Integrated Photonics Devices and Circuits
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Lecture – 08
Fundamentals of Lightwaves: EM Waves: Plasma Dispersion

Hello everyone, today I am going to discuss about plasma dispersion. This plasma dispersion effect in metals particularly in semiconductor is very important, we use plasma dispersion in silicon waveguide for designing high speed modulators. So, we need to understand what is that plasma dispersion? It is nothing but basically the carrier concentration dependent refractive index or dielectric constant and that is what we need to derive.

And we can we need to concentrate on how electromagnetic wave propagates in metals in semiconductors contributing to certain kind of frequency dependent refractive index as well as frequency dependent losses. Let us see.

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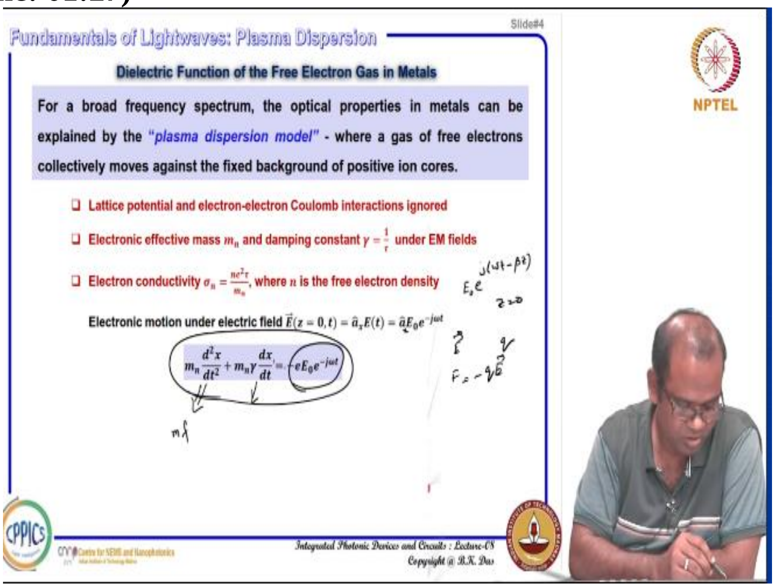

So, far we have learned actually for a broad frequency spectrum the optical properties in metal can be explained by the plasma dispersion model. Again, when I say metals not like exactly metals, it is basically semiconductors also which can be considered sometimes like a metallic properties when you dope heavily that also we can consider. So, in this plasma dispersion where we a gas of free electrons we assume that it is a very heavily doped high concentration precarious will be the free electrons for example.

That collectively moves against the fixed background of positive ion cores that is true for both it can happen for so called a transverse electromagnetic wave as well as longitudinal wave we are considering earlier that plasmonics wave can also take place, but transverse wave also can see some kind of electron motion in the vertical direction. If you consider the wave propagation. So, 3 different interesting point you will note first.

The lattice potential that means you know any crystal whether it is a metal and semiconductor and crystalline structures or whatever amorphous structure you can consider for metals also, but, you see that there are some points where there will be heavy core ion will be there and surrounding electrons will be their relatively free. And this potential you know that periodically in a crystal whether it is a metal or semiconductor this positive ion cores will be there and that will contribute per electron some kind of periodic potential.

So that potential and electron-electron Coulomb interaction ignore for this thing because there are you can consider that sometimes we have mentioned that it is a heavily doped semiconductor or a lot of precarious in metals are there normally, their effect actually is dominant compared to whatever ion cores are there. So, that is why we can ignore for the moment and electronic effective mass to consider those types of electron free carrier like a never bound to the lattice points.

So, we consider effectively but they are bound and we need some approximation to consider each consider them as a pre carrier. So, that consideration is imposed on mass which we call as effective mass of electron which is not equal to your mass of free electrons that can be different inside the material electron mass when you consider of course they are like a free that is one type of model. Normally in semiconductor physics they use. And damping constant when it were considering free it is not completely free.

Again we are considering approximately free if little bit damping is there, well propagation and that damping takes place because of collision sometimes when it moves and sometimes some collision happened because of the lattice heavy ionic points and then that thing considered as a some kind of you can say that average time or mean free time between 2 successive collisions, it is called relaxation time or scattering time.

And then you can say that coalition per unit time that is actually you can consider probability collision probability per unit time is known as a damping constant 1 / tau. And electron conductivity we have explained earlier instead of ne square tau / m n we are just considering effective mass you can consider effective mass, n is the electron density. This thing we have learned earlier this conductivity I just borrowed directly on there.

And then you just think about electron motion under electric field at you can consider z = 0 normal electric field we consider is e 0 e to the power j omega t - beta z propagation direction. Normally, if I consider a particular point z = 0 around z = 0 just if you have a material medium you just fix a coordinate point z = 0 in one dimensional case you are considering and you can consider always this would be a x electric voltage oscillating along x direction.

So, in that case we can consider this equation of motion as this equation where first one basically you are considering as a just mf it is like a mf if they are acceleration, electron acceleration, one dimensional acceleration that force and because damping constant is there, so, you can consider it is something like that dx / dt equal to velocity and velocity by gamma if you are considering 1 over tau that means, mB by tau that means, rate of change of momentum per unit time that is also contributing some force.

And that force that damping force as well as pre movement force total accelerating force together actually coming because of the force imparted by the electromagnetic wave that means, if you have a electric field on charge a q then force is equal to you know that q times E and normally if it is electron which is a negative direction electrical if it is positive direction electron will tend to move in the opposite direction opposite force will be there. So, that is a negative sign there.

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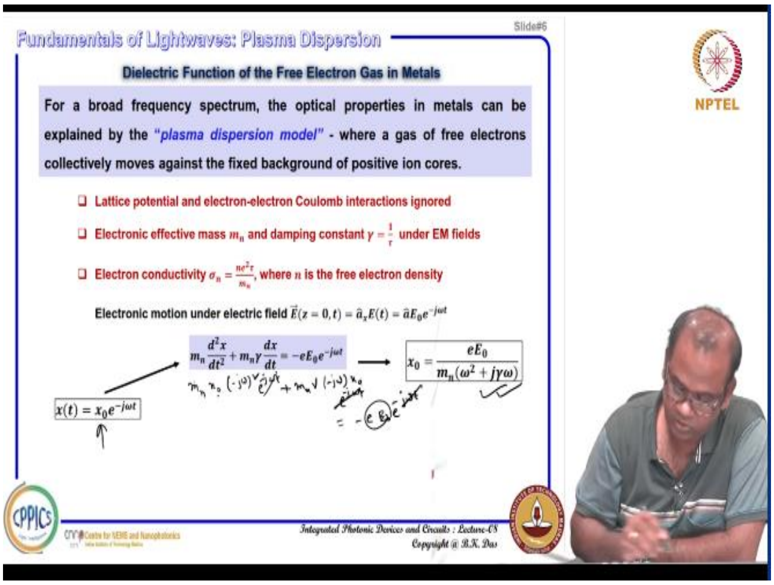

Now, what will be the solution of this equation just we are considering z = 0 and we are focusing only on one electron what happens to one so, called free electron what happens that has some kind of damping constant will be there and force will be there because of the electromagnetic wave and this one we can consider here. And this solution we can consider something like this one dimensional case that is why I am not considering the vector here.

So, if you consider this one in substitute here, so what you get? You get here after substituting you can find what is the value of x 0 because, we have assumed that this electric field when it is in motion x 0 whatever time dependent thing also will be this one in this case. And in this case x 0 can be a complex just if you substitute here it is coming like a complex. You substitute here and then you can find out e to the power j omega t for example, here you will be getting the x 2 / d 2 also m n here.

And then you can have x 0 and minus j omega square and then e to the power j omega t and then this one would be m n gamma and minus j omega and x 0, times e to the power j omega t and right hand side your minus e E 0 e to the power -j omega t, e to the power -j omega t, e to the -j omega t and minus j omega t cancel. So, in that case I can find out x 0 value here x 0 here, so x 0 is E 0 right hand side E 0 divided by this one minus sign, minus sign will be cancelled.

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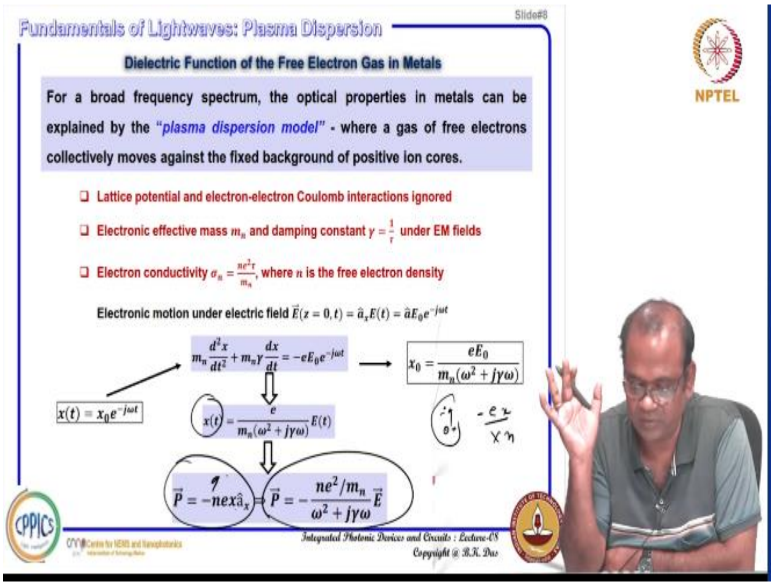

So, now with this x 0 is there now what we get what would be the x t value I just put this x 0 here, then I get x t, x t equal to this one. E 0 times e to the power j omega t I can write this time dependent. If now, you may not consider that it is only one frequency it can be time dependent, whatever electric field is oscillating and frequency is there. So, you can write frequency domain at certain frequency you would be writing something.

Now, what is this? This is actually we know that suppose a positive ion core is there and electron is moving in certain direction x direction it is moved. This is the electron position here. So, this is plus this is minus, so x position displacement is there. So, in that case it is a dipole and dipole you know it whatever the charges their charges E and displacement is x and dipole moment is for a particular atom dipole moment is charge times momentum and this negative sign is that because the dipole moment direction is the opposite.

Because electrical if you give one direction electric normally dipole moment we consider positive to negative in the opposite direction so, negative sine minus f. Now, if you are considering this is only one electron you were considering if they are our electron density of I n. Then you can have n times ex that is the dipole moment per unit volume, n is the per unit volume density of number of electrons electron density I would say n so, this is I n.

So, I have x solution is there we are considering all electrons are moving like this collectively that type of displacement happening. So, in that case I can substitute this x value here, then the dipole moment per unit volume is called as your polarization density that polarization density comes into your Maxwell's equation. So, I have just written polarization density equal to this one that is actually for a given frequency it is proportional to E.

So, this type of things we are considering that polarization density is just a linear to electric field we are just considering all solutions are here now linearly dependent. This solution simple solution is coming like this.

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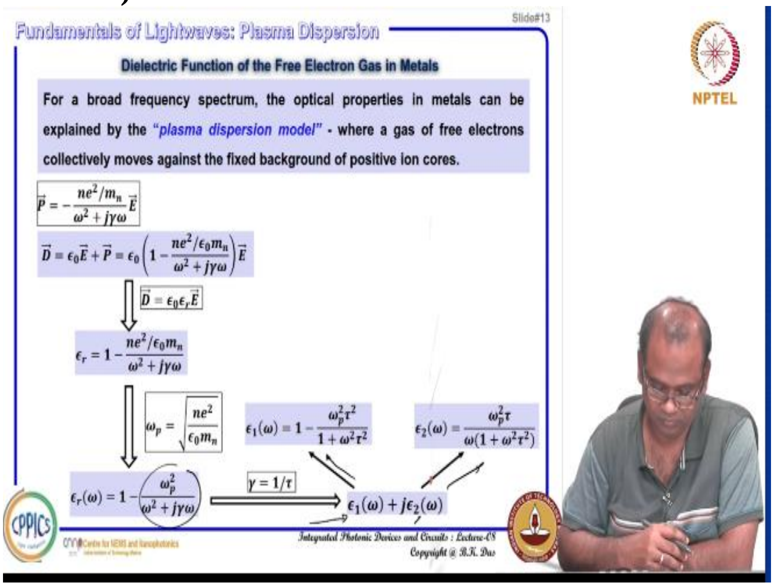

Now, I have just written down that one here same thing polarization density. And now displacement vector we know that displacement vector is this one epsilon 0 E what is coming from external things and P polarization density P is this expression. So, it is simply you can modify like this straightforward just putting here P you are putting here directly minus sign is there epsilon 0 will be common. So, then epsilon 0 comes here m n.

Now, so if I just think that D equal to epsilon 0 epsilon r E that is D equal to epsilon E you will write. So, that means, I can write epsilon r from here if this one is the epsilon r, D equal to epsilon 0 epsilon r E. So, epsilon r will be this one just within bracket whatever is there that is actually epsilon or dielectric constant of 3 electron with some damping constant is there they have considered.

Now, we just consider because numerator if you see here this thing they are actually all are fundamental constant only thing is that density number of electron density that is given there. We just consider that is actually omega p square this one is omega p square will come that is actually known as plasma frequency why it is called plasma frequency that will be clearer after some time. If we consider that we get a very nice equation like this omega p square.

So, omega p square it is actually proportional to number of carrier density. So, carrier density more omega p will be more. Now, we know that this gamma equal to one over tau already earlier discussion we have mentioned and then this is epsilon r and you see this is a complex. So, we can decompose into real part and imaginary part and little bit of math one step algebra you can find whatever the real part and whatever the imaginary part epsilon 2, in terms of omega p tau, I have just replaced gamma is 1 / tau.

So, we have real part imaginary part just you have to multiply here omega 0 - j gamma omega then you can get denominator is you can decompose real part and imaginary part simply and the real part will be like this and the imaginary part will be like that straightforward. I multiply this one in the top and also I have to multiply this one in the bottom.

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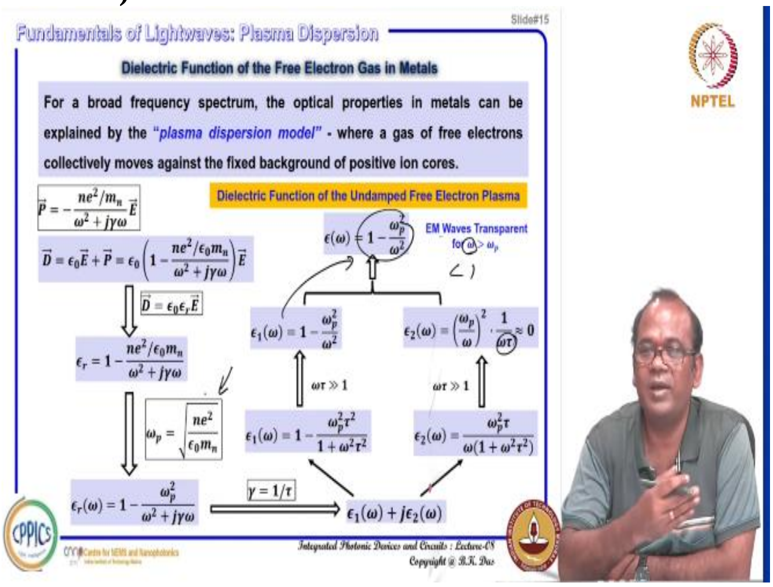

So, now we will be considering a certain region omega tau larger much larger than 1 that can happen if gamma is where is gamma = 1 / tau is very small or omega frequency is very large. Whatever the things if you just consider omega tau much larger than 1 typically it can happen also with large frequency we are considering. Then in that case this omega tau this is very large this 1 you can ignore if that is large omega tau is much larger.

So, in that case this one will be reduced to so, I can ignore 1 so, omega square tau square, tau square tau square cancel omega p square / omega square this one will come. And here if I ignore 1 then omega times omega square omega 3. So, omega p square / omega 3 square and then tau tau will be cancelled. So, we will be getting 1 over omega tau. Omega p square here 1 omega is there that should be omega square.

For electromagnetic wave, if you are considering omega greater than omega p this is approximately equal to 0 because this is actually very large. So, you can just reduce to imaginary part can be smaller only the dielectric constant will reduce to real part only like this one will be dominating for example, but this type of situation if that happens. And omega greater than omega p that means, this will be less than 1. So, in that case your electromagnetic wave will have some dielectric constant which is completely real.

And if it is completely real then refractive index also will be real and it will be propagating electromagnetic waves will be propagating without any loss. So, that is why it is called electromagnetic waves that means, you need to have omega at least greater than omega p, omega p can be calculated like this. So, you can get your plasma frequency so, called a certain frequency greater than plasma frequency your electromagnetic wave can propagate without any loss it is a transparent that is interesting.

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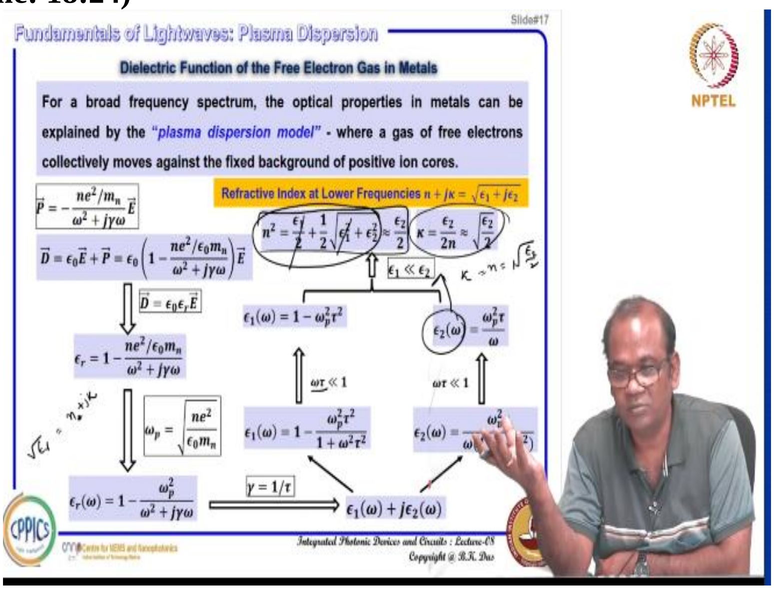

Now, other region we are considering up to here it is same and instead of omega tau greater greater than 1 very small frequency if you are considering for example, or certain tau and tau is their tau value is there that is somehow less. So, what we can consider in that case if it is small this part we can ignore, this part we can ignore. If we ignore that, then this real part will reduce to this one and the imaginary part will reduce to this one.

In this case, if you see omega tau that means omega is much much lower that means epsilon 2 will be much higher compared to epsilon 1. So, if you consider epsilon 2 much higher compared to epsilon 1. Then earlier you know that n square expression we have derived earlier, because square root of epsilon r equal to we can write n 1 + j kappa and that is why epsilon 1 and epsilon 2 we can relate to the n and kappa. And they are we have derived what is the relationship between n square and kappa.

Since epsilon 2 is much greater than epsilon 1 these expressions for refractive index real part of the refractive index can be approximated as, so we can put this one cancel, this one cancel. So, epsilon 2 / 2 approximately, because epsilon 2 is dominating. And kappa we know epsilon 2 / 2n. So, n value is the square root n equal to square root of epsilon 2 / 2 because n square expression is this one if you just put kappa also same. So, that means you are getting n and kappa will be same in this particular case.

So, what I mean to say that when omega is some hot very smaller frequency may be long wavelength, for longer wavelength case this kappa and n real part and imaginary part of the refractive index they are almost similar.

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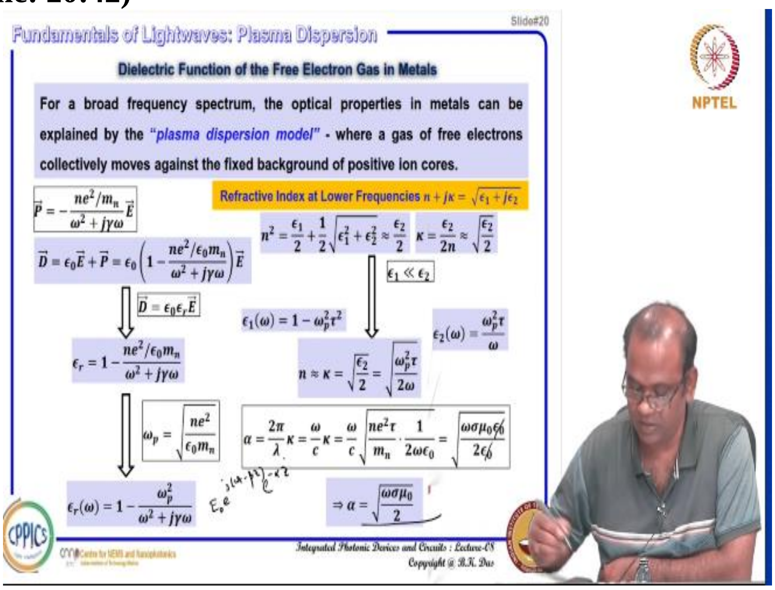

This is a little bit reverse situation I am considering when epsilon 1 less than much much lower than epsilon 2 that is what we have considered earlier. Then n = kappa equal to same thing both are same whatever written. Now, epsilon 2 alone I have this one I just put down here square root of omega p square tau and 2 is there 2 times omega I just put down. And now next thing is that alpha, alpha we know that is the attenuation coefficient for the electromagnetic field propagating inside the material that is 2 pi / lambda times kappa.

Or omega / c that is also we have derived c c multiplied 2 pi c / lambda = omega and c will be there this one. Omega c and kappa expression put down here. how what we are put omega by c kappa equal to this one and omega p expression is this one we are p square if I just put down what I get? I get very nice equation. So, alpha expression is reduced to this one, let would simplification what you get here this one.

So, mu 0 epsilon 0 epsilon 0 epsilon 0 cancel so, omega sigma mu 0 / 2 that is alpha, alpha is attenuation coefficient you know that E = E 0 e to the power j omega t - beta z e to the power -alpha z. So, amplitude how it is attenuated as it propagates. So, attenuation coefficient we find this one.

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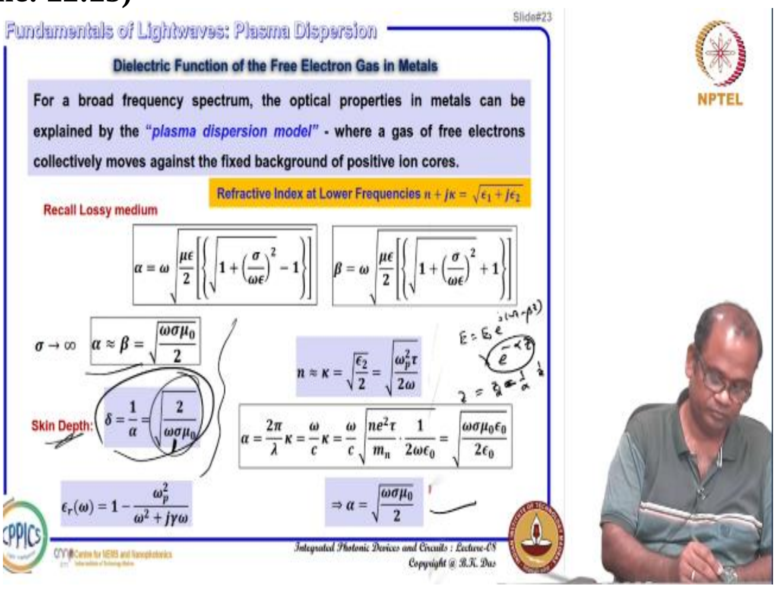

You recall Lossy medium, Lossy medium we would call earlier we have discussed we have discussed gamma = alpha + j beta space dependent electric field in Fourier domain we have solved and alpha expression for Lossy dielectric medium this one and beta we have solved this one and then for sigma is very large if you are considering infinity sigma tends to infinity means if it is large or maybe you can consider sigma is greater than omega epsilon, some values there.

Then I can ignore this one I can ignore this one compared to this one then alpha value will be alpha = beta equal to. So, basically at lower frequency or very high conductivity that region the alpha value real part of the propagation constant and imaginary part of the propagation constant they are identical. So, skin depth when it is propagating e to the power for example, E equal to your writing is E 0 e to the power j omega t - beta z times e to the power - alpha z attenuation coefficient, z = 1 / alpha beta.

That is actually you can consider z d, z = z d = 1 / alpha, then this will become 1 over e it will be a minus 1 that means your amplitude will drop 1 / e, 1 / alpha distance if you find 1 / alpha, alpha is the per unit length what about the attenuation. So, 1 / alpha distance it travels then your amplitude will drop to 1 / e that is actually called skin depth that electromagnetic wave certain frequency or very high conductivity if it travels high conductivity and you can have optical frequency also.

Then, you can think of the optical frequency after traveling to this much length, it will be attenuated to 1 / e that is called actual skin depth. So, if you have a source material medium it cannot travel first depending on this value. That means, if your sigma is very large in this expression if you see sigma is very large, then your delta will be small otherwise suppose sigma is fix if you are increasing your frequency in that region.

Then your delta also would be small, higher the frequency it cannot penetrate that is why optical frequencies if you see in the metal it normally reflects, it does not pay cannot actually propagate longer distance. So, same thing here we say that this is actually alpha whatever you are getting here this value and this value same and skin depth we can just find out the 2 different treatments, we have used Lossy dielectric medium.

They are also you can find out what is the skin depth whatever expression you get and here considering plasma dispersion effect also you get similar expression for the proportion constant, loss coefficient I mean to say.

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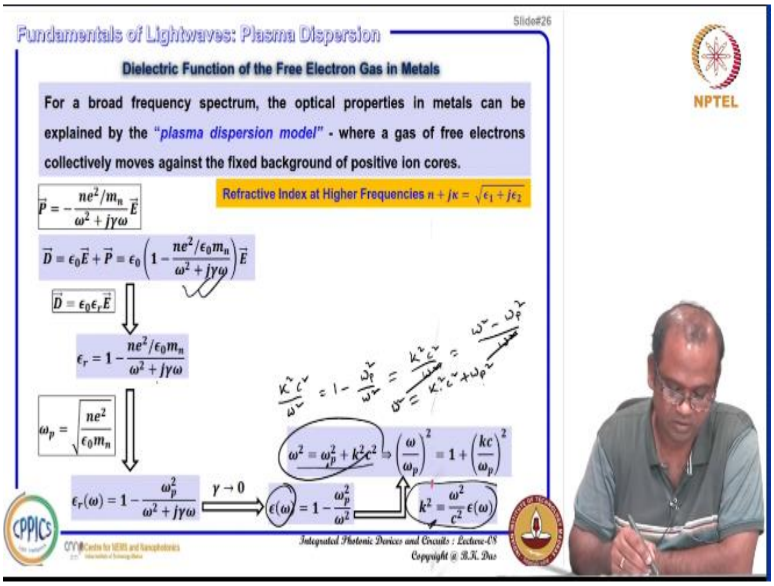

Now, refractive index at higher frequencies I would like to find out now, refractive index loss coefficient is fine, but refractive index we can express like this and that can be just consider square root of epsilon r dielectric constant whatever dielectric constant you get square root of that you will get the refractive index. So, I have written this one some little bit differently I am just considering you just assume that for higher frequencies this gamma that means this damping coefficient is small compared to your frequencies.

Then here if you are just putting gamma = 0 so, omega square will be very large so, in that sense this part I can ignore, then I can obtain this expression that dielectric coefficient that can be expressed as frequency dependent 1 - omega p square omega square. So, now, what is happening I know that for transverse wave that dispersion relation k and omega relationship we can find out we have already discussed earlier that is actually k square = omega square / c square into epsilon omega I know that.

So, if I use this one and from this expression if you insert this one, then you get this one, this expression you get because epsilon omega we will just write here epsilon omega = k square c square omega square that means, I am just writing k square c square times k square c square divided by omega square = 1 - omega p square / omega square. So, this part is nothing but I am getting from here epsilon omega k square c square /omega square and right hand side 1 - omega p square omega square.

So, I can write this one equal to k square c square / omega square = omega square - omega p square / omega square, omega omega cancel and then k square c square + omega p square =

omega square. So, omega p square k square c square that is the expression I have written here. And that one you can actually write if you just divide normalize by omega p square then omega over omega p whole square and this one will be one kc / omega p square that is the what expression I get.

So, this expression I am just considering that you are considering at higher frequencies, I want to find out the refractive index. And that refractive index I am just starting from this whatever the plasma dispersion I have considered polarization as a density all these I am just putting down here and I am getting this thing.

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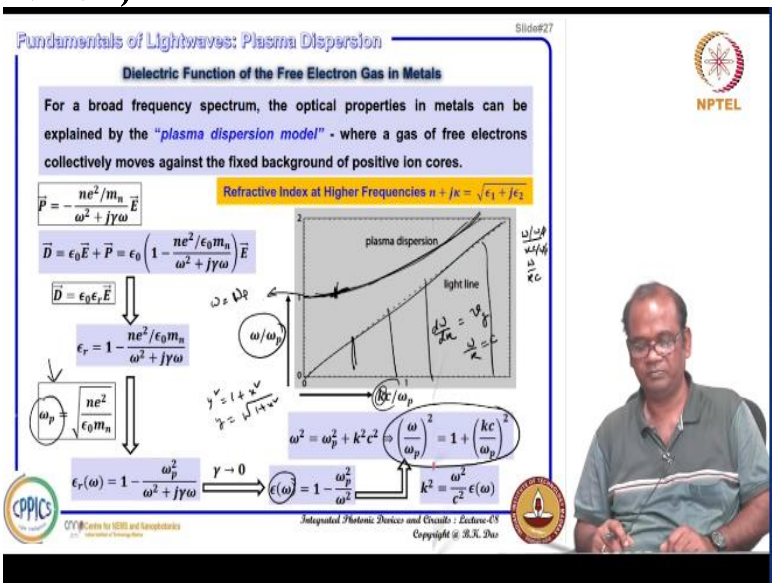

And then if you see this expression if I just tried to plot, it only looking like you see this y axis is omega over omega p that means, this one actually we can write y square = 1 + x square. So, when x = 0 I will be writing y equal to square root of 1 + x square. When x = 0, y = 1 we are putting there. So, for omega / omega p exactly equal to 1 there so, this means this point is omega = omega p exactly at so called plasma frequency.

So, called plasma frequency then omega / omega p 1 then kc / omega p that will be 0 so, that time nothing is happening. So, exactly k is actually appear to be 0. So, you will be getting this equation if you plot you just vary omega p is a constant this is a constant value depending on the constant or ne square epsilon 0 m n you get a constant value just put down here then you get dispersion relation. Now, if you see in this expression omega / omega p = kc and this curve if you see here, whatever value the ratio this is nonlinear basically.

So, I can say that as a function of frequency your population vector k is going to changed while vector is going to be changed. So, normally, at any point if I just tried to find out d omega / dk that is actually called the group velocity. So, group velocity here is very small, because the slope is less d omega / dk if you are just considering this one and this one at any point, just take a slope here, and then you get a group velocity and lower and slowly to be increasing.

And then one interesting point is that, in this case, I am just taking this picture from a textbook, this is called light line, what is that light line that means this omega omega p and kc / omega p they are actually so linear. So, every point d omega / dk is nothing but omega / k = d omega / dk omega / k which is the slope will be that is nothing but c that is the velocity of light.

So, normally this is a light line whenever I am considering that any point you make omega / dk or omega / k they are same. Omega / k will be if I just take a slope this omega / omega p divided by kc / omega p. So, omega p omega p cancel and omega k / omega k, c will be the slope. So, in that case it will be getting linear that is when you are considering light line that means, when the light wave propagates in free space if it is free space nothing then it will be just omega versus k dispersion relation will be like this.

But inside material medium is still looking like this, it will follow this type of so, in that case it is highly dispersive as a function of frequency you are way better it is dramatically changing particularly at exactly at omega = omega p if you see here exactly at omega = omega p your epsilon omega will become 0 and we have learnt earlier that when dielectric constant is 0 in that case no transverse wave exist instead of that longitudinal wave comes into picture.

So, electron the wave will be propagating as if it is not like a transverse wave electromagnetic wave will be existing as you will see that electron wave will be oscillating as a collectively to oscillate that is actually called plasma oscillation and that is the reason this will omega p is called plasma frequency.

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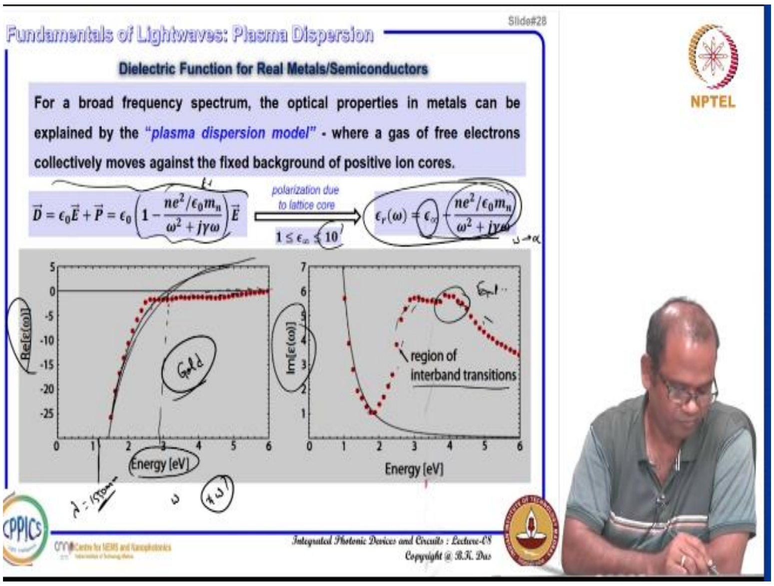

Now, this is nice, so we have discussed that considering your dielectric constant because of the free electrons, free carriers considering sigma. How is the polarization? It is polarization density created because of the displacement of electron motion we have considered under the influence of electromagnetic wave. But in practically you know the semiconductor for example or metal heavily doped semiconductor you have plenty of free carriers are there, but apart from that you have actually practically you have the ion core.

This ion core will haps can also create some kind of dielectric polarization separate can be completely separated from you balance electrons. So, core electrons that can also create some kind of local dipole moment. So, that looks like dipole moment can contribute to your overall dielectric constant that is the reason even though we have just discussed plasma frequency plasma oscillations everything.

But actual metal if you just consider then you will see that this core actually contribute some kind of dielectric constant that is why this equation whatever we have derived here polarization here this particular thing dielectric constant is supposed to be epsilon r that needs some modification and that modification experimental it is found that somehow we have to add instead of one. So, called epsilon infinity typically it is found that it should be within 10, 1 to 10 it must be more than one.

Because that is actually additional dielectric constant is contributed because of the ion core. So, we can have this one, the expression can be modified like this, why it is infinity? If see if omega tends to infinity whatever dielectric constant is there that means at very high frequency, then right hand side will be completely negligible, negligible is small. So, the effect of plasma all those type of things, this part will go. So, whatever leftover that we can call as a dielectric constant E epsilon infinity.

So, your actual expression will be like this in practical situation. In this simple model, whatever we have considered the plasma dispersion model ion core we have not considered contribution of the ion core just to get the contribution, but you should know it is a kind of model, but the practical situation there would be something like that. So, with that model you can do, you can repeat with experimental results.

For example, here, it is a gold material, I have taken again from the textbook gold material, if you see this actual dielectric constant as a function of frequency, frequency means energy here we are just considering energy when frequency is omega then put on energy h cut omega. So, this h cut omega energy is called energy of the photon electron volt. So, 0 1 2 so on. So, about 1.12 electron volts here actually.

It is lambda equal to around 1550 nanometer wavelength that is the curve communication with the optical communication window. So, 1.12 and so on. Now, if you see that when your energy is being increased more and more energy is increased for gold particularly then what happens, you can actually find imaginary part as well as your real part. So, this one it will have a real part imaginary part along with that epsilon infinity will be there that is more than one but less than 10. It can be higher than 10 also something like that.

So, if we just add the real part from here and here you combine that together as it is increased energy is increased towards the visible, this is 1550 nanometer and we are increasing energy means your wavelength is decreasing that means you are going towards visible then you see that experimental results says that the real part of the dielectric constant is actually happening negative value and around this point this is the value here about free electron volt or so, this is actually almost saturating higher energy and saturating to 0.

Some model it is fitted with here it is swing like this, but practically when you measure the real part is almost close to 0 is happening. So, when close to 0, what is happening for higher energy as your higher frequency when it goes that time, it is something like that real part is 0. So, you can see like some kind of 0, real part 0 means basically it cannot propagate only the imaginary part will be there, that imaginary part here it is shown here, because of this imaginary part actually, you can see that will be attenuated.

And that imaginary part also you can see as a function of energy, this is actually dropping like this. But you will see certain kind of experimental measurement these dots these are actual experimental measurement, this experimental measurement shows that imaginary part is increasing at higher energy and that is that cannot be modelled with this one this happens because of the interband transition. Within the band the high energy photons will be absorbed by electrons and it can also create it can see some losses.

So, this is basically absorption photon will be destroyed and electron will be it will capture the photon energy to get its own higher energy within the band that is why it is called interment transitions. So, this is the real part of the dielectric constant this is the imaginary part of the dielectric constant.

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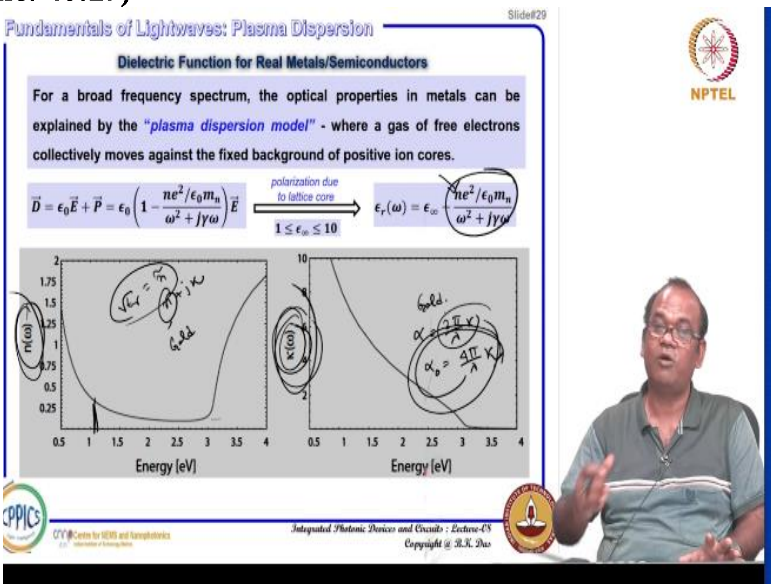

And you can also measure say refractive index also, because square root of epsilon r = n this one that can be considered as this one. So, this n omega if you are seeing this is for gold again for material metal, this is also gold from the textbook this picture I have taken, you see the real part of the refractive index it is dropping as you increase the energy as you increase the energy means, you are actually decreasing the wavelength.

So, this is actually called dispersion actual dispersion that is extracted from the results shown from here. So, whatever value you are getting here this model using this model, you can extract using this expression real part of the refractive index it is showing like that and imagine it that is called extinction coefficient you know alpha = 2 pi / lambda times kappa if you know the kappa then you will know the attenuation coefficient.

And then from the Beer's law, what is the loss coefficient loss constant you are getting that will be 2 times 4 pi / lambda kappa. So, once you know that then you can find that how it will be attenuated. So, this is shown for gold same type of consideration you can consider for semiconductor for silicon also if you are just considering silicon crystalline silicon or intrinsic semiconductor silicon or conducting silicon a little bit of doped silicon.

If you are using as a core material for waveguide for example part photonics integrated circuits, in that case also if it is doped or carrier concentration, if you are increasing conductivity you are increasing then also you can see this type of loss and refractive index both. You see, you can have refractive index real part of the refractive index and the imaginary part of the refractive index both actually depends on this value.

And the in this value if you see n is there what is that carrier concentration that carrier can be free electron that can be free holds also. So, for a given n you can have a certain value for example, you are operating at energy here, it is you are considering for example, silicon not gold for example, and you know that you have a certain refractive index. Now, if you change the concentration then the real part and both the real part and imaginary part will be changed.

So, that means, your real part of the refractive index also will be changing and imaginary part of the refractive index also will be changing. So, that particular property is very much you can exploit particularly in semiconductor. Semiconductor you know, you can actually have a some kind of diode junction and junction region, you can actually control the carrier concentration by bias control. So, if you can give a bias electrical bias you can control the carrier concentration.

And if you can control the carrier concentration both real part of the refractive index and the imaginary part of the refractive index can be changed. Real part of the refractive index change resulting to a phase change and imaginary part of the refractive index change resulting to a loss. So, that means, you can actually control the carrier and you can control the loss coefficient as well as refractive index space constant, space actually can give you some modulation phase modulation and this one so, called extinction coefficient control can give you attenuation, you can have a variable optical attenuator.

So, this is the basic thing is exploited for photonic integrated circuit using CMOS compatible silicon photonics platform. So, very important when we will be discussing photonic integrated circuit, particularly modulator where actually convert electrical signal into optical signal. When you design a modulator, that time this thing actually very important, you have to think about how to control the carrier and how to control the refractive index and how fast you can do that that is how modulator design.