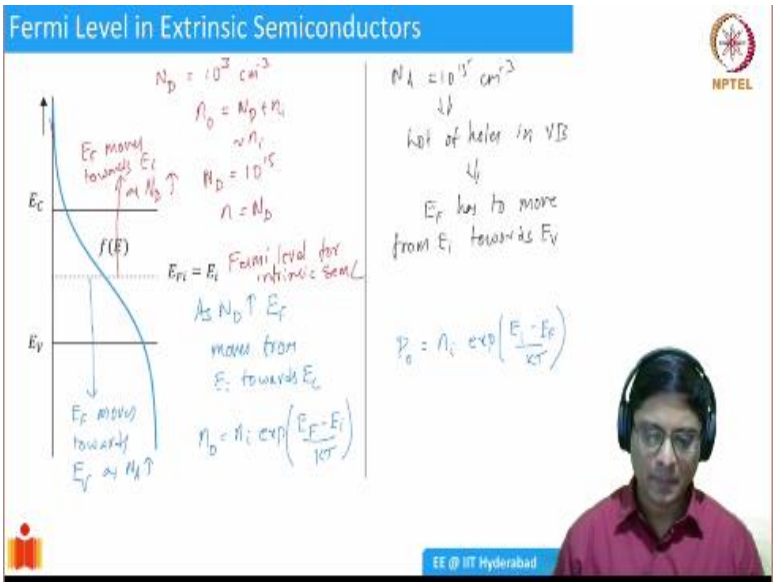

But we want to understand. You know, we wrote the expression straightaway. But let us understand how fermi level changes in a extrinsic semiconductor. And then assuming let us say that T = 300 room temperature. So, let us start with first, what happens if I introduce some dopants. Let us say N_D donors I am introducing. I introduce $N_D=10^3$ cm^{-3}. It was a very, very small amount of donors.

So I have 10^{22} states in my silicon lattice per centimetre cube. But I am only introducing 10^3 donors. If I do that, what do you think will happen? Well, even when I do not introduce now, we are n_0 is going to be basically your N_D, which is the dopant introduced electrons because there is also going to be some contribution from n_i. You can think of it sort of like that. I mean, I am just simplifying it a lot.

But anyway, this is going to be roughly what it is. So, it is still going to be n_i. It is not about the matter if you introduce very, very small doping concentrations. Because 10^3 versus 10^{10}, so 10^{10} is going to win. It is n_i. So, if you want to dope a semiconductor, you can not put a doping of 10^2 or 10^3 and expect something. The range of doping that we have to do is something in order of 10^{15}, at least; 10^{15}, let us see.

If I take 10^{15}, what do you expect will happen. Now, it is going to introduce lots of 5 orders of more holes than what is there in the intrinsic. Now, in this case, simply n is going to be N_D. So, the donor atoms are going to dominate. You know that is whole thing that will dominate. That is what will give you a lot of electrons.

So, now, what happens physically? If you are putting lots of electrons here, you know lots of electrons on this slide, what happens to the Fermi function? Fermi function should give you the distribution of electrons. So, previously, there was equal probability of finding electron on the conduction band and a hole in the valence band. That is why the Fermi function was at E_i.

But now you have added externally a lot of electrons. When you add a lot of electrons, what you need to do is this Fermi function moves upwards now. So, in some way, basically the distribution has to move upwards. So that your E_F basically, E_F is defined as a energy at which your probability is half. So, the probability half is what you need to get about. So, since more electrons have been introduced in the conduction band, the E_F has to go higher.

Higher means, it is going to go towards the E_C. So basically, what if you use, introduce now, $N_D = 10^{18}$. I increase the number of dopants. If I do that, E_F is going to go further high. So, as N_D increases E_F moves from E_i. E_i is basically the intrinsic level in the centre where the intrinsic semiconductor E_F is going to be at the middle of the bandgap. From there it starts moving towards E_C.

That is what happens. And that is why when I said in the last $n_0 = n_i\, e^{\left(\frac{E_F - E_i}{KT}\right)}$. So, when I introduce N_D, when I introduce donor atoms, I know that my Fermi level is going to go upwards. As it goes upwards this quantity n_0 is going to increase and that is what we know will happen? So, logically you could think of it like this now what happens if n_0 is increasing?

What should happen to p_0? What will happen to the equilibrium density of holes? So, to understand that what we need to do is, we have to also write p_0. So, what will we be p_0? p_0 is going to be well, let me come to it. Just let me take a slide view. I will talk of acceptors and then we will talk about this one more time. So, let me put a divider. So now let us think of what happens when you have acceptors?

Same story. If $N_A=10^3$, 10^4, it does not matter. N_A has to be something large, as $N_A = 10^{15}$ cm^{-3}, then what happens? You are essentially creating lots of holes. You know this implies lot of holes in valence band That means you are essentially removing electrons from the valence band.

So, your probability of finding an electron is going to come down. You know F(E) has to come down. The Fermi distribution has to go lower. So, implies E_F has to move from E_i. E_i is your reference always you know, intrinsic. From intrinsic as you add more and more acceptors E_F has to move towards E_V. So, let me erase this is. So, you see in intrinsic this is basically Fermi level for intrinsic semiconductor.

As you start adding dopants, let us say you are adding electrons dopants, N_D. You are adding dopants N_D. As you add N_D you are adding electrons and therefore, E_F moves towards E_C as N_D increases. Similarly, as you keep adding holes you are adding acceptor ions. If you substitute silicon atoms with the acceptors then you create holes. And therefore, your E_F should come down. E_F moves towards E_V as N_A increases.

So, this is how the Fermi level will go. So, what will be p_0? p_0 is simply going to be n_i. It is a starting point and if the Fermi level moves below E_i, p_0 will increase. If it moves above E_i, then p_0 will reduce. So, this is going to be exponential, E_F is going to be higher now, E_F minus E_V. E_i is going to be higher. So, E_i - E_F by KT.

$$p_0 = n_i \, e^{\left(\frac{E_i - E_F}{KT}\right)}$$

So, basically as Fermi level goes closer and closer to E_V, the term in the exponential will increase.

The exponential term will increase and so, you get more and more holes. If the E_F goes above E_i, then the exponential term will reduce. So, the p_0, the holes will reduce. So, now let me see. Let us take a few examples.

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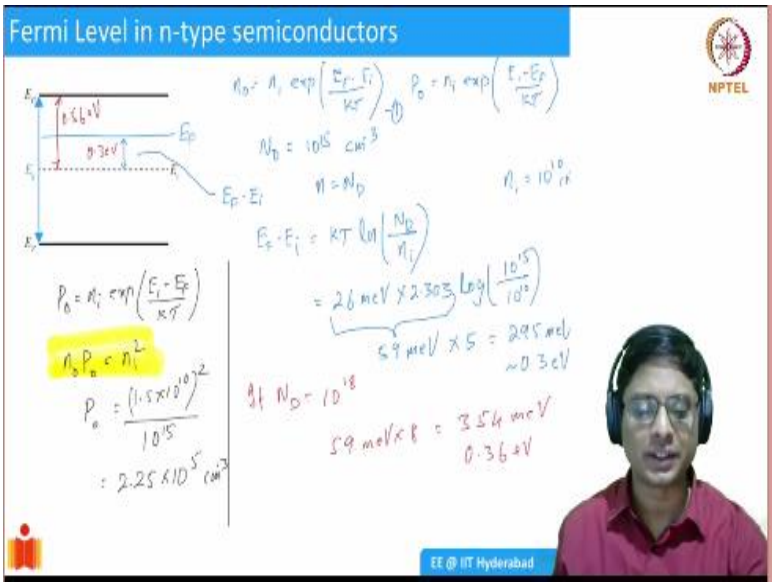

Let me do this. Let me just write down this formulas, I am just repeating it because you know we should also get used to these formulas.

$$n_0 = n_i\, e^{\left(\frac{E_F - E_i}{KT}\right)}$$

$$p_0 = n_i\, e^{\left(\frac{E_i - E_F}{KT}\right)}$$

So let us consider a situation where $N_D = 10^{15}$. This is an n-type, let us assume an n-type semiconductor and $N_D = 10^{15}$ cm^{-3}.

What will happen to E_F? So, what we can do is; what will the concentration of holes? Sorry, concentration of electrons? Well, it is much much more than n_i. So, n_0 is going to be I mean, we can forget the expression. We can even say that it is going to be; now let us do this slightly differently. We know this. So, what will be the distance of where we will, where exactly will be the Fermi level?

So we said for intrinsic semiconductor, the Fermi level is at this point in the middle of the bandgap. So, if I add more donor atoms, where exactly will be the Fermi level? That is what we want to compute. So, we want to compute let us say it is somewhere you know, somewhere in the middle, this is going to be somewhere here for a n-type impurity. What will be the distance we want to compute?

So, this distance we want to compute. How will you calculate that? Essentially, this distance is going to be $E_F - E_i$. To calculate this, what we have to do? Well, we have the expression.

$$n_0 = n_i\, e^{\left(\frac{E_F - E_i}{KT}\right)}$$

So, whenever you have n-type impurities, we know that essentially those are going to contribute because intrinsic contribution is going to be less.

The electron hole pairs from the; intrinsic contribution is going to be less. So, $n=N_D$. If sure, this is a 10^{15}. And E_F - Ei, I can simply take the logarithm of the expression on the top here, you know. Let us call it equation 1. I can take the logarithm of equation 1. And so basically what I get is KT times ln of, natural logarithm of n_0, n_0 is going to be N_D by n_i, like this.

$$E_F - E_i = KTln\left(\frac{N_D}{n_i}\right)$$

This is an expression. Now, this KT is basically going to be in the energy units, it is what is boltzmann constant times and temperature, but you have to write it in energy units. And we told you multiple times that this is going to be room temperature, it is going to be 26 milli electron volts. This is 26 meV into natural log is half. So, let us write it in terms of log 10. So, $N_D = 10^{15}$. Now I want to make my life simple.

So I will say n_i equal to, I am not taking the exact number. $n_i = 10^{10}$, I will take because if I take 1.5 extra then I have to do exactly the calculator, which I do not have right now. So, I will simply taka it as 10^{10}. If I do this, what do you get? Well, I sort of you know, have memorise this expression. This is basically 26meV ×2.303 = 59 meV. This is the number.

So, this is going to be $log\ log\left(\frac{10^{15}}{10^{10}}\right) = 5$ .So, $E_F - E_i = 59 \times 5 = 295meV \sim 0.3eV$. This is how you can calculate, to know what is the concentration. So, this is going to be what now? This distance we calculated is 0.3 electron volts. The whole distance is how much? This is basically .56eV because bandgap was 1.12. So, just the half bandgap, ($E_g/2$) = .56. So, we are sort of come halfway into the towards the conduction band. If you dope10^{50} , you have come halfway into the conduction band. Same thing you could repeat for 10^{18}. If you know if $N_D = 10^{18}$, what would happen? Now we have already done most of the calculation.

So it is simply it is going to be 10^{18} divided by 10^{10}, so 8 orders of magnitude. So, I know I leave it for you to verify. But it is going to be 59 meV × 8. That is going to be 60 ×8=354 meV=0.36 eV. So, as you increase, you see 10^{15} was already at point 3. But as you go higher

and higher, the Fermi energy moves slower, it does not move as quickly as your concentration increases.

This has some implications. We will show you something in that. So, well this is electron concentration that is fine. What is the hole concentration? Well, hole concentration is basically going to be

$$p_0 = n_i\, e^{\left(\frac{E_i - E_F}{KT}\right)}$$

So, we can calculate it, but instead you know I have a very, we can do a simple thing here. So, we should remember always under equilibrium

$$n_0 p_0 = n_i^2$$

You can verify this. $n_0 p_0 = n_i^2$ So, p_0 is going to be, once you know what is n_0, p_0 can be directly calculated. I mean, bear with me. Now I will do the exact calculation just for the heck of it. So I will say, $n_0 = N_D = 10^{15,}$

$$p_0 = \frac{(1.5\times10^{10})^2}{10^{15}} = 2.25 \times 10^5 cm^{-3}\cdot$$

So, you see what happened as you introduce dopants, n-type dopants the equilibrium concentration of electrons increased and the equilibrium concentration of holes decreased. So that always, you remember this. You know you have to remember it very very well that $n_0 p_0 = n_i^2$ under equilibrium conditions. This cannot be violated.

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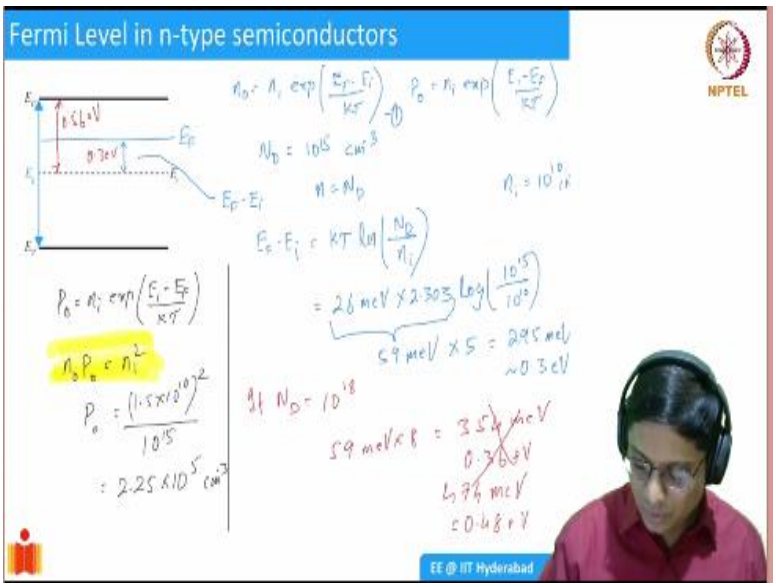

You could do the same math for acceptors. You could consider what will happen when N_A, let us say you put sorry. If you consider $N_A = 10^{15}$ cm^{-3}, so, if you want to calculate N_A sorry. If

you have accepted impurities, your $p_0 = 10^{15}$ because most of them are holes we are introducing into the valence band,

We do not have to we can ignore the contribution. We can ignore the intrinsic contribution. So, $p_0 = 10^{15}$. So, from that again, you know, if you do the same math, it will come out to be, the Fermi level is going to be .3 below E_i. In this case,

$$p_0 = n_i e^{\left(\frac{E_i - E_F}{KT}\right)}$$

E_i- E_F is going to be implies, now we did the math last time, it should be the same.

This is going to be 0.3 eV. So, what this is telling you is, if we are going to introduce acceptor ions, your Fermi level is below by a distance it is going to be 0.3 eV. As you increase acceptor ions, the Fermi level has moved below. And if you take 10^{18}, what would happen? It would go to 0.36 I think I put this expression is wrong. Because 60 × 5, 8 5's are 40, 48. So, it should be wrong.

I took it as 6 times. It is wrong. You just verify. This is going to be 8 ×6=48. 48 minus 6, so it is going to be 474 I think. I mean, this is going to be 0.48 eV. Please check it out.

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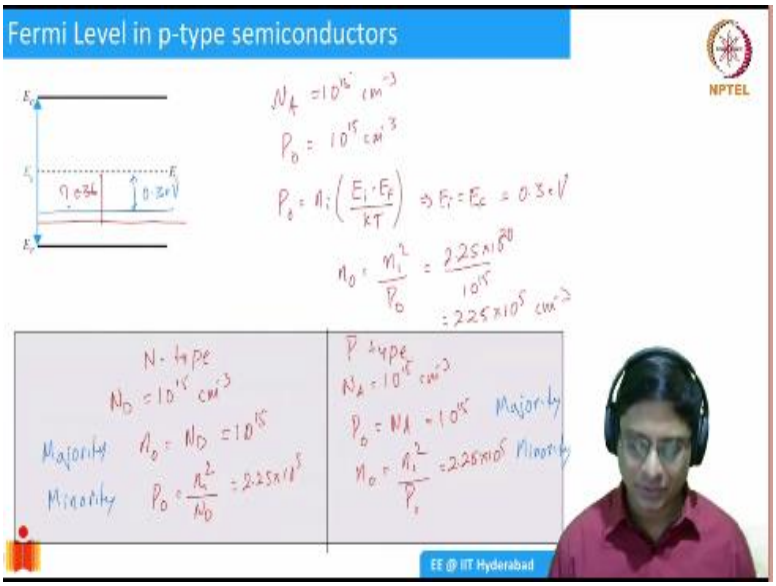

And here also check it out. I will put a question mark. I think that is what it should be the same story repeats. And once you know p_0 what is the electron concentration? $n_0 = \frac{n_i^2}{p_0}$. That is going to be $n_0 = \frac{2.25 \times 10^{20}}{10^{15}} = 2.25 \times 10^5 cm^{-3}$. So, remember now, let us say let us summarise this.

You have n-type and p-type. You also have $N_D = 10^{15}$ cm^{-3} where $N_A = 10^{15}$ cm^{-3}. Then this implies $n_0 = 10^{15}$ cm^{-3}. I am just leaving out. If $p_0 = 2.25 \times 10^5$ cm^{-3}. Here, p_0 is let me write it like this. Let me write it in the expression form so that you can use it.

This is going to be $n_0=N_D$. $p_0 = \frac{n_i^2}{N_D}$ Here $p_0=N_A$ which is basically, you are introducing acceptors. So, we are going to be having N_A. And $n_0 = \frac{n_i^2}{p_0}$. So, in an n type semiconductor, we call electrons as majority carriers because those are the ones which have the highest concentration and the holes are called us minority carriers.

You see the numbers if you had calculated. This was $n_0=N_D =10^{15}$. And this was $p_0=2.25 \times 10^5$. You see majority carriers are the most significant number. The concentration is high minority carriers are smaller in concentration. Similarly, here it was $p_0=N_A =10^{15}$ and $n_0 =2.25 \times 10^5$. So, if you have p-type semiconductor the majority carriers are holes and minority carriers are electrons.

So, this is a distinction between n type and p type. Let us see if it works. So, I just you can use this as a reference you can calculate multiple times multiple in I just tried some examples.

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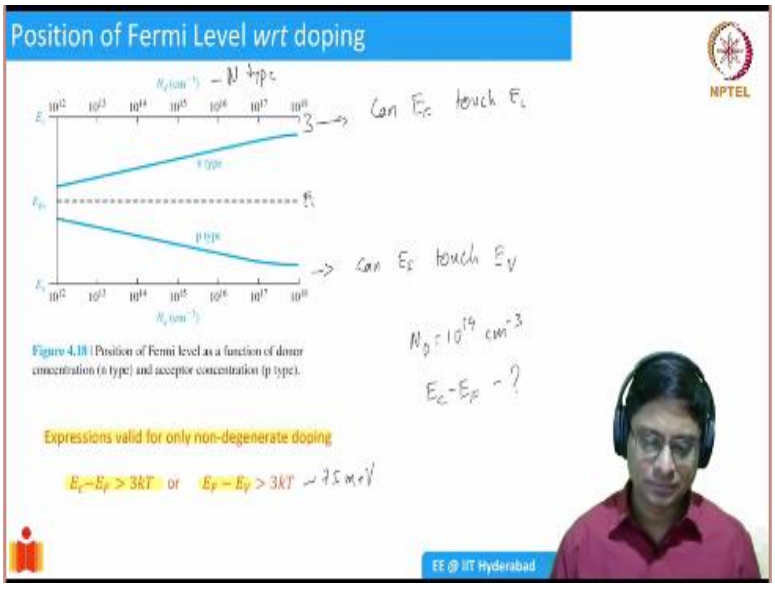

So, well I just wanted to show you a few more things wherein we calculated this already. So, as you increase your doping, you know this is a graph of E_F versus doping. So, if you have n-type semiconductor, this is an n-type semiconductor. When you have n-type semiconductor as

the doping N_D is increasing. As the donor concentration is increasing your Fermi level starts out closer to E_{Fi}. This is E_i. You can call it E_i E_{Fi}.

These are all same. So, it starts closer to that and as you increase the concentration it tends to move towards the conduction band. The opposite behaviour is shown by the valence band. So, it starts out at closer to end and then moves towards the valence band. I want you to actually note one extra one you see this distance. The question here we can ask is can E_F touch E_C or here can E_F touch E_V?

So, essentially what you are saying is if you keep doping high enough, can the Fermi level go further? Because it sorts of say seems like saturating but will it go further into the conduction band? There is a very long answer to that. That it will actually go into the conduction band or into the valence band as possible for E_F to go in. But these expressions cannot be calculated with the expressions I have given.

These concentrations or this E_F levels Fermi levels cannot be calculated with the expressions I have given you. Because the expressions that we have shown in the class today are only valid for what is known as degenerate doping. So, the degenerate doping means you have the distance between E_C and Fermi level has to be greater than 3kT $((E_C - E_F) > 3KT)$. Similarly, distance between Fermi level and E_V has to be greater than 3kT$((E_F - E_V) > 3KT)$.

3 kT is basically going to be roughly 75 or 75 milli electron volts 0.07. If the E_F goes closer than that, then the expressions we have used to break down the approximations do not work anymore. So, we will not get correct results. So, we will not even bother. There is no point in calculating. You know you could put 10^{19} , we can check out how much is you know, please do that as an exercise.

Take $N_D = 10^{19}$. Just take it that way. For this calculate $E_C - E_F$. How much it is? Or you know. You can expect to calculate this and see what happens. If the $((E_C - E_F) < 3KT)$, then the calculation is not going to be correct. I mean, you will get some numbers, but they are not going to be correct. Because the approximation we make in that case is actually breaking down. So let me just give me a minute.

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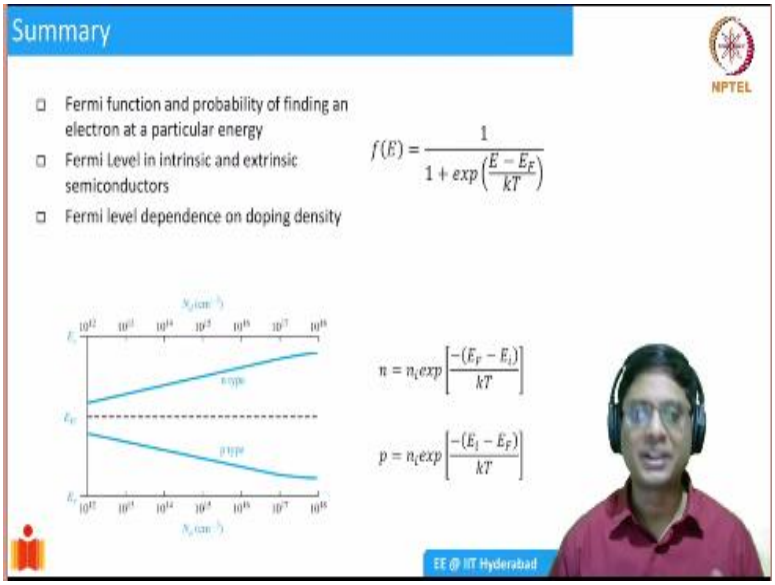

So I thought of doing a few more things like it looks like I am running out of time. So, I just want to summarise now. So, what we did today is we introduced the Fermi function. We talked about the probability of finding an electron at a particular energy level, which is essentially the Fermi distribution and then based on that we defined the Fermi level for an intrinsic semiconductor.

Once we did that, we understood what happens to Fermi level as you change doping. Maybe try to take some test cases, you know, if you have dopants, n-type dopants what happens to Fermi level? If you have p-type dopants what happens to Fermi level? And we calculated that. So that is what is summarised on this slide here. So, please take your time and try to figure out how to write these equations correctly.

So in your exams, it can be you know multiple forms. You can solve a problem in many different ways. Whatever works for you, I mean, there is nothing like a way or a wrong way. So, it is going to be fine. As long as you get the correct answer, it is fine. So, you can use this expression or you could use n equal to N_C expression that also will give you the same answer.

Because you know how much is $N_C - E_i$. So, you will get the same answers, but whatever you know is convenient for you. Please you know, whenever I show you the summary slides, please make sure that you are comfortable with the information provided on the summary slide because that will be useful in the next couple of lectures.

So, with that, I would like to stop here today. And have a great day and I look forward to seeing you in the next lecture. Bye.