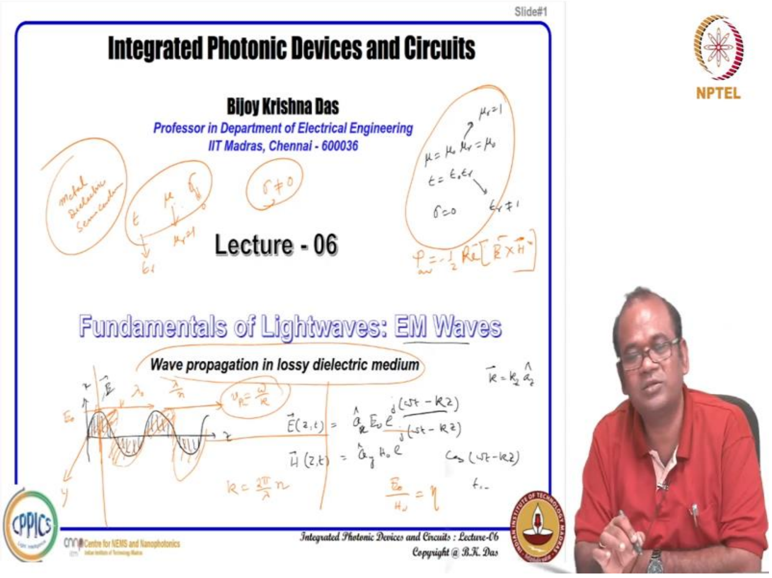
Integrated Photonics Devices and Circuits
Prof. Bijoy Krishna Das
Department of Electrical Engineering
Indian Institute of Technology - Madras

Lecture - 06
Fundamentals of Lightwaves: EM waves: Wave Propagation in Lossy Dielectric Medium

(Refer Slide Time: 00:17)



Hello everyone, we have already learnt how electromagnetic waves behaves in free space behaves means using Maxwell's equation we have seen that how one can derive a plane wave solution which can propagate freely in free space and homogeneous dielectric medium where sigma we consider 0 epsilon we consider epsilon 0 epsilon r and mu we consider mu 0 mu r = mu 0 nonmagnetic material where we consider mu r = 1 and epsilon r not equal to 1 mu r = 1 we consider.

So, in that case we have got a solution like say electric field say if it is propagating in a certain direction z, t we can consider then we can say that electric field can be propagating along x direction and it can have a constant amplitude and you can think of monochromatic wave if it is positive direction propagating k, z and magnetic field we can be present as a y and is something like this e to the power j omega t - kz it is positive direction propagating minus kz.

And if it is reverse direction minus z direction so this thing will be plus minus will become plus that is how we have solved and we have shown that k will be basically k is nothing but k vector we consider like that k z, a z direction if it is orthogonal direction and in this case if you just see that wave if it is just to take z direction wave which electric field if you just consider this is your x direction.

And then x direction you can see that it can probably like this sinusoidally because real part will be just cos omega t - kz. So, if you just take at any instant of time t = 0 you can see electric field will be like this, this will be your electric field will be oscillating like this and this will be oscillating like this and if you see that magnetic field if magnetic field if you are just considering say let me consider the another color something like this.

Then this is your y direction magnetic field then you can say that magnetic field will be oscillating something like this, this is in y, z plane magnetic field will be oscillating in y, z plane we are just showing in y, z plane basically this is this is going along y direction this is your magnetic field, magnetic field oscillates like this and you can say that this electric field in this particular case electric fields and magnetic field it is in phase.

When electric field is increasing magnetic field is also increasing and when magnetic field decreasing electric field is also decreasing. And if you try to see this one this peak to peak that is actually it will be lambda, lambda 0 if it is free space and if it is medium then lambda / n that is what we have seen that one important part is that the solution itself is showing sinusoidally and each time this value is actually weighing maximum to E 0.

That is one instant, next instant of time this space will be moving in this direction with the velocity that is called space velocity will be omega / k, k equal to as you will know that we have defined earlier k = 2pi / lambda. If it is material medium it will be 2pi / lambda over n. So, the phase velocity we can consider omega / n that means this phase will be traveling with a velocity like this traveling wave.

And interesting part is that the maxima as it travels it does not change E 0 always it is coming back to E 0 and by plus minus E 0 it will vary from plus minus E 0 magnetic field plus minus H 0 obviously E 0 and H 0 they are also related by eta that is what we have learned in the previous lecture. So that means if it is a homogeneous dielectric isotropic medium isotropic means that material property does not depend on direction.

And homogeneous means material property does not depend on space position. So, homogeneous isotropic medium you have seen that the magnetic field, electric field is propagating without any disturbance that will always come back and forth E 0 + E 0 - E 0 + H 0 - H 0 obviously you should keep in mind that E 0, H 0 always perpendicular orthogonal direction.

And this wave can carry energy that carrying energy we have defined that p = 1/2 average if you are just consider average you just consider a real part of E cross H star. So that is your average energy will be and that average energy if you see here at this point if you see whatever the average energy at a particular distance if you see that average energy flow will be constant.

So that means in free space and homogeneous medium according to the material property we have used here the electromagnetic wave propagates without any attenuation no loss, source can be somewhere and from the source electromagnetic wave is generating and that electromagnetic wave is propagating in free space or homogeneous medium that homogeneous dielectric medium without any loss the energy can flow.

And it can flow as long as you want as you can desired and as it can travel as a function of time. So that is what the idea now what we would like to discuss here now practically we do not get such thing of course free space is there but you do not handle with free space you do not use free space to design your device free space is free space you have to design or fabricate manufacture your device using some material.

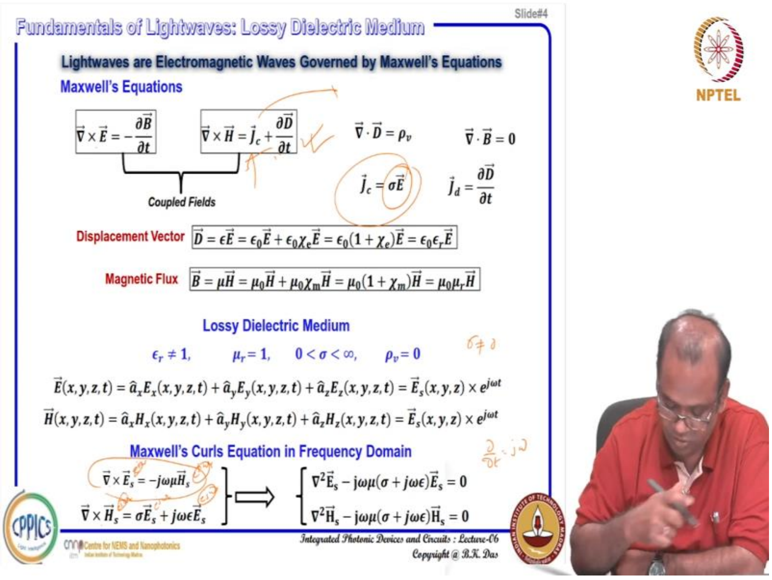
And that material as I said that those materials are can be categorized in terms of whether they are metal, whether they are dielectric completely dielectric and then they can be semiconductor also conductor, semiconductor so on. So, these 3 materials medium is there so in this 3 material medium you can actually define material properties epsilon, mu, sigma by choosing different set of value.

But or in other words you can say that metal has a certain value range of values for epsilon, mu and sigma dielectric will have obviously dielectric as we consider ideal dielectric sigma = 0 and semiconductor will see sigma equal to 0 or not. So, far we have treated that is constant sigma = 0 and considering mu r = 1 mu 0 is there epsilon 0 will be there and epsilon r it can be 1 or anything in that case no loss is there we have discussed.

So, now we will just consider a wave is propagating in a lossy dielectric medium dielectric where we have consider ideal dielectric where sigma = 0 now we are considering it is not really ideal dielectric but some sigma equal to there not equal to 0 is there in that case this is not ideal dielectric if presence of sigma can actually hampered the propagation of electromagnetic wave or not that is what we would like to see.

And it will show that since sigma not equal to 0 you will see that this electromagnetic wave will be attenuated if sigma not equal to 0 it will be attenuated as it propagates. So, there will be some kind of losses will be there. So that is why when I am just considering sigma not equal to 0 and all other parameter I am considered epsilon r some value is there mu r some values are there. They are actually dialectic we can consider a somewhat non ideal that is lossy which means I am just considering sigma not equal to 0.

(Refer Slide Time: 09:54)



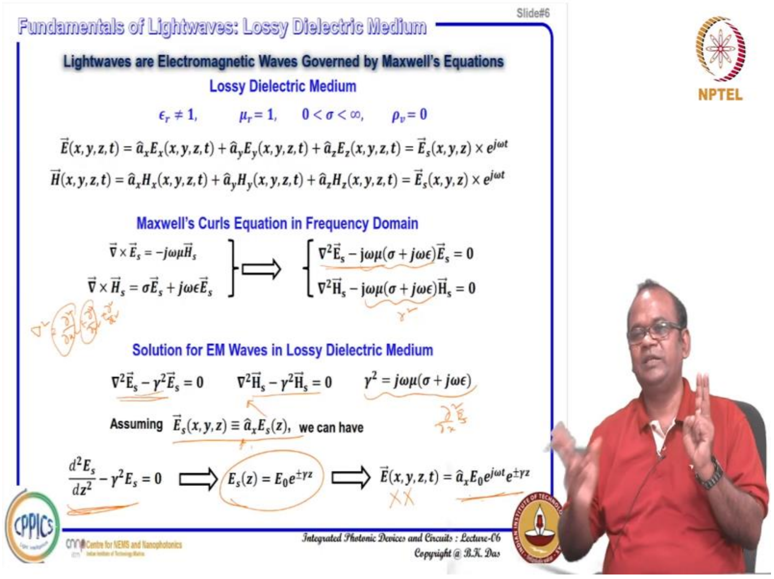
I just start with Maxwell's equation once again whatever we discussed in the previous thing I just repeated here in a material medium this is the displacement current, conduction current and this is the charge things are there everything is there. Again we will be considering this thing sigma we are considering I think we should say much, much lower than infinity sigma is ideal conductor normally we can consider sigma tends to infinity high conductivity.

Otherwise it is much, much less than that in practical material never becomes infinity but you can have sigma value close to 0 and sigma value close higher than much, much higher than 0. So, we are considering some values is there mu r again we are considering non magnetic material, dielectric non magnetic material epsilon r has some value and same way we can just consider the solutions are like that x, y, z dependent, space dependent and time dependent variations this type of solution we can find from the Maxwell's equation for dielectric losses.

So, if I just use sigma not equal to 0 then I have to put here J c = sigma E I have to insert here J d I have to put here then the Fourier domain or frequency domain Maxwell's equation can be written as like this because del del t we know that del del t = j omega, j omega we are putting curl E s, E s j omega mu and then this one will be there j omega can be written here and from this one we can write sigma E s will be there also and j omega epsilon E s will be there.

So, this one del del t and j omega epsilon s and this one equal to simply we are writing this thing this value and e to the power j omega t time you can write here e to the power j omega t here also e to the power j omega t they can cancel. Here also e to the power j omega t, here also e to the power j omega t here e to the power j omega t they can cancel. So I get only the frequency domain time dependent part can be canceled on the space dependent solution only thing is that sigma non 0 it brings this parameter now new parameter new atom we are getting there.

(Refer Slide Time: 12:15)



So, I just repeated here how it looks like this one this equation this 2 equation I can represent something like this by you take it curl both side and this one curl here both side then you get wave equation like this if you are taking curl this term we can write something like that curl of this one we can say that this one minus grad square E s and right hand side will be minus j omega mu curl of H s that can be written as this one equal to base value equal to 0 I have considered charge free.

So, I am considering del square E s = - j omega mu so I can say that that can be del square E s = -, - cancelled. So, I can say j omega mu sigma + j omega epsilon E s that gives me del square E s - j omega mu sigma + j omega epsilon E s = 0 vector sign you should not miss that is what I have written here this is the equation we get here. So, similarly we start from here we get this equation.

Now you see we are using in this one I am using as gamma square I just representing that one then I can express the electromagnetic wave equation this one like this gamma square E s similarly this one will be like this simply I am just representing this one as a gamma square. Now same way I can try to get a solution so if you just think in this particular case I have just for simplification purpose.

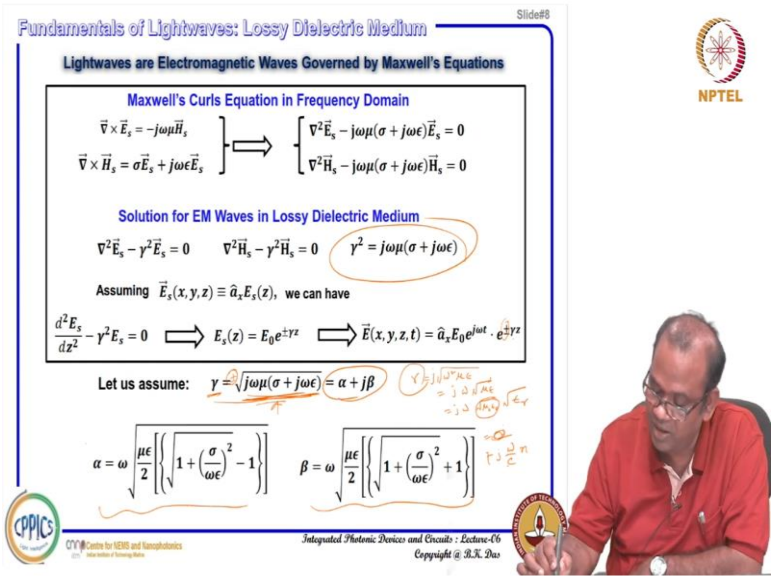
If I consider that the electric field only have x component, z component of the electric field is 0 just without any loss of generality you can consider that just for understanding purpose we are just considering electric field component has only alone x component. And also we can consider that, that is electric field earlier we have seen that it can be considered that it is x, y independent it is only propagating along z direction.

It can be varying I would say varying only z direction and x, y direction it is remained unchanged you can consider. So, if it is remained unchanged I can say that only electric field z dependent so these 2 assumptions we are considering for understanding purpose. So, if I just substitute this one here then I know that if I just try to get del x E s because this del square is nothing but del square del x 2 plus this one plus.

So, since the field is x independent, y independent so these 2 term will go so will end up with only z dependent terms and in that case I am just not using the vector sign also because both sides vector sign which is x direction so it is reduced to a scalar equation basically. So, for this we can have a solution very interesting solution like this just plus minus gamma z so you get a solution ultimately x dependent I am not considering.

I am not considering y dependent just z dependent and forcefully I consider that electric field is oscillating along the x direction. So, if it is oscillating along x direction there I have 2 choices it can propagate that wave can propagate in z direction or y direction. We will see in this type of situation what happens since we see that something exponential time coming like z like this.

(Refer Slide Time: 16:33)



Then I can say that this is just taken same thing whatever derived earlier up till here we just got this thing. So, we have this thing and then time dependent part I just put j omega t there also you have just written this thing time dependent part this one I will put. So, this is written and now we later but move here gamma I have seen that gamma square is equal to chosen like this gamma square is a complex.

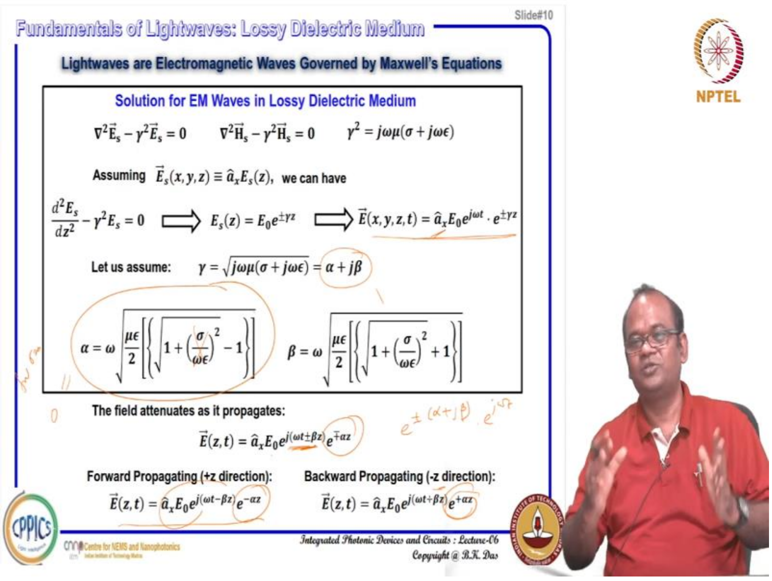
And gamma value square root you have to take since we have taken plus minus I have not put here plus minus sign I have not put here because all the solution itself is coming plus minus over gamma. So, here I am just putting gamma since it is a complex value here in terms of material property remember that if you are putting sigma = 0 then you see sigma = 0 if you put gamma value will be equal to square root of omega square mu epsilon and 2 to the power - 1 means we will be getting j.

So that means you get j omega times mu epsilon and mu epsilon means j omega mu 0 epsilon 0 and epsilon r and mu 0 epsilon 0 if you square root if you take that is nothing but it is minus j omega / c and epsilon r = n. So, this is basically c / n so gamma is appearing like a k value. So, once you put sigma = 0 it is a k like a homogeneous medium and free space similar type of solutions you will be getting.

So, in this case sigma not equal to 0 that is why we have to deal with it. Since it is a complex we are just considering the solution the square root if you take it will have the real part imaginary part alpha is the real part and beta the imaginary part. And if you just to do a little bit of algebra here you can try to find out alpha value if you just make a square root here just squaring both sides and if you do some trick you can find alpha value in this form.

And beta value you can try this thing few steps I just omitted here the straightforward any textbook in electromagnetic you can find that. So that means I see that gamma is you know is a complex value. And this gamma appears in your solution and it has a real part imaginary part like this.

(Refer Slide Time: 19:15)



The field attenuates as it propagates how is that? You see now I have electric field this one I am writing z dependent part E 0 consider x direction I am writing e to the power j omega t + - beta z - + alpha z. So, if I am just writing this one alpha + j beta I am just writing plus minus we are writing e to the power + - alpha + j beta into e to the power j omega t. So, you are writing e to the power j omega t, j beta you are writing.

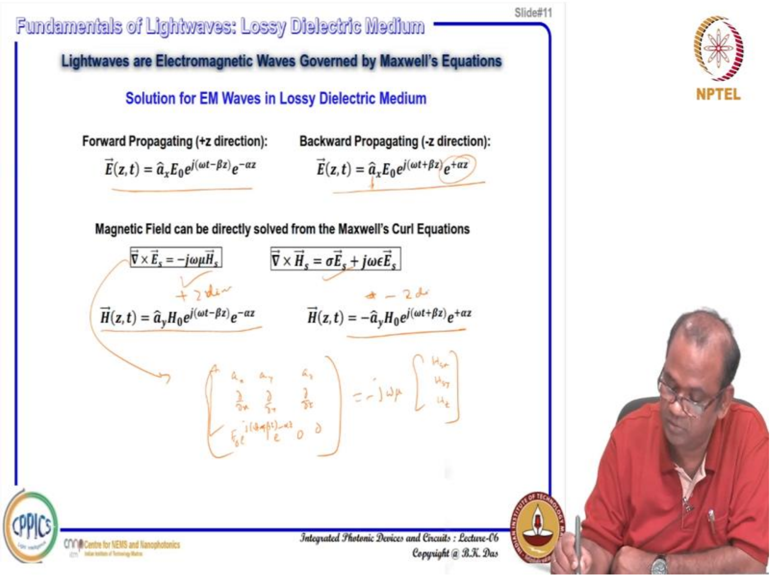
And alpha z part I am just putting it outside because this is just like a phase it will be a j omega t + - beta z and this part is somehow alpha z real value plus or minus. So, if you consider plus or minus you can always consider always you know that the space part if you are considering plus and minus, minus you are considering that means why it is propagating along forward direction positive z direction.

And negative it is like a wave propagating along if you are considering negative that means it is a negative forward direction positive backward direction. So, here it is written something like that negative means positive direction and backward propagating the minus z direction and minus z since minus z I can use here alpha z because z if it is minus that will attenuates and here I can write also z equal to positive direction positive attenuates.

It will oscillating but as it propagates it will be oscillating this is the difference. So, if sigma not equal to 0 you see that you are still getting a plane wave solution you can assume a plane wave solution but it is attenuating in nature as it propagates alpha is a positive value positive number that can be defined like this then you can get this on but this check here if sigma = 0 if you put this one will go.

That means 1 - 1 0 alpha will become 0 so for sigma = 0 alpha will be equal to 0 then in that case this will miss this will miss so it is a solution like plane wave things. So, once sigma is there you can say that the wave starting from Maxwell's equation I can say that the wave attenuates as it propagates.

(Refer Slide Time: 21:53)

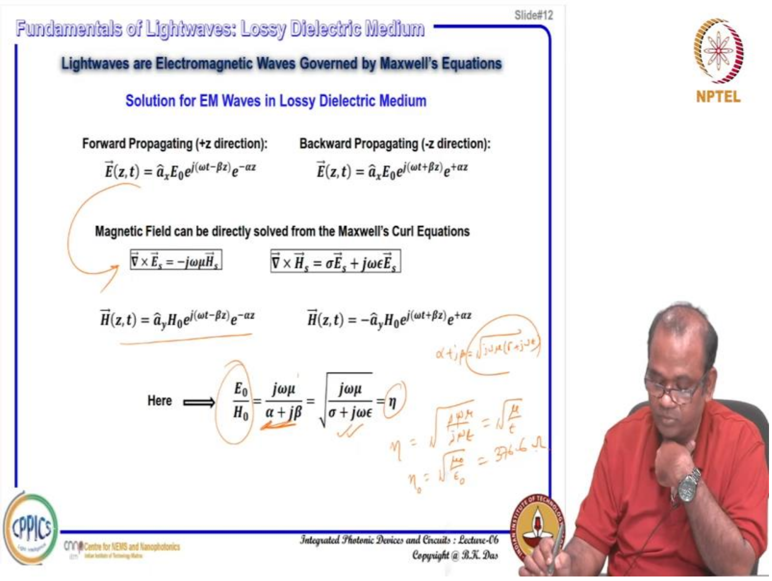


Now so I have just written down forward direction reverse direction. So, for our propagating wave here reverse propagating wave here and plus alpha whatever this is because negative direction that is why it is a lossy thing I write plus should not be confused that it will be amplifying. Now magnetic field can be directly solved from the Maxwell's curls equation once I began the electric field I can find magnetic field here.

This is the x direction I can solve this one for example curl E s if you just see you can just something like that a x, a y, a z dou - dou x, dou - dou y, dou - dou z and then I have magnetic electric field I have only x direction I have E 0 e to the power j omega t + beta z e to the power - alpha z this is the thing 0, 0 .So, this is the thing and the right hand side I can put minus j omega mu and I can write H sx maybe you can consider H sy, H sz this is the 3 components.

So, if you solve that I can find the magnetic field will have in the y direction. If electric religion the x direction magnetic field can be considered y direction this is a positive z direction and this is a negative z direction.

(Refer Slide Time: 23:36)

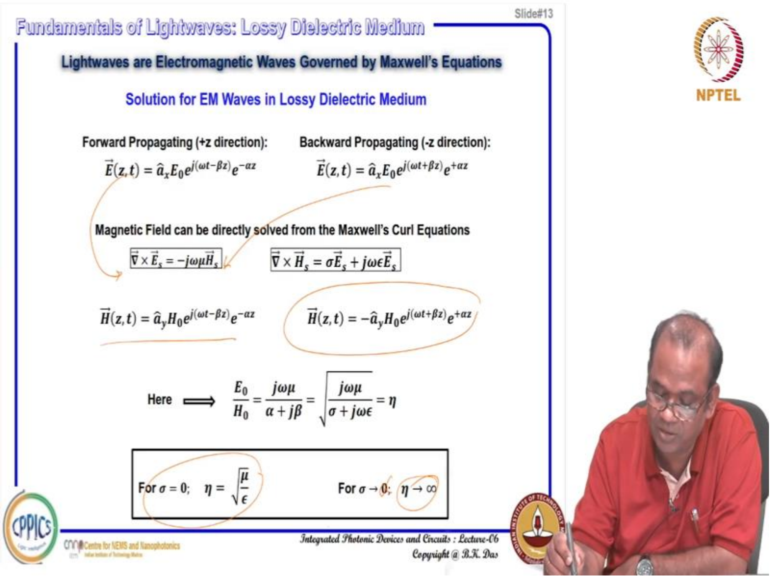


Now if you see from here I have the from this equation also if you solve one more thing you will be getting similar to earlier E 0 / H 0 when it is lossy medium sigma = 0 that will be coming like this you just solve this one substitute here after substituting this one here for example then you get magnetic field solution at the same time if you just take a ratio it will be E 0 / H 0.

And if you know that ratio if you are taking that is also since sigma not equal to 0 alpha will be there. And if you just put alpha + j beta to explanation we know that alpha + j beta is nothing but we have j omega mu sigma + j omega epsilon. So, instead of that if I just put this one then ultimately we are getting these equations and that is actually we say that E 0 / H 0 there is a characteristics impedance intrinsic impedance for the material medium.

So, now it is sigma value is there so eta is there if I just put sigma = 0 then eta will be you can consider j omega mu / j omega epsilon j, j cancel omega, omega cancel that is actually mu / epsilon that is simple. That means dielectric lossless dielectric medium sigma = 0 if it is a free space then eta 0 will be just simply mu 0 / epsilon 0 mu 0 is Henry per meter 4pi into 10 to the power - 7 then you get 376, 377 something like that this is ohm. This is all so that is what we know.

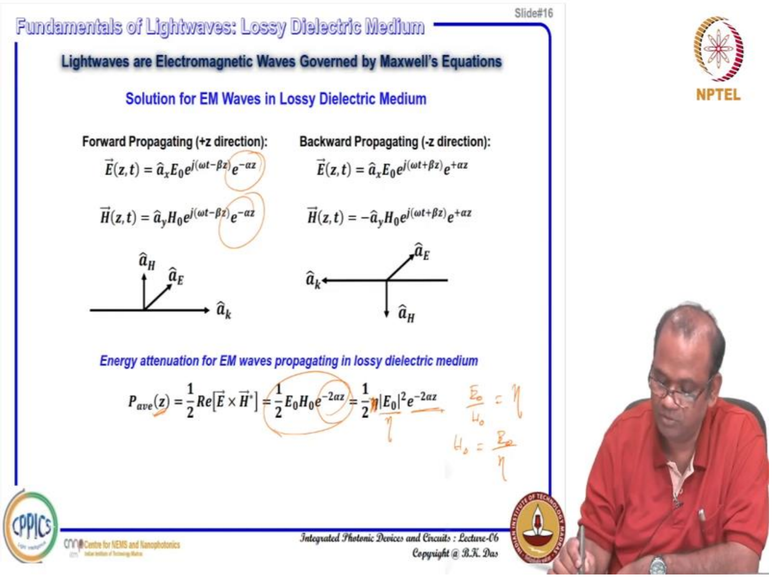
(Refer Slide Time: 25:16)



So, now we have electric field and we have magnetic field forward propagating, reverse propagating and one important thing you should be noticing that in the reverse direction when you solve whenever you are solving this thing using this one you are substituting this electric field here directly and solving then your magnetic field will be you are getting within minus sign and if you are solving this one here you are getting magnetic field with a positive sign.

That means whenever you are getting electric field with a forward propagating wave then your magnetic wave will be in phase together electrical and that is the only positive x direction and this also oscillating in the positive y direction but whenever it is you are considering reverse direction propagation that is beta is considered instead of beta - beta you are considering that is why plus beta j then minus sign will be there. So that thing you should be keeping in mind.

(Refer Slide Time: 26:19)



That is what I have just tried to say. So that means if your wave is propagating in this direction then this is your positive x axis that is electric that is actually x axis for example if you are considering a E this is E field that is like this and this is magnetic field in a positive wave direction this is actually x axis this is y axis this is z axis for example along this one you have magnetic field.

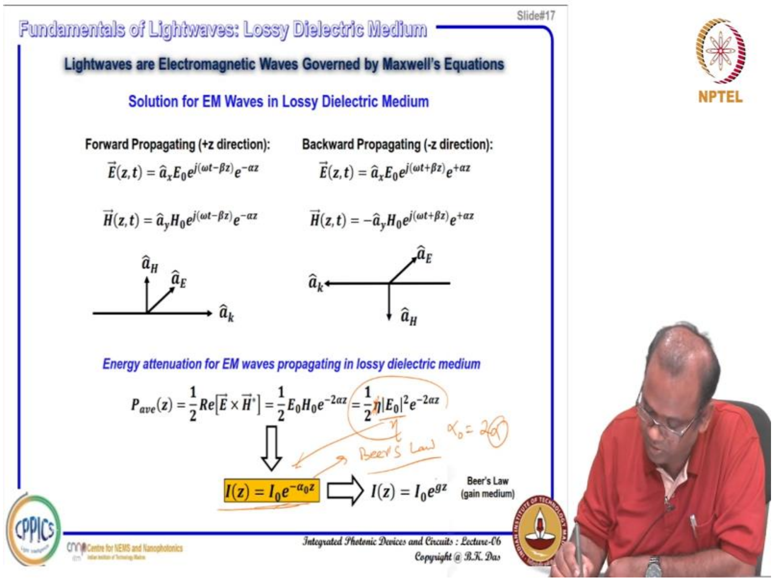
So, wave it is propagating like this so electric field will be oscillating like this and magnetic field will be oscillating like this so on in the x direction. So that is how it will be propagating at instant of time you can find out what is the electric field phase is known you know what is the magnetic field, which position and what is the value? But if it is reverse direction propagating in this direction you see electric field you can consider x direction.

But magnetic field you are showing like this it is reverse at an instant of time same instantly if you are showing that that electric field propagating negative direction magnetic field direction has to be changed this is very important is sometimes you know how it is defined for example right hand rule you can consider if this is a electric field all the fingers, 4 fingers I am showing that the electric field and magnetic field x direction magnetic field is in this direction then it is propagating this direction.

If this is electric field and magnetic field this direction downward then it will be propagating in this direction. So that is how it is I obtained to represent here and again if you just try to find out energy flow same way p = E cross z because it is z direction propagation I have just written z then you find interesting formula like this E cross H because in E also you have minus alpha z H also you have minus alpha z if you multiply then you are getting minus 2 alpha z.

And you know that E 0 / H 0 = eta so, H 0 I can be represent like H 0 = E 0 / eta E 0 / eta. So, this is actually a mistake I would suggest that this should not be there. So, it will be E 0 / eta 1/2 E 0 square / eta e to the power -2 alpha z. So, this is a type error I am sorry for that so this means that energy as it propagates it see some kind of attenuation. Electric fields and magnetic field also attenuates with alpha value because of the sigma so energy also you will be seeing that as it propagates energy will be dropping.

(Refer Slide Time: 29:22)



So, then this thing normally we can represent like this, this is again this would be eta E 0 was fine and then this one can be represented as I z I 0 e to the power alpha0z. So that means it is intensity so this can be written as amplitude square is basically power flow per unit time per unit area and if it is per unit time sometimes it is energy flow per unit area it is sometimes called intensity.

So, I write I z = I 0 e to the power alpha 0 where alpha 0 = 2 alpha basically that means this alpha is the amplitude attenuation and alpha 0 is the intensity attenuation and we can represent like this and this is basically known as Beer's law, Beer he derived first much before the Maxwell's equations. So, by experimental observations that if it is lossy medium the intensity how it is dropping it is calling exponential law that is known as Beer’s law.

And this is now we can show that from the Maxwell’s equations you can see that but one important part you should be keeping in mind that this alpha 0 sometimes it is attenuating lossy medium but you can have a gain medium also. So, the light signal electromagnetic wave as it propagates it can amplify. So that amplification also can be represented by Beer’s law instead of alpha 0 you can put a gz where g is the gain coefficient.

So that means this alpha we have represented in terms of if you see previously alpha expression sigma this is alpha expression this is your sigma, alpha, epsilon, mu everything is there that is alpha that is why you are getting a negative alpha as it propagates it attenuates but if you have a gain medium so how to represent gain medium and trade the Maxwell's equations then you have to modify this epsilon mu sigma such that.

It will be giving some gain that means gain medium mathematically you can represent again in terms of omega mu epsilon by suitable change in dielectric constant suitable change in conductivity suitable change in magnetic all these things can be done. So that means again I am emphasizing that Maxwell's equations is macroscopic Maxwell's equation whatever we have discussed that can be useful very easily for any structures any material medium.

And you can have lossy medium you can have a lossless medium you can have a gain medium still you can actually utilize whatever the solutions we are getting from here. Now we will see that since sigma non 0 we have explained now we will be trying to understand that just not lossy medium if it is just a metal practical metal we are using just dielectric with some loss medium is not sufficient towards ideal metal, ideal conductor, ideal semiconductor then what would happen?

Because these things as I mentioned earlier again and again that any photonic integrated circuits all these material is very important to realize different types of devices. So, electromagnetic wave, light wave how interacts with those material medium you need to know.