

Integrated Photonic Devices and Circuits
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Lecture - 07
Fundamentals of Light waves: EM waves

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Slide#1

Integrated Photonic Devices and Circuits

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Lecture - 07

Fundamentals of Lightwaves: EM Waves

Wave Propagation in Metals and Semiconductors

$E_g = \frac{E_0 - E_0}{(1 + i2\pi\sigma/\omega\epsilon_0)}$

$\mu > \epsilon_0$

$\sigma > \omega\epsilon_0$

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Hello everyone so far in the previous lecture we have discussed about how electromagnetic wave propagates please increase space and lossless dielectric medium as well as lossy medium. So, when we discussed about lossy medium we just additionally consider that sigma that is conductivity is not equal to 0 some conductivities there and using that actually we could find out that the propagation constant could be modified as alpha + j beta.

It will have a real part and complex part imaginary part real part is alpha that contributes to loss as it propagates and beta actually contributes the imaginary part contributes to the phase that is your phase constant as it propagates. So, that is what we have discussed in the previous lecture. So, now, we will be discussing about electromagnetic waves or wave propagation. Light wave propagation I mean to say in metals and semiconductors.

So, as you know there is a some distinction some differences are there in metals and semiconductors. So, metals you know that if we just think of a band diagram suppose this is at any x axis is something partition and then y axis if you just consider energy of electrons inside the metal or material then we know that there is a band always energies, energy of electrons inside metals it is like a band.

So, from here to here, this is called band anywhere any energy states it can take electron and you can have sometimes there is a gap. So, called band gap and then you can have another band also. So, you can think of some other band, if you just think about this thing. So, this is band gap and then it can have another energy bands will be there. So, typically in electron in metals particularly if you just consider metals.

If you see the top most band all the energy lines energy solutions whatever you get within the band, they may not be occupied and you can see that up to this point everywhere you can see anywhere the electron is there. So, some kind of electron density you can consider n will be there and that will be occupied up to some levels and above that in this region for example, slowly, slowly it will be reduced the density and higher energy state following so, called Boltzmann distribution.

So, you can think of that density more density will be at the lower energy levels and slowly, slowly it will be reduced at the higher energy level. So, density of states accordingly and according to your so called Boltzmann distribution or permit Dirac distribution you can say that will be distributed. So, in that case normally any electric field if you apply that can be electromagnetic wave there can be some E_c to certain fields.

Then electrons can actually energize and they can be easily movable, they can conduct current that is why some metal you will see that the conductivity σ whatever they consider your σ that will be very large that can be large basically for metal. However, for semiconductors if you see that the gap there is a clear band is there, but sometimes what happens that the gap is a little bit it is a small not so large.

If it is small then what happens the upper 2 bands if you are just considering this one and this one what happened some electrons from the this is supposed to be completely filled this band some electrons, it can go to the higher band were supposed to be empty at low temperature, but at room temperature electrons can go. So, you can see some kind of electrons here occupying at room temperature and here some empty space will be empty point will be there some vacancy will be there.

So, in that case, normally these are electrons energized from the room temperature, lattice vibration etcetera it can take. So, you can say some carrier electron is here and here you know huge amount of energy bands energy solutions will be there. So, electron can occupy and they can actually freely move up they can little bit energize also they can move. So, they can contribute here electron will contribute in that this is called conduction band.

So, called I can write conduction band, this is conduction band and this can be called as a valence band. So, in semiconductor you know conduction band and valence band will be there and this will be your so called bottom of the conduction band energy and this can be called as a top of the valence band energy and we call this $E_g = E_c - E_v$. For example, in case of silicon this $E_c - E_v$ equal to about 1.12 electron volt.

So, that is the band gap. So, it will be silicon up to here you can see some kind of band that is called conduction, valence band and then there is a gap of up to 1.12 electron volt then you can have a conduction band. So, some electrons can easily overcome this 1.12 electron volt at room temperature and that they can occupy in the conduction band. So, that is why it can be somehow some carrier will be there in the conduction band and some holes. So, called holes this will be called as holes will be in their valence band.

So, in that case both electron and hole can contribute. So, we can say for electron we can write n and for hole we can write σ . So, we can have conductivity. However, this conductivity you can control also if you for example, if it is silicon what happens you can have a some kind of dopant like pentavalent silicon normally pentavalent atoms are their crystalline structure.

So, if you adjust to doping silicon with some for example, phosphorus pentavalent then this fifth electron, electronic states normally comes very close to conduction band energy solutions comes very close to conduction band. So, in that case these energy separation from the bottom of the conduction band it is very small. So, easily that electron can contribute to the conduction band and you can get a lot of electrons in the conduction band free electrons.

So, in this way this will be called an n type semiconductor and thus you can contribute conductivity in σ_n . Again if you doped atoms like boron, then you can get boron you know this is a trivalent atom. So, you can see when it is just a bound inside the silicon crystal

that means, occupying a state sub silicon then the fourth electron is actually shortage in boron, boron in valence electrons there are 3 normally silicon valence electrons are 4.

So, that fourth electron, which is not available in boron that will occupy a state somewhere here just above the valence band. So, in that case some electrons can easily energize and occupy that energy and it will be attached to boron. That is why negative ions will be immobile negative ions will be created and in return in the valence band you see empty space. So, some shortage of electrons.

So, that is how we can say that those were electrons are not available they are holes basically. So, in that case it will be called p type semiconductor and you can also contribute conductivity in holes. So, that is how basically whether it is a metal or semiconductor, then they can be just approximately consider as a material with having some kind of sigma which actually contributes to similar equivalent to your whatever we discussed earlier dielectric, lossy dielectrics.

So, some sigma value you have to consider not equal to 0 you have to consider and you have to treat your electromagnetic wave accordingly. So, there is a one third situation you can think of. Suppose the same thing you are not dope silicon for example, if it is not doped at all it is intrinsic and very little amount of electrons will be available in the conduction band and very little amount of holes will be available in the valence band. So, the sigma will be slightly lower it is kind of not so high like conductor.

So, it is not doped at all whatever carrier you will see in the conduction band and valence band that is because of the thermal excitations. So, in that case, if any electromagnetic wave propagates here, which electromagnetic wave if it is propagating through the material, this is your position I am saying that everywhere energy electrons will be occupying this band and conduction band there will be 3 things will be there.

So, you pay electromagnetic waves propagating the light for example, electromagnetic wave light wave if you know light wave you can imagine quantum mechanically that is a stream of photons and that stream of photons normally $h \nu$ energy will be $h \nu$ equal to $h \nu$ and sometimes you can call $h / 2 \pi$ times $2 \pi \nu$ and $h / 2 \pi$ we call it h cut and $2 \pi \nu$ means angular frequency h cut. So, that is the energy of a photon.

If this energy of a photon you can calculate energy of a photon if $\lambda = 1550$ for example, 1550 nanometer then this energy you can consider basically this can be considered as hc / λ . So, C the Planck constant and λ then you can calculate the energy if this photon energy $h \omega$ is less than E_g band gap then that electromagnetic wave it will be actually can propagate inside the semiconductor also.

If the photon energy less than band gap by the way then it can propagate as if it is like a lossless dielectric medium. So, even though it is a semiconductor like silicon, germanium, gallium arsenide if you are treating and if it is undoped at all completely undoped intrinsic material and any electromagnetic wave consider if you consider a stream of photons and photon energy is just less than band gap energy then that semiconductor that particular semiconductor it can be silicon it can be gallium arsenide it can be germanium.

That can be considered as a just lossless dielectric medium. However, when it propagates of course, what happens if $h \omega$ greater than E_g , it can happen because you notice higher the electron in the valence band can absorb the photon energy and photon will be destroyed and then that electron can go up. So, you can create 1 electron here and 1 hole here, 1 electron will be here and 1 hole will be in the valence band. So, electron hole pair will be generated.

So, in that case you will see some kind of certain kind of absorption. So, one thing is the absorption I will be talking because that absorption and sometimes also what happens if a certain condition is created the electron can reoccupy the hole and can emit again photon. So, this type of absorption, emission can happen also photon absorption photon emission can happen and that is normally quantum mechanical phenomena.

We will be discussing that later we will be discussing when we shall discuss about laser diode and photo diode. For the moment we can consider that semiconductor as a dielectric material for electromagnetic wave having energy photon energy less than band gap and in that case, you know that this can be considered as a dielectric dope, but never, you have to consider also there are some electrons because of the thermal excitation etcetera will be there.

That electron in the conduction band and some holes in the valence band they can contribute some kind of conductivity. So, that conductivity you can consider it is not completely lossless dielectric, it is a kind of lossy dielectric material medium. So, with this understanding that the conductivity band gap transparencies, all those types of things, if we consider, then we can think that electromagnetic wave when it is propagating inside the material, whether it is a metal and it is a semiconductor how that behaves.

So, normally it is slightly different than what we have discussed, when we consider the lossy medium that σ is there. The reason being both metals and semiconductor metals you know that what you call that the conductivity can be very large and semiconductor also if you dope or even room temperature it has certain kinds of conductivity. So, depending on the conductivity you can see that certain frequency range it may transmit.

And sometimes you can see that the velocity of electromagnetic wave phase inside the material within metal and semiconductor depending on the conductivity and so, on, they will travel with different velocity different frequency, different velocity. So, they will there will be some kind of material dispersion all these we think we need to discuss because we have to deal all the semiconductor as a dielectric material, semiconductor dielectric material, metal.

All these things you need for developing your photonic integrated circuit especially when you talk about CMOS electronics and CMOS compatible photonics circuit you need to deal all these materials. So, we need to know how they react how they behave all these material as a function of frequencies as far as lightweight propagation is concerned. Let us move on.

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Fundamentals of Lightwaves: Metals & Semiconductors Slide#7

Lightwaves are Electromagnetic Waves Governed by Maxwell's Equations

Maxwell's Macroscopic Equations

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \nabla \times \vec{H} = \vec{J}_c + \frac{\partial \vec{D}}{\partial t} \quad \nabla \cdot \vec{D} = \rho_v \quad \nabla \cdot \vec{B} = 0$$

Coupled Fields

$$\vec{D} = \epsilon_0 \epsilon_r \vec{E} \quad \vec{B} = \mu_0 \mu_r \vec{H}$$

Metals are in general:

Non-magnetic: $\mu_r = 1$

Conductivity very high: $\sigma = \frac{ne^2\tau}{m}$

Typical electron relaxation time: $\tau \sim 10^{-14}$ s

Following charge conservation:

$$\nabla \cdot \vec{j} = -\frac{\partial \rho}{\partial t} \Rightarrow \nabla \cdot \vec{j}_{int} = -\frac{\partial \rho_{int}}{\partial t} \Rightarrow \nabla \cdot \vec{j}_{int} = \nabla \cdot \left(\frac{\partial \vec{P}}{\partial t} \right)$$


$$\vec{j}_{int} = \frac{\partial \vec{P}}{\partial t}$$

electronic charge and mass

$e = -1.6 \times 10^{-19}$ C

$m = 9.1 \times 10^{-31}$ kg

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So, you know, if you want to discuss about so called light waves or electromagnetic waves governed by Maxwell's equation that is constantly continuously insisting and even though these are the Maxwell's equations, I am just repeating here. So, that these equations you can always follow and refer following the discussion when I discuss different phenomena. So, now, you see, metals are in general, these are the property whatever we discussed so far it is a non-magnetic material.

So, here when I am just considering $B = \mu_0 \mu_r H = \mu_0 \mu_r H$ magnetic field this most of the time whatever metal we use, they are non-magnetic that is I mean what should be equal to 1 you can directly you can use that and conductivity which I consider that is your very high, but that conductivity you can have an expression what is that that is n is the density of electron free electron I mean to say and e is the electronic charge and τ is the tau is a very important thing.

This is basically called the relaxation time or whenever you apply electric field sometimes what happens even though you are not applying electric field, the electrons metal in the metal they are like a free they are just like a Brownian motion and they will be when they are just randomly moving around, they can actually somehow sometimes it can scatter out of the lattice points and that is the collision they can collide with the lattice points.

So, in that case, it will suddenly it will stop its motion and then again it will start taking up the energy. So, that type of situation in time be taken between 2 collision is called the relaxation time tau. Typically that is 10 to the power -14 second in case of meta where

density is very high and m is of course, we can say that mass electronic mass and if it is free completely free, it is 9.1×10^{-31} kg and sometimes you can modify this mass because of the inside material medium.

We have a little bit potential so called periodic potential will modify the mass that will be called effective mass. In that case, m^* can be higher than this or can be lower than that depending on this dispersion relation inside, I am not going into that right now. So, now you know we can find the conductivity of you know the density of electrons and also the relaxation time there are various experimental techniques you can find that you can use them to find out the conductivity.

Here I have just given 1 table directly taken from textbook, where we see that aluminium copper, gold and silver these are the material normally used typically used in any electronics CMOS electrons particularly aluminium and copper is very popular they are frequently used in CMOS industry, if you see the P carrier concentration electron P electron concentration it is in the order of 10^{18} to 10^{22} per centimeter cube it is very large and copper is slightly lower.

However, if you see conductivity see major or calculate depending on the effective mass as well as τ , you see copper conductivity is higher relatively higher. This is actually 3.5×10^5 say per ohm per centimeter per ohm means sometimes it is called mho sometimes it is called Siemens per centimeter and so on. Different units you can use, but typically in SI units we can just write like this. So, these are the conductivity and this is a much much higher than it is then 10^5 .

In case of dielectric where no free electrons are there this is completely 0. So, you can imagine this can contribute a lot of losses inside the material medium just simply you can conclude and if you are just considering silicon normally in silicon you can go doping concentration in the order of 10^{19} to 10^{20} per centimeter cube. Normally, in a crystal silicon crystal the atomic density is in the order of 10^{22} .

So, you can replace maybe every 100 silicon atoms 1 boron or phosphorus you can replace still you can maintain a silicon crystalline structure and then once you dope you can get this type of concentration. So, this type of concentration also you can put and you can try to find

out the so called conductivity typically in case of semiconductor conductivity you are writing like this $n e \mu$ so called mobility. What is that mobility? Mobility actually expressed by this one.

So, you know drift velocity of electron insight crystal it is proportional to electric field you apply because if you apply electric field electron will start moving and that is called that is somehow called drift velocity and that will be no typically it is proportional, but this proportional constant we write as mobility. Electric field and drift velocity. So, once you know the mobility then you can write any μ normally this μ stands for $e \tau / m$ that is what you are getting $\sigma = n e^2 \tau / m$.

So, in case of semiconductor and metal you can treat alike as long as you are considering the carrier concentration doped semiconductor or some intrinsic and also metals with high concentrations electron, free electrons etcetera you can be you can treat it alike. So, now one important conclusion here we will just try to show that in case of metal or semiconductor. You we know that any electromagnetic wave propagation inside the material medium we need to know 3 parameters σ , ϵ and μ .

In case of semiconductor or conductor we know that this is nothing but μ_0 . $\mu_r = 1$ and normally this we consider ϵ or ϵ_0 and σ that is what under consideration. We will so that when electromagnetic wave propagates particularly at higher frequencies, optical frequencies that time the σ and ϵ are they are somehow they properties somehow somewhat blurred you can actually relate σ with ϵ_r depending on the σ you can actually estimate what is the ϵ_r actually.

Frankly speaking electromagnetic wave propagation in metals or semiconductors, heavily doped semiconductors whatever you can consider doped semiconductor doped semiconductor they can be explained by either of ϵ or σ that is what the conclusion we will see. So, let us move on really quick modify towards that point, because you know in a semiconductor and some metal you have us talking about a lot of free electrons.

So, that free electrons when it moves, it is not completely free of course from the metal or semiconductor metal or conductor because electrons in that case, if it is completely free, it will come out outside the metal somehow wondered. So, we actually when we are talking

about the Maxwell's equation particularly Gauss equation here there we actually decompose the charge basically.

So, divergence D equal to charge density that we consider as a 2 part one is called charge external and charge internal you can think of something it is a free or something incoming excess carrier related to that and ρ internal that is related to somehow it is part that is bonded to lattice points or somewhat internal polarization you can think of. So, we can call it external charge internal charge and similarly.

Particularly ρ v we are considering a dielectric or free space that some charges they are in that case we can consider ρ external or whatever things comes and internal the internal charge will be instead of ρ v, we are just decomposing into 2 parts and similarly, if we see this equation, you have current density which is actually so called σ times e that is actually the conduction current and that is how you are getting the current density.

That current density also you can think of about some kind of external current density as well as internal current density typically J c, we can just think of the external and we are just adding internal. So, we are just trying to see semiconductor and conductor this 2 part of charged density can happen when electromagnetic wave is propagating one is external and other is internal.

So, then this displacement vector what we are showing here expression we have discussed earlier. From there, if we just click a bit move on just that, we know that divergence D equal to divergence of D if we just try to write then divergence of this part also. So, if you just put down that thing, then you can say divergence both divergence D you are writing divergence D plus equal to divergence of E ϵ_0 plus divergence of p that is both sides.

If you are taking divergence and if you are just considering this ρ total charge; then we can say that this divergence E basically $\epsilon_0 E$ that actually gives you the total charge. So, this will be like this divergence D equal to if you are writing that is total charge then divergence equal to ρ total / ϵ_0 . So, if you are just writing this one as total charge with the electric field and ϵ_0 whatever the things are coming that is actually total charge.

This total charge actual contribution coming from your internal and external that is what the understanding modelling. So, if you are just putting this one here like this, then obviously, divergence ρ that means the polarization density and its divergence if you are taking that is nothing but $-\rho$ internal. So, ρ internal whatever we are trying to define that internal charge that is because of the polarization density.

Polarization density means, you are just applying electric field then you have a electronic a little bit displaced with respect to the atom and then you can create a dipole moment dipole moment per unit volume that is actually called polarization. That is that polarization happens because of the charge separation. So, you if you are taking divergence of ρ at any point, if you are just talking divergence ρ what about the polarization.

You have you are just taking divergence then you can just consider that is nothing but the charge internal charge and negative charge that is how you can just incorporate ρ external ρ internal. So, because of the polarization density you are having additional charge that is how must be added to the external charge that is the whole idea. Now, if you are just considering continue to equation you know charge conservation you know charge current density and you have current density that means divergence of \mathbf{j} .

How much current is diverging out if you are at any point if you are trying to find at that particular point suppose all the current density if you are just putting divergence at any point, then what you can see that, that particular point what is the charge rate of change of charge density. So, the rate of change of charge density that must be negative that should be reduced, that is why you see the current diverging out.

So, we write divergence \mathbf{j} equal to $-\text{del } \rho / \text{del } t$. That is how we call it as charge conservation. So, if you are just again know that ρ equal to ρ external current equal to divergence \mathbf{j} , \mathbf{j} equal to \mathbf{j} external and \mathbf{j} internal in that case if I write this one per internal case then I can say that $\text{del } \rho$ internal / $\text{del } t$. Whenever I am taking internal current density I have to write internal charge.

So, that means, this internal charge variations we can say that we have already discussed earlier here ρ internal is a divergence ρ . So, if I just put here divergence ρ $\text{del } \rho / \text{del } t$ so, I can write that divergence I can take it out. So, this one I am writing $\text{del } \text{del } t$ times divergence

of p and minus sign will be there. So, what I can say this this one I can take it outside and because it is a space dependent del equal to del del x del del y del del z 3 different direction and del del t I have taken.

So, this one I can write here. So, comparing these 2 I can easily write that j internal equal to del p del t. So, internal current density we can say that it is a polarization rate of change of polarization density rate of change of polarization density we can define as a internal current density.

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Fundamentals of Lightwaves: Metals & Semiconductors Slide#10

Lightwaves are Electromagnetic Waves Governed by Maxwell's Equations

Maxwell's Macroscopic Equations

$$\vec{\nabla} \cdot \vec{D} = \rho_v \quad \vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \vec{\nabla} \times \vec{H} = \vec{J}_c + \frac{\partial \vec{D}}{\partial t}$$

Coupled Fields

$$\vec{D} = \epsilon_0 \epsilon_r \vec{E} \quad \vec{B} = \mu \vec{H} = \mu_0 \mu_r \vec{H}$$

$\vec{D} = \epsilon_0 \vec{E} + \vec{P}$

Since optical response in metals strongly depends on frequency, we need to consider nonlocality responses in terms of space and time by generalizing linear relationships to:

$$\vec{D}(\vec{r}, t) = \epsilon_0 \int \epsilon_r(\vec{r}-\vec{r}', t-t') \vec{E}(\vec{r}', t') d\vec{r}' dt'$$

$$\vec{J}_{int}(\vec{r}, t) = \int \sigma(\vec{r}-\vec{r}', t-t') \vec{E}(\vec{r}', t') d\vec{r}' dt'$$

By taking Fourier transform w.r.t. plane waves defined by $\int e^{i(\vec{k}\vec{r}-\omega t)} d\vec{r} dt$, we obtain relationships in Fourier domain:

$$\vec{D}(\vec{k}, \omega) = \epsilon_0 \epsilon_r(\vec{k}, \omega) \vec{E}(\vec{k}, \omega) \quad \vec{J}_{int}(\vec{k}, \omega) = \sigma(\vec{k}, \omega) \vec{E}(\vec{k}, \omega)$$

$\vec{J}_{int} = \frac{\partial \vec{P}}{\partial t}$

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Let us see, let us move on. So, I have just written down all these things concept. So, what is the internal current density and divergence p equal to this one this may be useful for further discussion that is why I have just written this side and let us see this conclusion that is a very interesting point. You see, since optical response in metals of course, doped semiconductor I should be also consider doped semiconductor is like a metal.

Because concentration can go up to 10 to the power 19 20 so on strongly depends on frequency we need to consider non-locality response in terms of space and time by generalizing linear relationship. So, you understand that your electromagnetic wave or electric field is an oscillating electric field associated to electromagnetic wave it is oscillating and if it oscillates very fast then what happens some effect in a material medium that polarisation density which is the result of electric field oscillation.

That can be that you cannot consider very local phenomena, because something you are electrical these oscillating here, but material will response it will take little more time and as the electromagnetic wave propagates inside the material medium, so you can think about that something happening in surrounding region that also will affect polarisation density elsewhere.

So, that is why we redefine we have to consider whenever we are talking about very high frequency optical frequencies, then this dense displacement vector we have to consider this thing you know this is ϵ multiplied by E $\epsilon_0 \epsilon_r$ is ϵ . So, that is what this one will consider this one ϵ that is $\epsilon_0 \epsilon_r$. But what we have to do we have to integrate suppose I want to find out displacement vector at position r at instant time t .

Then I have to consider all other position vectors. These are that vectors, r' and you have to consider also t' every other points what is the electric field that also contributes to the dielectric constant ϵ . So this is r' and P' is the variable, I am just integrating all positions electric field I am integrating and that how you are getting here you have to take convolution.

Then you can get the displacement vector. Similarly, for current density, we know σE σ times E is the everywhere you are bearing r dr dt and you are trying to find out what is the value at r and t conductivity so this in principle, you have to take this one for your consideration of displacement vector and current density here we are considering internal current density as we discussed earlier.

And then you will see by taking Fourier transform with respect to plane waves, you know plane waves we are considered we normally consider e to the power $j\omega t - k \cdot r$ that is a plane wave you can write E_0 and you can have some polarisation and so on this is power of propagating plane wave and you can also write forward propagating plane wave like this e to the power $j(k \cdot r - \omega t)$.

So, this can be a plane wave because the power propagating the sign I have changed only minus sign I have just multiplied. So, that also will be considered a polar propagating earlier we have discussed this. So, what we have to do if we just take a Fourier transform of this then

position and time 2 variable Fourier transform if we adjust considering position is the real space there that will actually convert it into k space.

That means k you normally you know $k = 2\pi / \lambda$, λ is the reciprocal space λ length the space and time will convert into frequency. So, in Fourier domain the convolution is nothing but just simple multiplication. So, ϵ_0 , ϵ_r you just take a Fourier transform of this one and that that will be k ω and so r will be converted in k ω and e will be k ω so just multiplication it is just as simple principle of convolution and Fourier transform.

Similarly, internal current density you can write like this. So, in frequency domain not a Fourier domain we are Fourier this is actually whatever we consider in time domain considering non localization and here we are considering just Fourier domain that if we know that what is the dielectric constant as a function of frequency and wave vector and electric field as a function of k and web vector that then we can just write like this a straight forward.

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Fundamentals of Lightwaves: Metals & Semiconductors Slide#13

Lightwaves are Electromagnetic Waves Governed by Maxwell's Equations

Maxwell's Macroscopic Equations

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \vec{\nabla} \times \vec{H} = \vec{J}_c + \frac{\partial \vec{D}}{\partial t} \quad \vec{\nabla} \cdot \vec{D} = \rho_v \quad \vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{J}_c = \sigma \vec{E} \quad \vec{J}_d = \frac{\partial \vec{D}}{\partial t}$$

Coupled Fields

$$\vec{D} = \epsilon_0 \epsilon_r \vec{E} \quad \vec{B} = \mu \vec{H} = \mu_0 \mu_r \vec{H}$$

$\vec{D} = \epsilon_0 \vec{E} + \vec{P}$

$$\vec{D}(\vec{k}, \omega) = \epsilon_0 \epsilon_r(\vec{k}, \omega) \vec{E}(\vec{k}, \omega) \quad \vec{J}_{int}(\vec{k}, \omega) = \sigma(\vec{k}, \omega) \vec{E}(\vec{k}, \omega)$$

$\rho_{tot} = \rho_{ext} + \rho_{int}$

$$\vec{J}_{int} = \vec{J}_{ext} + \vec{J}_{int}$$

$\vec{\nabla} \cdot \vec{P} = -\rho_{int}$

$$\vec{J}_{int} = \frac{\partial \vec{P}}{\partial t}$$

$\frac{\partial \vec{D}}{\partial t} = \epsilon_0 \frac{\partial \vec{E}}{\partial t} + \vec{J}_{int} \quad \frac{\partial}{\partial t} \equiv j\omega$

$$j\omega \epsilon_0 \epsilon_r(\vec{k}, \omega) \vec{E}(\vec{k}, \omega) = j\omega \epsilon_0 \vec{E}(\vec{k}, \omega) + \sigma(\vec{k}, \omega) \vec{E}(\vec{k}, \omega)$$

Fundamental relationship between dielectric constant and conductivity

$$\epsilon_r(\vec{k}, \omega) = 1 + \frac{j\sigma(\vec{k}, \omega)}{\epsilon_0 \omega}$$

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Now, let us try a little differently you take this one and take time derivative partial derivative because D can be space dependent as well as time dependent we just take a partial derivative. So, both sides partial derivative then it will be $\text{del } p / \text{del } t$ and $\text{del } p / \text{del } t$ we know that is nothing but the current density internal current density. So, we write down here and then $\text{del } \text{del } t$ we know that are plane wave E to the $j\omega$ t barriers like $j\omega$ t $\text{del } \text{del } t$ $j\omega$ we have used earlier.

Also and if we use that one then what I write del del t j omega epsilon 0 epsilon r k omega E k w. So D value we have already explained here this one and j also we already defined like this in Fourier domain. So, del del t I have written j omega epsilon 0 I have written epsilon 0 as it is and then del del t j omega = E k omega E of k omega and then j internal = sigma of k omega and E of k omega sigma t.

So that is how we can define and with a little modification you just E E cancel, E cancel here then you can j omega j omega cancel you can just write the j omega here epsilon 0 epsilon 0 cancel you can write epsilon 0 here. So you get a nice expression like this. So this is the thing I was trying to explain in the beginning the fundamental relationship between dielectric constant and conductivity if you know the conductivity.

Then you can know the dielectric constant at a particular frequency by the way this conductivity can be also you can think of sigma 1 j sigma 2, if it is real part and complex part, then you can also consider this epsilon r can be represented here. So that means you can either you need to know what is the value of sigma or you need to know what is the value of epsilon r. So either of them is useful to explain the polarisation internal polarisation and electromagnetic wave whatever they dispersion relations that is I am going to discuss a little while later, hopefully it is going good.

(Refer Slide Time: 40:48)

Fundamentals of Lightwaves: Metals & Semiconductors Slide#17

Lightwaves are Electromagnetic Waves Governed by Maxwell's Equations

Maxwell's Macroscopic Equations

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \vec{\nabla} \times \vec{H} = \vec{J}_c + \frac{\partial \vec{D}}{\partial t} \quad \vec{\nabla} \cdot \vec{D} = \rho_f \quad \vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{J}_c = \sigma \vec{E} \quad \vec{J}_d = \frac{\partial \vec{D}}{\partial t}$$

Coupled Fields

$$\vec{D} = \epsilon \vec{E} = \epsilon_0 \epsilon_r \vec{E} \quad \vec{B} = \mu \vec{H} = \mu_0 \mu_r \vec{H}$$

$\epsilon_r(\vec{k}, \omega) = 1 + \frac{j\sigma(\vec{k}, \omega)}{\epsilon_0 \omega}$

Spatially local response corresponds to $\vec{k} \rightarrow 0$: or at very long wavelength regime (in comparison to lattice constant)

$$\epsilon_r(\vec{0}, \omega) = \epsilon_r(\omega)$$

- At very low frequencies ($\vec{k} \rightarrow 0$), ϵ_r is usually used for the description of the response of bound charges to a driving field, leading to an electric polarization, while σ describes the contribution of free charges to the current flow.
- At optical frequencies (high frequency regions) however, the distinction between bound and free charges is blurred.
- For highly doped semiconductors, the response of the bound valence electrons can be lumped into a static dielectric constant $\delta\epsilon_r$, and the response of the conduction electrons into σ' , resulting into:

$$\epsilon_r(\omega) = \delta\epsilon_r + \frac{j\sigma'(\omega)}{\epsilon_0 \omega} \Rightarrow \delta\epsilon_r \rightarrow 1; \sigma' \rightarrow \sigma' + \frac{\epsilon_0 \omega}{j} \delta\epsilon_r$$

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So, now I have written down that expression what about that sigma and dielectric constant their relationship you know specially local response corresponding to k = 0 what is k, k we know that k = 2 pi / lambda. So, lambda if it is very large then k value will tends to be 0 or at

very long wavelength region that is what I mean then particularly that λ if you are considering with respect to the lattice point whatever the electron motions etcetera.

We are considering that λ if it is much larger than that spacing's it can be considered a homogeneous medium as if. So, in that case we can say that $\epsilon_k = 0$, $k = 0$ means I am writing 0 ω , simply we can write $\epsilon_r \omega$. So for long wavelength long wavelength means, you know long wavelength means frequency is sort, sort of frequency range so, you can simply write that.

So at very low frequency ϵ_r is usually used for the description of the response of bound charges to a driving field leading to an electrical polarisation while σ described the contribution for pre charges to the current flow that is the low frequency when it is happening. So, we can consider this one you can just distinguish that $\epsilon_r \omega$ that is actually like a result of your polarisation dipole moment etcetera it is created that contribution.

But the conductivity part you can just distinctly you can consider that there that actually contributes to your current, current can flow. But at optical frequencies when ω is very high, the distinction between bound and free charges is blurred you cannot distinguish because this polarisation actually because of the bound states and pre electrons that will be contributing to the conduction.

So earlier for low frequency they can respond separately conduction current as well as your polarisation etcetera you can think but for high frequency this thing will be you cannot really distinguish the difference I will discuss that that is going to be discussed, for a high doped semiconductor highly doped semiconductor that is again and again I am saying that is like a metal the response of the bound valence electrons could be lumped into static dielectric constant ϵ_r .

Whatever the bound electrons are there in semiconductor you know there will be free carrier and then bound electrons will be there also they can also concept they can actually result into certain kind of dielectric constant. So, that thing if you are considering ϵ_r and the response or the conduction electrons σ does resulting into what you are getting you are saying ϵ_r this is a lump 1.

So, instead of 1 we are just adding delta epsilon r that is actually but the bound electrons and whatever the sigma it is there we have written or you can say that delta epsilon you do not need to put that bound state will little bit modification that you can put as it is 1 earlier whatever it is written here, 1 we can write but the sigma prime but conductivity that can be modified like this.

If you can modify this one and then if you just put here sigma value then it will be giving you same results. So in that case you do not need to consider what is the additional epsilon r because of the bound states are not only thing you have to consider your conductivity is changed. That is it. So, that is the whole idea, some textbook they consider something like this.

(Refer Slide Time: 44:44)

The slide, titled "Fundamentals of Lightwaves: Metals & Semiconductors" (Slide#25), presents Maxwell's macroscopic equations and their application to metals. It includes the NPTEL logo and a photograph of a man in a green shirt.

Maxwell's Macroscopic Equations:

$$\begin{aligned} \nabla \times \vec{E} &= -\frac{\partial \vec{B}}{\partial t} & \nabla \times \vec{H} &= \vec{J}_c + \frac{\partial \vec{D}}{\partial t} & \nabla \cdot \vec{D} &= \rho_v & \nabla \cdot \vec{B} &= 0 \\ \vec{D} &= \epsilon \vec{E} = \epsilon_0 \epsilon_r \vec{E} & \vec{B} &= \mu \vec{H} = \mu_0 \mu_r \vec{H} & \vec{J}_c &= \sigma \vec{E} & \vec{J}_d &= \frac{\partial \vec{D}}{\partial t} \end{aligned}$$

Metals are in general:

$$\epsilon_r(\vec{k}, \omega) = 1 + \frac{j\sigma(\vec{k}, \omega)}{\epsilon_0 \omega}$$

$$\epsilon_r(\omega) = \epsilon_1(\omega) + j\epsilon_2(\omega)$$

$$n = \sqrt{\epsilon_r}$$

$$\tilde{n}(\omega) = n(\omega) + j\kappa(\omega)$$

$$\epsilon_1 = n^2 - \kappa^2; \epsilon_2 = 2n\kappa$$

Recall Beer's Plane Wave Propagating along +z direction:

$$\vec{E}(z, t) = \hat{a}_x E_0 e^{j(\omega t - kz)}$$

where $k = \frac{2\pi}{\lambda} \tilde{n}$

$$\vec{E}(z, t) = \hat{a}_x E_0 e^{j(\omega t - \beta z)} e^{-\alpha z}$$

$$P_{ave}(z) = \frac{1}{2\eta} |E_0|^2 e^{-2\alpha z}$$

Beer's Law:

$$I(z) = I_0 e^{-\alpha_0 z}$$

Handwritten notes on the slide include: $\alpha_0 = \frac{4\pi}{\lambda} \kappa$, $\alpha_0 = \frac{2\pi}{\lambda} \kappa$, and $\kappa = \frac{\epsilon_2}{2n}$.

Now, so metals I just pass it a little farther is a very interesting part. So, I am just considering that whatever sigma or epsilon r whatever you are getting this is the expression you can just say that it must be because j is there. So it has a real part imaginary part clearly is there. So, I can say that sigma can be sigma prime that can be considered like a real part imaginary part that real part will be again adding here.

So you can have a real part and imaginary part that is what and then next level I know that refractive index is square root of epsilon r that is what we discussed earlier, then we can say that here in this particular case because it is a complex we say that complex refractive index can have a real part of the refractive index and the imaginary part of the refractive index this imaginary part of refractive index sometimes it is called extinction coefficient.

We will discuss that a little later this that is a special name extinction coefficient and then you will see that this ϵ_2 if you just compare you know that square root of this one square root of this one that means, you are getting this one square root if you are getting this one, so you need to square root this one. So normally in complex numbers you will know that how to find out what is the square root if you are taking real part and imaginary part you can find out ϵ_1 you can have $n^2 - k^2$ ϵ_2 is $2nk$.

Just you square it then you just equate it if you just square it will be $n^2 - k^2 + j 2nk$. So if you square that is a dielectric constant that this means, ϵ_1 and this means ϵ_2 so that is ϵ_1 and ϵ_2 you are writing here. So what happens after that if you just compare this 2 equations you can try to find out only this one I think this one actually sufficient.

You know $n^2 - k^2$ $\epsilon_2 = 2nk$ you can try to find out what is the value of n . So n can be written simply just a little bit of algebraic step you can find out n ϵ_1 is equal to this one in terms of ϵ_1 ϵ_2 if you know the imaginary part and the real part of your dielectric constant then you can find what is the real part of your complex refractive index and what is the once you know that you can find out $k = \epsilon_2 / 2n$.

So that is $2nk$ whatever the n value is there you put down here then both n and k ϵ_2 they are actually can be derived once you know ϵ_1 and ϵ_2 the refractive index can be found out this standard equation you should keep in just from here you can just simple few steps and you can get that one. Now recall these plane wave propagating along z direction, I am just trying to explain that base law already we have you remember that we have just discussed we have discussed $I(z) = I_0 e^{-\alpha z}$ that is the Beers law.

I would try to see how it is it can be expressed by considering σ and this complex refractive index. So, this a plane wave and this k instead of normally $2\pi / \lambda$ I have to write in tilde, because you have a complex refractive index plane wave when it is propagating inside the material metal or semiconductor I have to write $2\pi / \lambda$ in tilde. So now if I just simply write there.

This can be modified as $a_x a_x E \not\sim E \not\sim j \omega t \beta z$ where and it would be $-\alpha z$ where $\beta = 2\pi / \lambda n$ this n and α because it is an imaginary part multiplied by j you can have $2\pi / \lambda \kappa$ you know β equal to basically $2\pi / \lambda n$ tilde $2\pi / \lambda n$ we are considering β here and α we are writing here, then I can just simply step forward we can get what is not so, difficult no rocket science here just a step forward.

Now we have also discussed the average energy flow pointing theorem pointing vector $E \times H$ and we know that the electrical and magnetic field can be orthogonal to each other if we just consider that thing then we do know that energy flow is this one $E \times H$ then it will be that I borrowed the equation from earlier discussion $1/2 \eta \int r^2 e^{-2\alpha z}$ and then what we get.

You see this one this one we can consider as $I \not\sim$ and 2α you can consider as $\alpha \not\sim$ and then we can consider intensity as it propagates the power or intensity, intensity is the power flow per unit area that is it intensity we just consider $I \not\sim e^{-\alpha z}$ that is actually your so called Beers law where this $\alpha \not\sim$ is the attenuation coefficient attenuation constant for the intensity of power and normal this α is the amplitude attenuation coefficient.

So $4\pi / \lambda \kappa$ this $4 / \lambda$ sometimes you can consider $2\pi / \lambda$ 2 times κ . So you can write $2\pi / \lambda c$ you can consider c you can consider $2\pi c / \lambda = \omega$, ω / c into κ . So sometimes it can be $\alpha \not\sim$ can be written at 2 times $\omega / c \kappa$ and α will be so called $\omega / c \kappa$. So intensity and amplitude they are just 2 times fine. So one thing is this should be n^2 I think this here you should see that n^2 expression should be corrected and n^2 and κ will be this one.

(Refer Slide Time: 51:21)

Fundamentals of Lightwaves: Metals & Semiconductors Slide#31

Lightwaves are Electromagnetic Waves Governed by Maxwell's Equations

Maxwell's Macroscopic Equations

$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$	$\nabla \times \vec{H} = \vec{J}_c + \frac{\partial \vec{D}}{\partial t}$	$\vec{\nabla} \cdot \vec{D} = \rho_v$	$\vec{\nabla} \cdot \vec{B} = 0$
Coupled Fields		$\vec{J}_c = \sigma \vec{E}$	$\vec{J}_d = \frac{\partial \vec{D}}{\partial t}$
$\vec{D} = \epsilon \vec{E} = \epsilon_0 \epsilon_r \vec{E}$		$\vec{B} = \mu \vec{H} = \mu_0 \mu_r \vec{H}$	

Regimes of Transverse and Longitudinal Waves

We have seen earlier that, it is sufficient to use either of ϵ_r or σ

$\epsilon_r(\omega) = \epsilon_1(\omega) + j\epsilon_2(\omega)$ OR $\sigma(\omega) = \sigma_1(\omega) + j\sigma_2(\omega)$

Considering the wave equation: $\nabla \times \nabla \times \vec{E} = -\mu_0 \frac{\partial}{\partial t} (\nabla \times \vec{H})$ $\vec{\nabla} \equiv -jk; \frac{\partial}{\partial t} \equiv j\omega$

$\Rightarrow \vec{\nabla}(\vec{\nabla} \cdot \vec{E}) - \nabla^2 \vec{E} = -\mu_0 \frac{\partial}{\partial t} \left[\epsilon_0 \frac{\partial(\epsilon_r \vec{E})}{\partial t} \right] \Rightarrow \vec{k}(\vec{k} \cdot \vec{E}) - k^2 \vec{E} = \frac{\omega^2}{c^2} \epsilon_r(\vec{k}, \omega) \vec{E}$

Transverse Waves (Electromagnetic Wave Propagation) $\vec{k} \cdot \vec{E} = 0 \Rightarrow k^2 = \frac{\omega^2}{c^2} \epsilon_r(\vec{k}, \omega)$

Longitudinal Waves (Collective Electron Oscillations) $\vec{k}(\vec{k} \cdot \vec{E}) = k^2 \vec{E} \Rightarrow \epsilon_r(\vec{k}, \omega) = 0$

This is signifying that longitudinal collective oscillations can only occur at frequencies corresponding to zeros of $\epsilon(\omega)$

Integrated Photonic Devices and Circuits - Lecture-47



Now regimes of transverse and longitudinal waves so transverse and longitudinal waves you know epsilon r I have written like complex real part imaginary part conductivity, you can also consider any one of them you can use that is fine and then next consider the wave equation you take wave equation this one karl both side then you can write mu 0 del del t karl h now, you know plane wave case null operator that is a del operator it is nothing but - j k.

So you are just considering e 0 e to the power j omega t - k dot r k dot or if you are considering the plane wave then null operator will be equal to - j k and del del t = j omega in this case particular case if you just consider that then what you get, you get like this and then this one will be better identity and the right side instead of karl of h you can say this one and sigma instead of sigma will be just writing epsilon r that is why I am just writing this thing one of them I am writing.

Because I do not need to write sigma additionally and then substituting this to this, this one will be k - j k you have to write - j k you have to write and k square. So if you just substitute there of this operator then you get left hand side this one and right hand side you will be getting this one you are taking mu 0 epsilon 0 = 1 over c square and del del t del 2 del t 2 will be there this will be actually mu 0 epsilon 0 epsilon r del 2 E / del t 2 and del 2 del t this will be equal to - omega square.

So if you just substitute there then you get this equation. So next thing what you get the wave equation transverse wave electromagnetic wave propagation if you just considering transverse wave that means, you know k and E propagation direction wave vector is

perpendicular to the E that means this will become 0. So in that case if this is become 0 that k square is equal to this k square is equal to $\omega^2 / c^2 \epsilon_r$ here it will be ϵ_r .

Now longitudinal waves; what is that longitudinal waves? So we know electromagnetic waves so called transverse wave but this is a special case when you can get a solution also. For example you consider the propagation vector and electric field H is parallel this type of solution can be considered also that type of situation mathematically you can consider this one can be suppose k is parallel to E in some situation.

What happens if this is the delay this one will become k square because that will not be 0 no more 0. So that means k square $E - k$ square E that is the left hand side. So this one and this one if you subtract that will be 0. So 0 left hand side and right hand side will be ϵ_r so looks like a little bit odd that 0 and ϵ_r so that means you can have ϵ_r must be equal to 0.

So in case of you are considering longitudinal waves that means your electric field is oscillating also in the propagation direction that time also you can think of certain solutions which are called which has to be this dielectric constant has to be 0 if you get a dielectric functions such that in certain points certain ω point k points the value is 0 that means that particular point will result into a longitudinal wave.

So that is why we conclude that this is signifying that longitudinal collective oscillations can only occur at frequencies corresponding to 0 of $\epsilon_r \omega$ interesting this collective oscillations occurs that means we are talking about conductivity electrons in most everything is there. So, I have if I have electromagnetic waves and that electromagnetic wave have had some frequency point where you get certain dielectric function and certain ω you are getting 0.

For that particular frequency what will happen you can see longitudinal wave normally collectively entire electron cloud you can assume that will be propagating and that is how we actually a new type of subjects topics came out that is also sometimes used in integrated photonics that is called plasmonics basically we will discuss that if time permits. So this is

all what we could discuss about the electromagnetic wave propagation inside metal or semiconductor.

And then what will be the transverse electromagnetic wave and what would be the dispersion equation this will be called as your dispersion equation because you know k wave vector how it is related but there is a chance that if it is metal and semiconductor longitudinal waves also can exist if the dielectric function had some solutions 0 at some frequencies.