Introduction to Semiconductor Devices
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Lecture – 2.4
Fermi Level in Extrinsic Semiconductors

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Hello everyone, welcome back. This is lecture 5 of the course Introduction to semiconductor devices.

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In the last lecture, we introduced a few concepts like effective mass and we introduced intrinsic carrier density and then we talked about how doping can be used as a means to control the carrier density in semiconductor devices. And in the end, we also studied a little bit about the temperature dependence of the carrier density in intrinsic semiconductor as well as extrinsic semiconductor.

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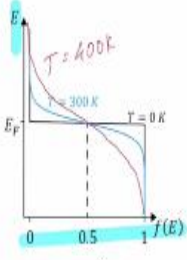
So, today we will try to get into a little bit more depth. We will try to give you a quantitative picture of what is happening. So, to start with let us we have to introduce a few functions, a few concepts. The first one is Fermi function. Fermi function is a function which is given here,

$$f(E) = \frac{1}{1 + e^{\left(\frac{E - E_F}{KT}\right)}}$$

So, this is a function that gives you the probability. So, f(E) gives the probability of finding an electron at a particular energy an electron at a energy E. Why is this important? Well, because you know electrons are distributed. We have this energy bands which are allowed states, but there is a certain amount of probability of you know which of them will be filled which has not filled. So, that probability is given by this Fermi function.

So, it plays a very crucial role. And in this function KT is anyway the Boltzmann constant and temperature. So, that will be units of energy and E_F, what is known as Fermi energy. You can also call it Fermi Level. So, E_F is a number basically an energy. So, how does this function behave ? Let us try to analyse the situation. So, let us consider, to begin with, let us say it is easier to do it at T = 0 Kelvin.

So let us consider that So when T=0, temperature is 0. What happens if energy is less than E_F? E<E_F. So, E_F implies this here, the exponential term in the denominator is going to be negative. So, $e^{\left(\frac{E - E_F}{KT}\right)}$ is in the denominator, and this term will be, the exponential term in the exponential is going to be negative, and because of which, essentially, the exponential will go to 0. Therefore, your f(E) the probability of finding an electron at a particular energy, if E<E_F,

that is going to be 1. It has a very high probability of finding an electron below an energy E_F. Now, what happens if energy is greater than E_F. So, if $E>E_F$, then this exponential term, the most important again, so now this is going to be $E–E_F$, so the numerator is going to be positive.

But then the denominator is going to be very small, so then essentially, this kind of goes to infinity, it tends towards infinity. Therefore, $f(E)=0$. So, fermi function at 0 degrees Kelvin behaves like a step function. Below E_F, it is going to be 1, above E_F, it is going to be 0. What happens at E_F? So for the moment, consider $E = E_F$.

When $E=E_F$, exponential basically, the parameter in the exponential is 0. That means exponential of 0 is 1. So, basically, $f(E)=1/2$. It is going to be exactly 0.5. So, this is an important thing to note. So, whenever your energy equal to Fermi energy, then the probability of $f(E)=1/2$. So, this also gives you one definition of Fermi energy.

It is basically you can define it as Fermi energy is that energy where $f(E)=1/2$. This is one definition of Fermi energy. What happens at higher temperatures? So, if you look at the expression for Fermi energy, at $E=E_F$, you saw that it is basically independent of temperature. So, this particular thing, this basically is independent of temperature.

So, $f(E)=1/2$ no matter what the temperature is. So, if you analyse it, and you actually you can plot it in MATLAB or Python, it looks now the graph is shown here, it looks like this. So, here I have deliberately, you know, flipped the axes. So, I am plotting $f(E)$ along X axis, and energy along Y axis. There is a reason, you know, why we do that. So, what you see is at $T = 0$, you have this step like behaviour.

At higher energies, there is no probability, E below E_F it is going to be probability one, very high probability. So, now, as temperature increases, the probability gets stretched. But always the $f(E)$ should go through half at E_F. So, if you keep the E_F constant and change temperature, it is going to stretch like this. And if I for example, if I take one more you know case where let us say I take a slightly higher temperature, so, that would have been like this.

It will stretch out further. Basically this will be equal to let us say $T = 400$ Kelvin. So, as we increase the temperature the fermi distribution stretches out. So, we can make a note here, so basically as T increases, $f(E)$ is stretched. Stretched around E_F. So, this is the characteristic.

So, how does this help us? This definition of Fermi energy is useful in denoting the operation in actual, in representing the operation of various semiconductor devices. For example, when we talk of PN junctions or MOSFETs we will always show a level which is called as Fermi level. That gives us a very, very easy way of understanding what is happening in the device.

So, for that we need to understand this concept of Fermi level very well. So, to do that, let us try to analyse a few situations. The first one we will consider is what happens to Fermi level in an intrinsic semiconductor. What would you expect? You know where? Where do you expect the Fermi level to be? So in an intrinsic semiconductor, you saw this already many times, you know, you have this electron hole pairs generated.

And at a particular temperature, let us say there is some electron hole pair distribution like this. So, what is the probability of finding probability of finding an electron at E_C? What will that be? We saw f(E) is basically the probability of finding electron at E. So, simply replace E by E_C.

$$f(E_C) = \frac{1}{1 + e^{\left(\frac{E_C - E_F}{KT}\right)}}$$

So now, what will be the probability of finding an electron at finding an electron hole? Let us not consider electron, let us consider the hole. Probability of finding a hole, at Ev. The probability of a hole is simply lack of electron. So, probability of finding a hole is basically the probability, probability of not finding an electron. The probability of not finding an electron is simply going to be 1– f(Ev).

f(E_V) will give you the probability of finding an electron. The probability of not finding an electron is going to be one minus that. So, this is what it is going to be, and if you well. So, now, so we have these expressions basically. It is the expressions. So, how will this help us find the Fermi level of an intrinsic semiconductor? So we use the relation we know that in an intrinsic semiconductor, n = P in intrinsic semiconductor.

We have not added any other dopants. So, basically there are only electron hole pairs which are produced and that should give us basically n = P. So, if n = P, basically the probability of finding an electron at E_C should be equal to probability of not finding an electron at E_V. So, basically, this relation implies f(E_C) = (1 – f(E_V)). So, this is the probability of finding a hole at E_V.

And that is equal to probability of finding an electron at E_C So, this should be equal. That implies you should have no I mean LHS is no, simply you have to rewrite it

$$f(E_C) = 1 - f(E_V) = \frac{1}{1 + e^{\left(\frac{E_C - E_F}{KT}\right)}} = 1 - \frac{1}{e^{\left(\frac{E_V - E_F}{KT}\right)}}$$

So, this right hand side can be further simplified.
And you know actually you can check it out it will turn out to be $\dfrac{1}{1 + e^{\left(\frac{E_F - E_V}{KT}\right)}}$.

Basically you manage the work with the fractions and you will see this immediately. So, this is a expression. So, what this is telling is this particular relation, this implies that you are the parameters in the exponential should be equal. So, that means (E_C –E_F)=(E_F – E_V).

So, that was just rearranging it. That implies,

$$E_F = \frac{E_C + E_V}{2}$$

So, what this is telling you is that E_F, the Fermi level for an intrinsic semiconductor lies exactly in the middle of the, E_F is basically at the middle of the bandgap. So, this is the conclusion we have from this analysis. So, sometimes you know we have various ways of representing.

So this is because this is only for an intrinsic semiconductor, we will actually give it a special name, we say that, we will call it E_{Fi} . So, when some sometimes we call it E_{Fi}, so E_F of intrinsic or sometimes will also referred to as E_i. E_i is basically the middle of the bandgap. So, many

ways of representing the same thing. So, this relation you know E_F equal to exactly the middle of the bandgap is almost correct. There is a small catch.

I will give you a problem in the homework, which will force you to understand that. There is a small deviation because of effective masses of the holes or electrons are different. So, there is a small change, I will give it as a homework. But anyway for all practical purposes, you can take the E_F to be in the middle of the bandgap for semiconductor devices. Good.

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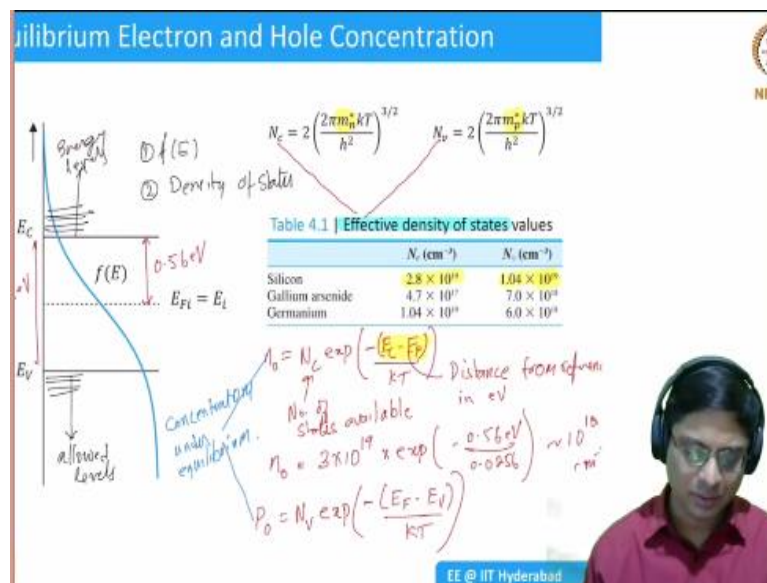
So, now, we are ready to talk about quantitative numbers of electrons and holes. So, far we have qualitatively mentioned the physics and we said how what should happen. Now, we are ready to do a quantitative. To do a quantitative analysis exactly turns out to be a little bit involved. So, what we are going to do is take a shortcut. We are going to give you a final lesson which is applicable in most of the scenarios.

Only when you go to certain very extreme conditions very high levels of doping, then what we are talking about today will not work. But I think that is really not necessary at this stage. So, to understand how to calculate these concentrations, we have to understand 2 things one is the Fermi function, which is essentially giving you the probability of finding electrons at various energy levels.

And the second thing that we need to understand is what are the available number of states. We all we have been talking about this energy levels. Essentially, we said this is a band. And you have energy levels available, energy levels. These are all distributed along the conduction band.

And similarly for the valence band as well you have a range of allowed levels. To understand what is the effective carrier concentration,

For me, you know, I like to think of it in terms of a classroom, you know. Think of a class a large class where maybe 100 plus, 200 plus students. And you want to calculate, you know, how is a, how many students are there. So, that would be something equivalent to calculating the number of electrons in the conduction band. The way to do that would be, you know, there is going to be always a distribution of probability of finding students.

Maybe the probability of finding students is, you know, high in the front of the class. But as you go towards the back back, the back benches, the probability might decrease. There is a distribution function, there is a probability distribution function. And also, the number of the number of seats in the tables can be thought of as the number of available states. So, if you look at it, the number of benches will be like, you know, maybe 2 students per bench or 2 students per metre, 3 students per metre or something like that.

You know. There will be a kind of density of states. And to calculate the exact number of students in the class, all I have to do is I have to take the density of states and multiplied by the probability function, and then sum it up. That will give you the exact number of students in a class. We do exactly similar thing here. What we do is we have a function log that is basically a function which is called as density of states function, which gives you the distribution of states in the conduction band and valence band.

And then we have to do a little bit of mathematics to actually solve that. So, instead, what we will do today is that you know, if you do it exactly, can be a little bit invert. So, I am not going to take that approach. But what I am trying to tell you is that the carrier density is dependent on 2 things. One is the Fermi function, I will just write it as a $F(E)$ and number 2, density of states.

These are the 2 things that are going to impact your carrier densities. So, instead of doing all that, we will abstract it out and we will introduce 2 quantities here. You should remember these quantities. So, which we call us effective density of states. So, those are the expressions for that is given here basically. So, this you see is N_C and N_V. N_C and N_V represent the effective density of states for the conduction band and the valence band respectively.

So, with that I mean you see that it has a bunch of constants, and one of the most important things is the effective mass here. The effective mass in the conduction band and the effective mass in the valence band. So, if you plug in the numbers, then you get the typical range of this effective density of states and these turn out to be about 10^{19}. It is the typical range of effective density of states.

But I must caution you here, these numbers are quite dependent on you know, where you take the data from. Because effective masses and you know, those are experimental or dependent on the experimental procedures. So, there can be some variations, if you look at one textbook might it might give you the effective density of states as some number, another textbook might be slightly different.

But it is they are all going to give you something in the ballpark of 10^{19} cm^{-3}, that will not change. So, fine, we have this effective density of states. So, what is the carrier density? What is the carrier concept? What is the concentration of electrons? What is the concentration of holes? Do that, what we will do is, we will just simply use the effective density of states and write an expression in this form.

And which is the carrier density? Or if you are talking about equilibrium. Let me make a distinction. Let me put n_0. When I say n_0, I am talking about equilibrium carrier density. This is not going to be valid for the non equilibrium situations. In equilibrium situations $n_0 = N_C$, I will talk about the non equilibrium situations in the next lecture or in the next week.

Next two three lectures we will be talking about some of this, non equilibrium institutions. For now, we are just saying that n_o basically n_o when I say electrons and equilibrium density of electrons.
$$n_o = N_C \, e^{-\left(\frac{E_C - E_F}{KT}\right)}$$
.So,many times I find that students tend to get confused with the sign in the explanation here. Let me try to tell you how I remember it.

So, the number of electrons is definitely going to be some fraction of the total number of states available. We are saying that the number of states available is basically N_C. So, this is a number of states. So, since your carrier density has to be smaller than this, the exponential has to be

negative. If it is positive, then actually It is going to be the electrons concentration is going to be higher than effective density of states, which is not realistic.

And essentially what we are doing is with reference to the conduction band edge, what is the carrier concentration? So, what we will say is basically N_C is basically E_C–E_F. E_F is somewhere in between the bandgap. It is going to be somewhere between E_V and E_C it is going to be somewhere. So, depending on the distance between the E_C and E_F, we can find out.

So for example, let us say if you take an intrinsic semiconductor let me Basically E_C, this particular thing here. This is essentially distance from reference in eV So, now, let us consider the case of an intrinsic semiconductor. If you take an intrinsic semiconductor E_F of an intrinsic semiconductor is at E_i. So, basically $n_0 = N_C$.

$N_C = 3 \times 10^{19}$, E_C – E_F is at E_i or in the middle of the bandgap. So, this is going to be minus E_g by 2. E_g is basically how much? In this case this band gap is going to be 1.12 eV($E_g = 1.12$eV) and this distance between E_C and E_i is going to be .56eV(E_C-E_i =.56eV).That is the KT = .0256 in eV units. $n_0 = 3 \times 10^{19} \times e^{\left(\frac{-.56eV}{.0256eV}\right)} \sim 10^{10}$

So, if you calculate this, this will be something in the order of 10^{10} centimetres. And if you remember in the last class I mentioned that ni = 1.5×10^{10}. So, this is where the slight variations in the numbers come about. The accepted value of ni = 1.5×10^{10}. But, there is no harm even if you simply take 10^{10} in the calculations.

But you have to you know, in the question generally we mentioned whether you have to use 10^{10} or 1.5×10^{10}. The answers are going to be close. So, you could do a similar analysis for even so n_0. So, we can also write a similar expression for P_0. So, let us say P_0 is going to be now we want to find out with reference to the N_V, with effective density of states for holes, which is N_V.

That is a maximum no states, you can think of. I mean not exactly. It is a maximum. It is a sort of maximum. So, number of holes is going to be something lower than that. So, exponential has to be negative. So, this is going to be minus and the distance of E_F from the reference level.

Reference level here is E_V. So, the distance I want to make sure that this quantity is positive, so highlighted is $\frac{E_F - E_V}{KT}$.

$$P_o = N_V \, e^{-\left(\frac{E_F - E_V}{KT}\right)}$$

So, this is your equations for carrier concentration for electrons and for holes under equilibrium. These represent as I already mentioned, concentrations under equilibrium. So, I mean, you might say that you know, I pulled the formulas out of a hat. I kind of did. But if I want to introduce that entire density of states formalism, it is going to take me 2 lectures or 3 lectures.

And I think that is not necessary at the introductory level. So, but at least you get the idea that the N_C and N_V are coming from basically the distribution of states in E_C and E_V. There is basically some, starting from E_C, there is going to be certain distribution and we make certain approximations and finally come up with this constants. That is effective density of states. It is as if the density of states is all, in all of the states are collapsed at E_C.

But anyway, we will revisit that if necessary in the later stages. So, this is how you can calculate concentrations.

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So, now we know what is equilibrium. Let me put this necessary $n_0 \, p_0$. Let me do that. So, what is the intrinsic carrier density? We know that intrinsic carrier density is basically $n_0 = p_{0} = n_i$ So, I can simply take a product

$$n_i^2 = n_0 p_0 = N_C N_V \, e^{-\left(\frac{E_C - E_F - E_F - E_V}{KT}\right)} = N_C N_V e^{\left(\frac{-E_g}{KT}\right)}$$

So, we saw that you know, we saw this expression previously. When I said that, you know ni is going to be proportional to (Eg/KT). In fact, I said it is going to be (Eg/2KT). The reason is, now we have to, if you do it, ni is going to be square root of this. So, there is going to be a 2 here. So,

$$n_i = \sqrt{N_C N_V}\, e^{\left(\frac{-E_g}{2KT}\right)}$$

And this basically a term which we have here, $\sqrt{N_C N_V}$ is the constant. In the last class, we said it is $e^{\left(\frac{-E_g}{2KT}\right)}$ when we said the slope can be calculated based on that. So, this constant is going to be $\sqrt{N_C N_V}$. So, now before I you know go forward, I want to use another form of this expression.

Basically, let me write I would rewrite this expression. Because while N_C and N_V, we are convenient, sometimes it is also convenient for us to look at, you know the distance from the E_i. So, what I will do is this expression formula for n, I will rewrite it, I will do it like this.

$$n_o = N_C\, e^{-\left(\frac{E_C - E_i + E_i - E_F}{KT}\right)} = N_C\, e^{-\left(\frac{E_C - E_i}{KT}\right)} e^{-\left(\frac{E_F - E_i}{KT}\right)}$$

So, when I do this, I can call this entire term. Basically, you have E_C-Ei is simply nothing but Fermi level of intrinsic semiconductor.

So I will call this as $n_i = e^{\left(\frac{E_F - E_i}{KT}\right)}$ So what we are doing is here, this is telling you the reference has changed now. So, basically, this reference level is E_i .That is where we are seeing the concentration of what is the concentration possibility of E_i? It is going to be n_i . And exponential. So, whenever you have a certain amount of doping, it is simply going to E_F is going to go away.

We will have to talk about how the E_F changes, but that term actually, for anti doping, it will increase. So, E_F will be going higher. So, this exponential term will actually become positive and overall you get that n_0 here. And similarly,

$$P_0 = e^{\left(\frac{E_i - E_F}{KT}\right)}$$

So, I mean, I am just trying to tell you how I remember it. You could also try to come up with your own ways of remembering it.

But It is essential that you remember these formulas because we will use them routinely. Come up with any logical approach of remembering this it should be fine.