

Integrated Photonics Devices and Circuits
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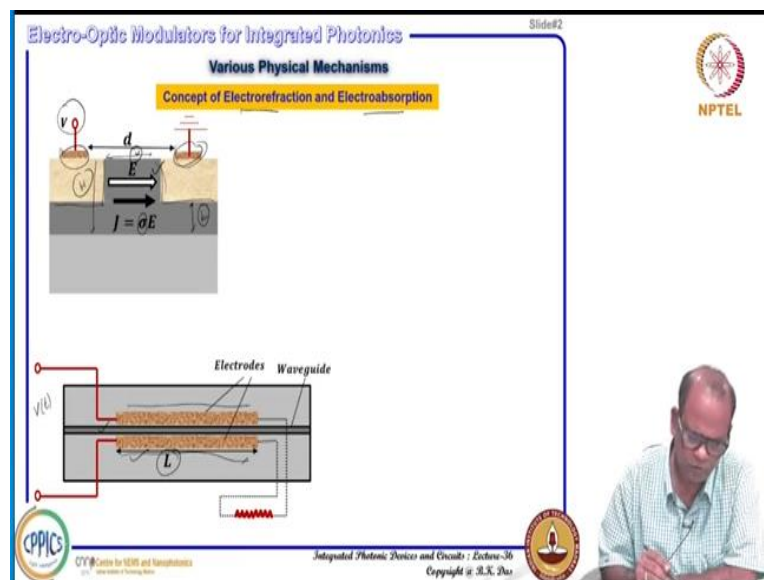
Lecture - 36

Electro-optic Modulators for Integrated Photonics: Various Physical Mechanisms

Hi everyone, today's lecture we are going to discuss various physical mechanisms specially, for integrated photonics purpose electro-optic modulators. So, first I will be discussing about 2 important phenomenon's that is called Electrorefraction and Electroabsorption I will give some conceptual idea what that means. And then I will be discussing bit of details on 2 mechanisms called Pockels Effect and Kerr Effect that is actually nothing but what I will I mentioned before that electrorefraction.

So, 2 electrorefraction methods one is called Pockels effect another is called Kerr effect and then I will be discussing about electroabsorption that is actually the effect is called Franz Keldysh Effect FKE we just mentioned and then finally, and most important one is the Free Carrier Concentration Effect, we just abbreviate it at FCCE, this method, this free carrier concentration if it can cause both electrorefraction as well as electroabsorption.

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So, let us try to get an idea about the electrorefraction and electroabsorption, so as I representative sketch, cross sectional scheme pay silicon waveguide for example, is shown here where this is actually the waveguide structure you can say this one as usual, we

discussed all of it and we know that this is actually your W . And this is your, this slab thickness is h that can be 0 as well, and this device layer thickness is H .

You can just use those parameters W , H and h to design a single mode wave guide structured that need not be silicon material, any material it can be silicon nitride any other material also. And you see, we have shown 2 electrodes across switch, we can give a potential that can be a DC that can be alternating signal or data. So, depending on the material here, in the core we can create electric field across the waveguide cross section E and if σ conductivity is present significantly.

Then also we can see some kind of current density, this is a cross sectional view and for a typical modulator phase modulator I would say it is actually designed with a pair of electrode structure as I mentioned here for which top view is shown here, it can be of length L , and you can actually apply some potential here potential difference or that it can be your data signal that can be a function of t . And then this depending on the length of the electrode structure if it is a very small one, you can just directly give your data.

And according to the data here refractive index will be changing and then refractive index change means propagating mode wave optical mode will see some kind of phase change. And you can use that phase change to detect either in the form of synthesizer detection, coherent detection or you can use that phase modulated signal to interfere another signal to get some kind of amplitude and intensity modulation that can be done either using your Mach-Zehnder interferometer or microring resonator discussed before. So, this is the overview.

Now, I am going to discuss about how this electro refraction and electroabsorption and can cause optical signal modulation. So, using electrorefraction I will show that actually you can change phase you can modulate phase as well as amplitude also. And using electroabsorption you can only attenuate or modulate your optical signal or amplitude.

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Slide#3

Electro-Optic Modulators for Integrated Photonics

Various Physical Mechanisms

Concept of Electrorefraction and Electroabsorption

$n = n' + iK'$

$n_{eff} \rightarrow n_{eff} + \Delta n(V) \Rightarrow \text{Electrorefraction}$
(PKE, KE and FCCF)

Devices can be designed for both Phase and Amplitude Modulation

$\Delta\Phi(V) = \frac{2\pi}{\lambda} \Delta n(V)L$

$\phi = \beta L = \frac{2\pi}{\lambda} n L$

$\Delta\beta = \frac{2\pi}{\lambda} \Delta n(V) L$

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Let us see what it is? Normally you know refractive index of any material of this waveguide material, core material I can represent in terms of real number as well as some imaginary value. Refractive index is equal to $n + iK$. If there is the attenuation this kappa or K whatever I say that is actually called extinction coefficient and n is the actual real part of the so called refractive index that is actually causing some kind of guided mode as well as your modulation if any effect causing effective index variation as your function of voltage.

For example, here voltage it is given it can be current, it can be anything we will see in course of time, then if refractive index changed as a function of input signal, voltage signal or current signal then it is called electrorefraction that means, you are actually applying electric field or electric current and then you are changing your refractive index that is why it is called electrorefraction.

And that type of phenomena can actually you can create through different type of effect one is called Pockels effect and then Kerr effect and then free carrier concentration effect because, depending on the material, some material you can see strong Pockels effect you can explore that. Some material waveguide material actually Kerr effect is dominating. So, you can use the Kerr effect to change the refractive index by applying your electric field or voltage and then free carrier concentration that means you know in the waveguide core region.

If you have certain kinds of free carriers it can be electrons, can be a hole and if you can change that carrier concentration that not only contributes to loss to the waveguide, but also it

can change the real part of the refractive index. So, real part of the refractive index change actually causing electrorefraction and then devices can be designed for both phase and amplitude modulation that is what I explained in the previous slide.

So, how it is done, you see, normally $\phi = \beta L$ you know L is the length of the device, β is the propagation constant normally we can write $2\pi / \lambda n$ times L . Now, this n is variable you can control either by means of Pockels effect or Kerr effect or free carrier concentration effect. So, for that if you change n that means, you are going to change the phase. So, if you have $\Delta\phi$ then you can have $2\pi / \lambda$ assumed that this is wave operating wavelength so called operating wavelength that is not changed.

And then only this one is changed and length is not changed for the device. So that is what it is mentioned here. So, in this way you can actually modulate your phase as you know you can modulate phase, you can have Mach Zehnder interferometer and you can create intensity modulation as well.

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The slide, titled "Electro-Optic Modulators for Integrated Photonics", is divided into two main sections: "Concept of Electrorefraction and Electroabsorption" and "Various Physical Mechanisms".

Concept of Electrorefraction and Electroabsorption: This section includes a diagram of a waveguide with electrodes. The top diagram shows a cross-section with an electric field E and current density $J = \sigma E$. The bottom diagram shows a top-down view of a waveguide of length L with electrodes on either side. Handwritten notes include $\phi = \beta L$, $\beta = \frac{2\pi}{\lambda n}$, and $\Delta\phi = \frac{2\pi}{\lambda} \Delta n L$.

Various Physical Mechanisms: This section lists the following:

- Refractive index: $n = n' + ik''$
- Electrorefraction: $n_{eff} \rightarrow n_{eff} + \Delta n(V) \Rightarrow$ Electrorefraction (PE, KE and FCCE)
- Devices can be designed for both Phase and Amplitude Modulation
- Phase shift: $\Delta\phi(V) = \frac{2\pi}{\lambda} \Delta n(V)L$
- Electroabsorption: $\kappa \rightarrow \kappa + \Delta\kappa(V) \Rightarrow$ Electroabsorption (FKE, QCSF and FCCE)
- Devices can be designed only for Amplitude or Intensity Modulation
- Attenuation: $\Delta\alpha(V) = \frac{4\pi}{\lambda} \Delta\kappa(V)$
- Intensity: $I(V) = I(0)e^{-\Delta\alpha(V)L}$

The slide also features the NPTEL logo and a small inset image of a person in the bottom right corner.

So, next thing is that electroabsorption, if any phenomenon any effect cause some change in extinction coefficient κ this one that means imaginary part of the refractive index, that can be changed κ to $\kappa + \Delta\kappa$ cell. And that can be function voltage function of current representative voltage it is shown here. So, in that case what you know, the extinction coefficient change means, you can actually change the attenuation or absorption or losses in the waveguide. So that particular phenomenon is called electroabsorption.

So, you are applying electric field then you can actually control the absorption of the guided light or loss of the in the waveguide. So that it is called electroabsorption. Again, this effect is possible this thing electroabsorption is possible in certain material which actually supports the effect called Franz keldysh effect. And this with a modified geometry, you can use also Franz keldysh effect by using some kind of quantum effects, then it is called quantum confined stark effect.

So that is the newest type of phenomenon people are exploiting forgetting a put proper design electroabsorption modulator. So, and then another method as you know the; free carrier not only changed the real part, it can also change the imaginary part. So, this free carrier concentration effect you can modulate the free carrier in the waveguide region, then not only you can change your real part, you can also change the imaginary part of the refractive index.

Once you can change the imaginary part of the refractive index you can think of electroabsorption modulator as well. So that means, FCCE actually contributes both electrorefraction as well as electroabsorption. So, using this electroabsorption mechanisms that can cause by means of Franz keldysh effect quantum confined stark effect or free carrier concentration effect, you can design amplitude or intense intensity modulation that I mentioned before that is kappa modulation normally you know that kappa means your $n + iK$.

So, if you just consider this $\beta = 2\pi / \lambda$ times this one, so in that case, you can have $2\pi / \lambda$ say n may be n effective that is actually causing your real phase and here another part it is coming $i 2\pi / \lambda$ times kappa. So, this part actually, because of the imaginary part you will know e to the power $j\beta L$ normally you write. So, if you are writing also j here so then it can happen that if you are multiplying in terms of β , β if you are replacing these plus these.

Then you can have some kind of losses that is called αL you can have. So that α if you have differential change in the extinction coefficient. Then you can have $\Delta\alpha = 4\pi / \lambda \Delta K$ and ΔK kappa is the function of voltage by 4 normally whenever we write e to the power $j\beta L$ that is a phase of the electric field, but whenever you are talking about intensity or power that means, you need to go for if it is amplitude is A A^* then you will be getting your and power.

So that is why we are just multiplying ϵ to the power $-\alpha L$ power $1/A$ to the power A star that is another αL . So, it will be $2\alpha L$ per power. So that is why there are 2 comes 2 multiplied here that will be $4\pi / \lambda n$ times ΔK . So, normally you know that the in that case voltage dependent attenuation you can get depending on the $\Delta\alpha$ value how much you are getting per unit length, extinction and attenuation and then length each L . So, you can find whatever the length.

So, accordingly depending on the voltage you can actually find what is the voltage dependent intensity or the output. So, in this way you can get intensity modulation or amplitude modulation so called power modulation.

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The slide, titled "Electro-Optic Modulators for Integrated Photonics", is labeled "Slide#5" in the top right corner. It features a schematic of a waveguide with two electrodes on top, separated by a distance d , with a voltage V applied. An electric field E is shown pointing to the right. A yellow box highlights "Electrorefraction: Pockels Effect and Kerr Effect". Another yellow box contains the equation for permittivity: $\epsilon = \epsilon_0 \left(1 + \frac{1}{\epsilon_0} \frac{1}{n^2} \right)$. To the right, there are handwritten equations: $\epsilon = \epsilon_0 \epsilon_r$, $\Delta n = \frac{1}{2} n^3 r E$, and $\Delta n = \frac{1}{2} n^5 K E^2$. The NPTEL logo is in the top right, and the CPPIC logo is in the bottom left. The bottom of the slide contains the text "Integrated Photonics Devices and Circuits - Lecture-36" and "Copyright © 2015, Prof. J. S. Rao". A small inset image of a man is visible in the bottom right corner of the slide frame.

So, now another we will start, so we know now what is meant by electroabsorption and electrorefraction both can be exploited for modulation purpose modulation of optical signal in a waveguide for integrated photonic applications. Now, let us go a little bit into details, how electrorefraction effect can cause certain kind of refractive index change a little bit of physics, what is the mechanisms? What is Pockels effect and what is Kerr effect that we would like to know first.

So, if we know what is Pockels effect and what is Kerr effect then we can also identify material in which we can actually get Pockels effect or Kerr effect depending on that we can say it is possible that for example, if we try to see that silicon waveguide for example, if silicon waveguide can have Pockels effect and Kerr effect then we can easily design a modulator using CMOS photonic integrated circuit technology so that is the issue. So, we

have to see the materials material should have certain kinds of effects. So, what is Pockels effect?

It is an electro-optical effect that means you will be applying some electric field you know that then you can have certain kind of refractive index change in a waveguide that is pursued. So, to see what certain material is required for Pockels effect and what certain material is required for Kerr effect or it can find the material also both can be also present so put that towards that purpose, let us try to see the actual physics.

So, you know, we have defined permittivity you know permittivity is nothing but epsilon we know that epsilon 0, epsilon r that is actually permittivity where epsilon 0 is the permittivity of the free space and epsilon r is the dielectric constant of the material. Now, we define permittivity eta which is nothing but epsilon 0 / epsilon. So, you have just writing the reciprocal of that one epsilon but you are multiplying with epsilon 0.

Then we represent that epsilon r and epsilon r is nothing but 1 / n square is the real part of the refractive index. So that means the inverse of dielectric constant we define as an impermittivity. So, eta is written like that.

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The slide, titled "Electro-Optic Modulators for Integrated Photonics", illustrates the physical mechanisms of electrorefraction, specifically the Pockels and Kerr effects. It features a schematic of a waveguide with electrodes separated by distance d and voltage V , with an electric field E applied. The slide includes the following content:

- Impermittivity:** $\eta = \frac{\epsilon_0}{\epsilon} = \frac{1}{n^2}$
- Tensor Representation for a small electric field \vec{E} :**

$$\eta_{ij}(E) = \eta_{ij} + \sum_{k,l} r_{ijkl} E_k + \sum_{k,l,m} s_{ijklm} E_k E_l$$
- Change in impermittivity:** $\Delta\eta \cong rE + sE^2$
- Derivatives:** $r_{ijk} = \frac{\partial \eta_{ij}}{\partial E_k}$, $s_{ijkl} = \frac{\partial^2 \eta_{ij}}{\partial E_k \partial E_l}$
- Indices:** where $i, j, k, l \rightarrow 1, 2, 3$

The slide also includes logos for NPTEL, CIPICs, and the Center for MEMS and Nanophotonics, along with the text "Integrated Photonic Devices and Circuits: Lecture-36" and "Copyright © 2013, 2015".

Now, what you can get this permittivity can be represented in tensor form for a small electric field in the material, certain material crystalline material you can think of if it is crystalline material you know normally it can be anisotropic it can be along the x direction it has certain property, along y direction it can have certain properties, z direction it can have certain

properties, you know the depending on the material property, we actually mentioned earlier that this epsilon r can be constant for isotropic homogeneous medium.

But it can be a tensor in case of anisotropic material that means, different direction it will see different type of dielectric constant, So, it can be represented as a tensor, so if epsilon r can be represented as a tensor so it can have eta which is related to epsilon r that can be also expressed as a tensor. So now, this eta which is related to epsilon r or refractive index, if that is actually changed by means of electric field.

In tensor form, we can write this equation that means, if you can think that if there is no field whatever the permittivity is a tensor that is ϵ_{ij} . Now, if you just apply a small electric field, you can actually express this ϵ_{ij} as a function of E in terms of Taylor series expansion. So you know, here we have just used ij what is those ij actual i, j, k, l we have used all of them in dices 1, 2, 3 that means, it represent 1 means x direction, 2 maybe y direction, 3 maybe z direction in Cartesian coordinate system.

That means, I can have x direction, can have y direction, can have z direction j also can have x direction y direction z direction k and l so on. So now, if we just represent that in a Taylor series expansion ϵ_{ij} so we introduce 1 coefficient. So that is actually also can be a tensor where that can be written as sensitive ϵ_{ij} we have written, r_{ij} we have introduced 1 more index that is k that can also run from 1 to 3 x direction, y direction and z direction.

That means, if your electric field is along x direction, along y direction, along z direction or it has all the 3 components, then all the 3 components will contribute some effect to eta. So, you can sum over k , k runs from 1 to 3 that is how it is r_{ij} whatever the r_{ij} value and then you can have all the k direction you can sum up if it is only one direction is there suppose k equal to say only 3 or x direction, then you can have only x, so you will be writing $r_{ij} E_1$ that can be written. If we have only y direction is there then I will be writing ij to E_y or E_2 .

If it is only z direction then it will be $r_{ij} E_3$ so 3 numbers will be there so here will be 3, so if all of them are there, then we have to write like this. So that is how if there should be some values of r_{ij} and then different direction if you are just using electric field applying electric field, then you can have contribution for the permittivity tensor. Similarly, next higher

order Taylor series we have just taken first 2 terms of the Taylor series, this is the main value and then Taylor series this one.

This is Taylor series expansion you know that differential term is there, differential variation is there for example, r_{ijk} is defined like that $\frac{\partial \epsilon_{ij}}{\partial E_k}$ that means, along k th direction what is the electric field variations and for that how much ϵ_{ij} is changing. So, that thing is that differential coefficient. Normally you know, normally anything we write along around electric field we can write something like that.

Then as a function of suppose electric field then we can write $\frac{\partial n}{\partial E}$ then we can see that how much field we are total we are applying that direction and then we can write $\frac{\partial^2 n}{\partial E^2}$, n can be ϵ_{ij} here and we can write $\frac{\partial^2 \epsilon_{ij}}{\partial E^2}$ and then you can have this number and multiplication if this is a ΔE this will be say ΔE^2 and so on this is the Taylor series. So, this thing since ϵ_{ij} is a tensor that is why we have some kind of tensor element r_{ijk} we have written and that can be represented here.

So, and then you are multiplying your additional field whatever $\frac{\partial n}{\partial E}$ means $E \cdot K$ your seen suppose, it is a second order Taylor series just a second term we are using that means, this $\frac{\partial^2 n}{\partial E^2}$ we can have $\frac{\partial^2 \epsilon_{ij}}{\partial E^2}$ because electric field can have can come from to this ΔE^2 then first the electric field can come from one electro particular field and another D can be coming from other fields.

So, in this way we can write that it is just it should be second order derivative this one and that can be written as $\frac{\partial^2 \epsilon_{ij}}{\partial E^2}$, $\frac{\partial^2 \epsilon_{ij}}{\partial E^2}$ and $\frac{\partial^2 \epsilon_{ij}}{\partial E^2}$ I have just introduced additional l so kl now 2 terms k can also run from 1, 2, 3, l also can run from 1, 2, 3 and so on. Now, this one we can simplify that suppose, you are just interested in a certain direction of electric field for example, you can simplify that one hugely.

For example only E_1 I am considering then only certain direction and then you multiply whatever value you are getting that is r , so this thing r coefficient I can write r times E and then this one I can write s times E^2 simplify it in a homogeneous isotropic medium certain things are happening then we can write this. This r and s again obviously, these are corresponding to first coefficient for the Taylor series and this one is s the second coefficients of the Taylor series fine.

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Now you see we do a little bit of maths here suppose, we say that this eta variation because of the electric field causing must be refractive index change. So that means when eta impermittivity is a function of electric field that means, refractive index is also function of electric field. So, let us say that by means of applied electric field you have a delta n amount of refractive index since happened in the core here.

So that means, I can say that what is the final refractive index because of the electric field and what was the original when $E = 0$ for example, that is your change in refractive index. Now, this part we can write like this. So, $\Delta n / \Delta \eta$ multiplied by $\Delta \eta$ suppose, n is changing I just want to see the rate of change of refractive index because of the eta change, how much eta change, how much contribution to the refractive index? Then I have $\Delta \eta$ amount of change can happen, this one Δn amount of change can happen.

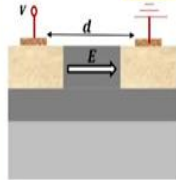
So, I write $\Delta n / \Delta \eta$ that is the slope multiplied by $\Delta \eta$ that is the thing we right hand side I keep as it is. Now this one I write $\Delta \eta / \Delta n$ because why $\Delta \eta / \Delta n$? I am writing inverse of that one because I have $\eta = 1/n^2$ is given $1/n^2$. If you eta is like this $\eta = 1/n^2$. So, $\Delta \eta / \Delta n$ equal to say that means I can write $\Delta \eta / \Delta n = -2/n^3$ so that means $-2/n^3$ to the power -3 . So, $\Delta \eta / \Delta n = -1/2n^3$ that should be equal to $-1/2n^3$ this one and cube that is what you have written. And then $\Delta \eta$ directly placed and then right hand side as usual just transfer here.

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Electro-Optic Modulators for Integrated Photonics Slide#9

Various Physical Mechanisms

Electrorefraction: Pockels Effect and Kerr Effect



Impermittivity $\Rightarrow \eta = \frac{\epsilon_0}{\epsilon} = \frac{1}{\epsilon_r} = \frac{1}{n^2}$

Tensor Representation for a small electric field \vec{E}

$\eta_{ij}(E) = \eta_{ij} + \sum_k r_{ijk} E_k + \sum_{kl} s_{ijkl} E_k E_l$ $\Delta\eta \cong rE + sE^2$

where $i, j, k, l = 1, 2, 3$ $r_{ijk} = \frac{\partial \eta_{ij}}{\partial E_k}$; $s_{ijkl} = \frac{\partial^2 \eta_{ij}}{\partial E_k \partial E_l}$

$n(E) = n + \frac{1}{2} r n^3 E - \frac{1}{2} s n^5 E^2$

$\gamma(k) \propto \frac{1}{2} r n^3 E$

Pockels Effect




$\Delta n(E) = n(E) - n$

$\Rightarrow \frac{\partial n}{\partial \eta} \Delta \eta = n(E) - n$

$\Rightarrow \left(\frac{\partial \eta}{\partial n}\right)^{-1} \Delta \eta = n(E) - n$

$\Rightarrow -\frac{1}{2} n^3 \cdot \Delta \eta = n(E) - n$

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Next what do we write? From here, we just try to write like this we are writing $n(E)$ meaning $\Delta\eta$ will be just replacing from here. So, I just substitute $\Delta\eta$ here and then we simplify refractive index as a function of electric field. So, I get this equation, so this is nothing but original refractive index, and because of the electric field you have this because of the r coefficient and electric field square because of the s coefficient that will contribute first term Taylor series.

Now, this one is called Pockels effect, so you can see that this Pockels effect that actually, if you are just considering refractive index change, if you are just considering up to here then if this one is negligibly small for example, then your $n(E)$ can be represented as $n - \frac{1}{2} r n^3 E$. So, n is the bulk refractive index of the material that is actually used in the waveguide core is the applied electric field then you know as a function of electric field, you can have refractive index change.

But if your electric field is because it is a vector, if it is reversed, sign is changed, then this should be plus. So that means, you can have your electric field dependent refractive index like this, when $E = 0$ you are getting just in that is written here and as you go for positive direction electric field, then your refractive index will be reduced negative direction it will be negative sign will come plus that will be increased.

So that means, this refractive index change is very much linear as a function of electric field you need a material where these r must be nonzero, it can be tensor also, it can have a tensor element because different direction electric field it will contribute differently but as a simple

example we have shown that r is fixed, s is fixed then refractive index will be changing as a function of electric field.

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The slide, titled "Electro-Optic Modulators for Integrated Photonics", discusses "Various Physical Mechanisms" for electrorefraction, specifically the Pockels and Kerr effects. It features a diagram of a capacitor with plates separated by distance d and an electric field E applied across a material. The permittivity is given as $\epsilon = \epsilon_0 \epsilon_r = \frac{1}{n^2}$. The tensor representation for a small electric field \vec{E} is $\eta_{ij}(E) = \eta_{ij} + \sum_k r_{ijk} E_k + \sum_{kl} s_{ijkl} E_k E_l$, with $\Delta\eta \cong rE + sE^2$. The refractive index is expressed as $n(E) = n - \frac{1}{2} r n^3 E - \frac{1}{2} s n^5 E^2$. The Pockels effect is shown as a linear relationship between Δn and E , while the Kerr effect is shown as a parabolic relationship. For Silicon Crystal, $r = 0$. The slide also includes logos for NPTEL and CPPICs, and a small video inset of a man speaking.

Now, again if you see that certain material, this thing is actually may not present because of the crystallographic symmetry etcetera it happen that this is not present, but you can have this is dominating. So, in that case, my refractive index can be written as n minus half $s n^3 E$ square. So, this is a constant that as a function of electric field whether it is positive or negative always the refractive index will be reducing. So that is the like a parabola, so it is like shown here so that is actually Kerr effect.

So, if you apply electric field either in positive direction or negative direction your refractive index is going to change and if that is happen, you will need a material again here s not equal to zero and as will be called as your Kerr coefficient and r will be called as your Pockels coefficient because coefficient. So, this is a Kerr effect and this Pockels. So, we have shown that it is possible to find a material where this Pockels effect can present or Kerr effect can present or both of them can present together simultaneously.

So, depending on the value of r and s you need to consider which one you should consider for your design or for your modulator design I would say. So, for silicon crystal $r = 0$, so that means, we are talking about silicon photonics, photonic integrated circuit, CMOS compatible, good waveguides, lower loss everything but however, this silicon Pockels effect is not present because silicon you know at the FCC crystal in crystallographic structure if you see the FCC crystal and most of the time the FCC crystals are centrosymmetric.

Centrosymmetric means if you are just getting a atom at a xyz coordinate you must get identical atom also presented -x, -y and -z position. So, in this case normally you know, since it is a centrosymmetric $n(E)$ must be equal to $n(-E)$ that is the E^1 function if you are applying electric field and if electric field is linear now, here it is soon linear, it is a reverse direction if you are giving it should be negative value. So, it should be increasing and positive direction it should be decreasing, but centrosymmetric case you have to so that $n(E)$ should be equal to $n(-E)$.

So, if this is the case but Pockels effect, so giving you this type of characteristics if this is happening then these type of characteristics would not be possible. So that means in case of silicon r must be equal to 0, so there is no Pockels coefficient. But normally most of the material including if you consider like a gas, like a liquid and silicon it is like an isotropic homogeneous medium, but this is coefficient is always present because you know, it is E^2 anyway it is symmetric.

So, electric field if you are increasing either minus or plus this is E^2 always become plus, so refractive index will be changed. So that effect is there in the silicon but you have to see what is the amount? How much field require whatever voltage required that can be actually peaceable for designing a modulator in silicon waveguide or not that is important question again. However, people already demonstrated this very high speed modulator using Pockels effect I will just discuss one slide on this Pockels effect for clarification purpose.

(Refer Slide Time: 31:57)

The slide, titled "Electro-Optic Modulators for Integrated Photonics", focuses on the Pockels effect. It features a diagram of a waveguide with electrodes separated by distance d , with an applied electric field E . The text states $\Delta n \propto E \Rightarrow$ Pockels Effect. The refractive index is given as $n(E) = n - \frac{1}{2} r n^3 E - \frac{1}{2} s n^5 E^2$. For LiNbO_3 , $n_o = 2.21$ and $n_e = 2.27$ at $\lambda = 1550 \text{ nm}$. The induced refractive index changes are $\Delta n_x = -\frac{1}{2} n_o^2 r_{11} E_x$, $\Delta n_y = -\frac{1}{2} n_o^2 r_{13} E_x$, and $\Delta n_z = -\frac{1}{2} n_o^2 r_{33} E_x$. The electro-optic coefficients are $r_{11} = 9.6 \times 10^{-12} \text{ m/V}$ and $r_{33} = 30.8 \times 10^{-12} \text{ m/V}$. A 3D coordinate system diagram shows the relationship between the applied field E_x and the induced changes in the refractive indices n_x, n_y, n_z .

For example, lithium niobate you know lithium niobate is a very good material ferroelectric material and very good electric-optic material where electric field actually can change refractive index linearly. So that means, I just if I write that if r is nonzero and s is nonzero simply we write refractive index in this form and normally in lithium niobate with this r value is very large compared to s value.

And in that case, normally refractive index changes proportional to the electric field that means Pockels effect is present. And in this way if you have a little waveguide structured like this, and if we apply electric field then if that electric field is in the only z direction you know lithium niobate crystal it is actually negative uniaxial crystal birefringence material. So, it has a crystallographic axis along z direction it can have x direction, it can have y direction.

Suppose any electromagnetic wave associated electric field oscillating along x direction then it will see one kind of refractive index called ordinary refractive index. And y direction if it is the electric field is oscillating then also refractive index we see that is exactly n_o that means x direction that means $n_x = n_o$ n_y also equal to n_o . However, electric field oscillating along z direction crystallographic direction then n_z is not equal to n_o but equal to n_e .

So that depending on the electro-optic coefficient, you can actually find even though you are applying support electric field along z direction. Then you can see that x direction suppose you are applying electrical in this direction, then along the x direction some refractive index will change meaning your electromagnetic wave if it is oscillating along these lines suppose in this direction it is applied field that can be DC that can be whatever low frequency electric field.

And then in that crystal if electromagnetic wave propagates along any direction, but it oscillates along x direction electric field then it will see a refractive index change of this amount and along y direction this amount, certain coefficient will called r_{13} that means, you know we have just mentioned r_{ijk} that is a Taylor series coefficient, electro-optical Pockels coefficient normally $r_{ij} = r_{ji}$. It happens this symmetric in that case week instead of 2 indices we can actually reduce it to 1.

Then you can get actually 6×3 tensor element you will be getting and then you can get 6 times 3 r coefficient will be there something like this r_{11} , r_{12} , r_{13} and so on, you will be

getting total 6 different elements you will be getting a tensor and out of them many of them will be I think large number of elements out of 18 that will be 0, but r_{13} will be there and r_{33} will be there, they are most important for electro optic modulator design lithium niobate.

And as you as I mentioned, if you are applied electrical field along z direction that means E_z , E_z is given then refractive index change along the x direction will be Δn_x this one along with Δn_y they are identical and along Δn_z they will be identical. And in this case, you know as I mention electric field if I applied electric field in this direction electric field is also oscillating along z direction, then the refractive index will be different not like x direction or y and that is actually called extraordinary refractive index.

And extraordinary refractive index for lithium niobate that means electric field normally without any field without this thing, if you see, if electric field oscillate along z direction at 1550 nanometre wavelength then refractive index will be 2.21. And if the electric field of the electromagnetic wave oscillates either in x direction or in y direction, then it will see refractive index that is called ordinary refractive index to no this is actually this one is n_o is the 2.21 this will be 2.13 that is a printing mistake here 2.13.

So that means, it is n_e is always less than n_o so when any material, if it is your birefringent that means, it has a 2 refractive index along the principal axis if ordinary refractive index is greater than n_e that is called positive uniaxial crystal and if n_o less than n_e that will be called as a negative uniaxial crystal. So, you should remember that this typo mistake here n_o is actually 2.21 and n_e around 2.13 at operating wavelength $\lambda = 1550$ nanometres.

So, keep it straight suppose, you take a lithium niobate material here any crystal you are just considering, suppose this is your z axis, this is your x axis and this is your y axis electromagnetic wave propagating through the material, if it is oscillating along the x direction operating at 1550 nanometre, it will see refractive index 2.1. If electric field oscillating y direction it will see refractive index also 2.21 that is ordinary refractive index.

If electromagnetic wave propagating in the crystal and electrical oscillating along z direction, then it will see refractive index of 2.13 which is actually called extraordinary refractive index and if this extraordinary refractive index is actually less than ordinary refractive index, this will be called negative I think you can see the textbook this will be called negative uniaxial

crystal and if it is n is greater than this one, then it will be called positive uniaxial crystal. So, lithium niobate according to the number it is negative uniaxial crystal that is fine.

And we found that from the handbook you can find also experimentally it was found that if along z direction we are applying electric field and you see the what is the refractive index change along the x direction if you know you know 3 then you can find out r_{13} , r_{13} is nothing but 9.6×10^{-12} meter per volt, you know electric field is if you just see the dimension refractive index is a dimension less electric field you know, the dimension of electric field is volts per meter.

So, r_{13} that r coefficient refractive index; must be meter per volt, so this is volt per meter and this is meter per volt, they must be cancelled. So, r_{13} is this one, but if you apply electro field along z direction, then your refractive index will be this one and other type of electro optic coefficient comes from this r tensor that will be actually 30.8×10^{-12} all the electro optic coefficient in the tensor if you see r_{ijk} whatever I was considering the r_{33} values the highest one that is 30.8×10^{-12} meter per volt.

So, if you just give significant amount of field then you can actually achieve certain kinds of refractive index direction lithium niobate that is actually very fast, if you just apply electric field it is almost instantaneously refractive index will change. That means, you can actually design a modulator which can respond very fast high speed modulator you can design

(Refer Slide Time: 40:21)

The slide, titled "Various Physical Mechanisms" under the heading "Electrorefraction: Pockels Effect and Kerr Effect", illustrates the physical mechanisms of electrorefraction in a waveguide structure. It shows a cross-section of a waveguide with electrodes on top, separated by a distance d , with an applied electric field E . The refractive index change is given by $\Delta n \propto E$ for the Pockels effect and $\Delta n \propto E^2$ for the Kerr effect.

For LiNbO_3 , the refractive index is $n_o = 2.21$ and $n_e = 2.21$ at $\lambda = 1550 \text{ nm}$. The refractive index change for the x , y , and z directions is given by:

$$\Delta n_x = -\frac{1}{2} n_o^3 r_{13} E_z, \quad \Delta n_y = -\frac{1}{2} n_o^3 r_{13} E_z, \quad \Delta n_z = -\frac{1}{2} n_o^3 r_{33} E_z$$

where the $e - o$ coefficients are $r_{13} = 9.6 \times 10^{-12} \text{ m/V}$ and $r_{33} = 30.8 \times 10^{-12} \text{ m/V}$.

A general equation for the refractive index change is given as $\Delta n(V) \approx -\frac{1}{2} n^3 r \left(\frac{V}{d} \right) f$, where f measures the overlap between electric field distributions of guided mode and applied external electric field.

Handwritten notes include $E \sim \frac{V}{d}$ and a diagram showing the electric field E and the refractive index profile $n(x)$ with a wave vector k and a transmission coefficient T_i .

Logos for NPTEL and IIT Bombay are visible in the bottom right corner.

By using that also 1980s are so that time refractive index change or phase modulator in lithium niobate waveguide could be demonstrated as well as intensity modulator could be demonstrated. In general if you just summarize this 3 equations as a function of voltage for example, if you are applying a voltage difference between these 2 points this point, this point for example, separation is E approximately electric field can be approximately written as V / d .

Applying electric field V / d according to the formula $E = - dv / dx$ that formula you can use so field is this one and you will know half the refractive index cube that is written r is the electro optic coefficient V / d and we add additional one gamma that is nothing bad because you know if you are applying an electric field here, suppose you have a waveguide for example, here waveguide design waveguide or lithium niobate normally it is done like this one titanium in diffuse waveguide substrate you dope certain area.

Then you can see doped titanium then depending on the titanium concentration refractive index change will be there and you can get a waveguide here and if you use electrodes like this and you apply a voltage across this one this can be grounded and if this is a d , then you know electric field will be something like this it will come perpendicular to the metal electrode and enter into this one and this will go like this, field distribution will be like this and this will be like this.

So, in this case basically electric field across the waveguide cross section where guided mode is there is not uniform. So, you need to see what is the overlap of the field you have applied with the guided mode electric field associated to the guided mode of the guided light. So that fraction that that overlap can be maximum if it is uniform, then the overlap will be 100% if that is not uniform, then it will be less than 100% so less than 1 gamma you can use overlap integral.

That means how much fraction of overlap is there of the external electric field with the guided mode that that also play major role for changing refractive index in a certain electric-optic material where Pockels effect is present. So, gamma measures the overlap between electric field distributions or guided mode and applied external electric field that is the expression.

(Refer Slide Time: 43:06)

Slide#15

Electro-Optic Modulators for Integrated Photonics

Various Physical Mechanisms

Electrorefraction: Pockels Effect and Kerr Effect

$\Delta n \propto E \Rightarrow$ Pockels Effect

for LiNbO_3

$$\Delta n_x = -\frac{1}{2} n_x^2 r_{11} E_x$$

$$\Delta n_y = -\frac{1}{2} n_y^2 r_{11} E_x$$

$$\Delta n_z = -\frac{1}{2} n_z^2 r_{11} E_x$$

$$n(E) = n - \frac{1}{2} \chi^{(2)} E - \frac{1}{2} \chi^{(3)} E^2$$

$n_o = 2.21, n_e = 2.21 @ \lambda = 1550 \text{ nm}$

where the $\epsilon - o$ coefficients:
 $r_{11} = 9.6 \times 10^{-12} \text{ m/V}, r_{33} = 30.8 \times 10^{-12} \text{ m/V}$

where F measures the overlap between Electric field distributions of guided mode and applied external electric field

material: $\Delta n(V) = -\frac{1}{2} n^2 r \left(\frac{V}{d}\right) F$

Electric Field

Output Signal

Input Signal

LiNbO₃

Modulator Curves

Phase Shift

Optical Signal

Electrical Signal

<https://www.soc.co.jp/>

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Now you see this is how lithium niobate modulator is designed and simplest structure I just took it from this website you can go through that website and you can also find that it is nothing but a Mach-Zehnder modulator you have input waveguide here and branched into 2 and then combined back again. So, at A you are just splitting into 2 half's and B you are trying to combine now, you have an electrode central electrode and both sides electrode is there.

These 2 electrodes if you are grounding, if you are applying a voltage in the central that means in this waveguide you will see electric field in this direction in this here you will see electric field in this direction. As you know if you electric field if you reverse the refractive index change will be positive and that means, if your electric field here in this direction and here in this direction, then what happens? Here if it is refractive index is increasing here actually refractive index will be decreasing.

So that means your phase difference is actually here phase is actually increasing and lower on phase is decreasing. So, you are getting 2 times effect so this is special dyeing design it is called push pull modulator design push pull that means, one arm you are pushing the refractive index in the positive direction or phase in a positive direction another arm you are just pushing the phase in the negative direction. So, ultimately 2 times effect you will be getting and you will be getting a compact device length maybe reduced.

So, if you can create overall 5 phase difference between these 2 arms then you can get constructive destructive interference and if it is zero phase difference and 2 phase difference

then it would be constructive interference if it is constructive interference you get the output signal 1 and if it is destructive interference then you will be getting 0, light will be dissipated in this way you can have your modulator.

Lithium high speed very high speed modulator first time it was designed with 14 Gbps transmission for long all communication purpose and nowadays this lithium niobate modulators are commercially available just 2000, 3000 US dollar many foreign companies they are manufacturing and they are selling. So, here it is just tried to give how it works, you know here if you see output normally we know that it is p output if it is a lossless case we have solved earlier $\cos^2 \pi / 2$ the phase difference by 2.

So, if phase is $\pi / 2$ that means it will be 0 and if it is $\pi = 0$ or 2π , then it will be in that power will be coming out so this is a cosine function. So, as a function of bias voltage this voltage if you plot the optical output you will see something like this suppose DC bias you are increasing. So, it will be just cosine sinusoidal or sinusoidal it will go 0 and sinusoidal going up and down. Now, what you would do suppose you set some bias point here, so that your output is at this point.

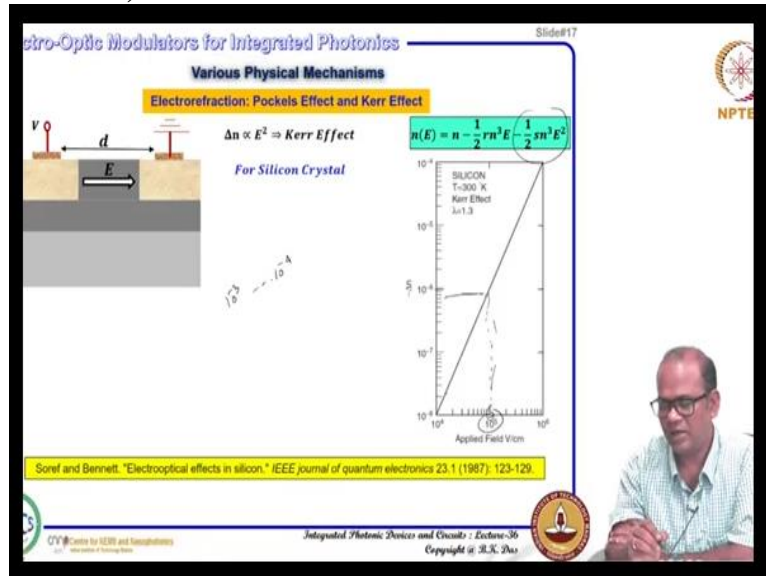
Now, this is your bias point BP, it is just mention as a BP now, at this bias point for after biasing this one, you support certain biases there and on the top of it you have your data for example, after bias point you have data stream is going 1 then 0 bit then 1 then again 1 and then 0 and this as a function of time this goes in here that means, I have a bias this is your bias voltage bias voltage here and from here, at this point of time your bias voltage is increased up to this.

So, once it is increased your output will be here at this point, so it is going here output will be at this point for example. And again when it is coming back to bias point then your output will be coming down to bias point here. So, that means, if your, bias point here and then you can actually changing your data from here to here voltage change that data one that gives you this power. And when your data is 0 that means you are coming back to bias voltage here, so your output signal will be 0.

So, 1 0 that means, this is your electrical data here it is an electrical signal and this will be your optical signal. So that is how electro-optic modulator is done. Sometimes it happens that

you are giving bias point at the quadrature point and then you can on the top of it you can modulate also here. Your modulation that modulation also here will be output depending on your data you will be getting.

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So that is an electro-optic effect and is mostly exploited in lithium niobate, lithium niobate modulators are commercially available and are used heavily, particularly in long-haul communication. However, as you know, we are talking about integrated photonics using a CMOS-compatible silicon photonics platform. So, in that case, silicon does not exhibit the Pockels effect because r is 0. So, what is the next if we want to get a modulator in silicon, obviously that is a let us see what is the Kerr effect.

So, the Kerr effect is what is dealt with here. This is your Kerr effect that is actually missing in silicon, so this one is there. So, if you use this one for silicon crystal, if you see, using this Kerr effect if you apply an electric field then the refractive index changes, the negative sign will be coming here, you see it is changing like this. So, if you apply an electric field up to 10 to the power 5 volt per centimetre here.

Then you can get a refractive index change in the order of 10 to the power -6 only. And 10 to the power 5 volt per centimetre field in silicon is close to the breakdown voltage. So that means electrons from the silicon atoms will be torn out, they will be snatched away and they will start flowing, it will create a lot of currents and then crystals will be damaged because of the heat generation etcetera. So, 10 to the power -6 refractive index change is actually if it is this type of change and that is again close to the breakdown voltage that is very bad.

So, normally we expected that at least 10 to the power -4 to 10 to the power -3 to 10 to the -4 in this region refractive index change is good enough for modulator design. So, normally for silicon this Kerr effect is present though, but it is very weak.

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Slide#18

Electro-Optic Modulators for Integrated Photonics

Various Physical Mechanisms

Electrorefraction: Pockels Effect and Kerr Effect

$\Delta n \propto E^2 \Rightarrow$ Kerr Effect

For Silicon Crystal

"There is uncertainty about the sign of the Kerr effect"

"The anharmonic oscillator model predicts a negative Δn "

"while a four-wave mixing experiment suggests that Δn is positive"

$n(E) = n - \frac{1}{2} r n^2 E^2 - \frac{1}{2} s n^4 E^4$

SILICON
T=300 K
Kerr Effect
A=1.3

Applied Field V(rms)

Soref and Bennett, "Electrooptical effects in silicon," IEEE journal of quantum electronics 23.1 (1987): 123-129.

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One thing we must understand that I have just taken this data from this paper Soref and Bennett their first time actually discussed in details electro-optic effects in silicon both in theoretical model as well as experimental results, but the commented one interesting thing there. There is uncertainty about the sign of the Kerr effect. So, here Kerr effect is negative and you know electric field square is there.

And this model this type of things, this refractive index change can be actually explained in terms of considering kind of an harmonic oscillator of the dipole oscillations, in that case this negative sign can be interpreted well and that is actually happened experimentally if you are applying electric field and that electric field will change some kind of refractive index that refractive index will be experienced by electromagnetic wave propagating through it.

However, while four-wave mixing experiments suggest that delta n is positive, so four-wave mixing is nothing but in silicon because of it has a some kind of E³ process normally if you see P polarization you can express normally $\epsilon_0 E + \epsilon_0 \chi^{(1)} E$ and then you have $\epsilon_0 \chi^{(2)} E^2 + \epsilon_0 \chi^{(3)} E^3$. So, normally polarization if you apply an electric field that is coming from electromagnetic wave and then you can your polarization normally

you know $D = \epsilon_0 E + P$ this polarization can have linear part and can have nonlinear part.

This nonlinear part it can be actually coming out of again Taylor series expansion that means, you can have because P normally we wrote $\epsilon_0 \chi$ times E normal polarization $D = \epsilon_0 E + \epsilon_0 \chi$ times E you can write $\epsilon_0 (1 + \chi) E$ and this $1 + \chi$ is known as ϵ_r $\epsilon_0 \epsilon_r E$ that is actually your D and that can be written as ϵE . Now, this thing, this additional part that is coming out of polarization, this polarization can have again no linear part that means, it can be proportional to E square and one term can happen E^3 square higher order term in Taylor series can come.

(Refer Slide Time: 53:29)

The slide, titled "Electro-Optic Modulators for Integrated Photonics" (Slide#19), discusses "Various Physical Mechanisms" for electrorefraction, specifically the Pockels and Kerr effects. It features a diagram of a waveguide with an applied electric field E and a voltage V across a distance d . The slide notes that for a silicon crystal, $\Delta n \propto E^2 \Rightarrow$ Kerr Effect. A graph plots the refractive index change Δn against the applied field V/cm on a log-log scale, showing a linear relationship. The equation $n(E) = n - \frac{1}{2} r n^3 E - \frac{1}{2} n^1 E^2$ is highlighted. Another equation, $\Delta n = \frac{3\chi^{(3)} E^2}{2n_0}$, is shown with a note that $\chi^{(3)}$ is the third order nonlinear susceptibility (for Si, it is $2.45 \times 10^{-18} \text{ m}^2/\text{V}^2$). A citation for Soref and Bennett (1987) is provided at the bottom.

So, in that case what happens? This is E square term that E square basically not for the external electric field you are applying but this E is the electric field associated with the electromagnetic wave propagating. So, if you have a very high amount of electric field associated with the electromagnetic wave propagating through the waveguide then that is E square term or E square means it is actually basically intensity electric field associated with the electromagnetic field if your square it can be normal intensity.

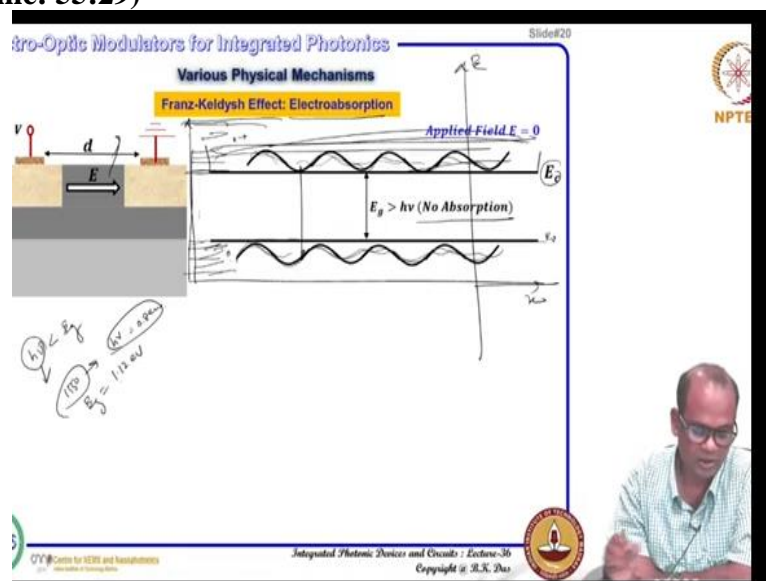
So, you can have some certain kind of refractive index change as well as. Here also E square comes, but that is E square you are considering external quasi static electric field you are applying for modulation purpose, but this electric field that is actually oscillating at a very high frequency. So, they are also this intensity because of the nonzero value you can have

refractive index change and that refractive index change will be always positive and that can be expressed in terms of power or non linearity chi susceptibility is there.

So, it can be shown if you if you are starting some nonlinear optic effect in silicon you can find any textbook you consult you can find that some refractive index change will be there, but in that case refractive index changes will be always positive. So, you should not confused with this type of Kerr effect and this type of Kerr effect, this Kerr effect we are huge exploiting by applying electric field externally and this Kerr effect is actually inbuilt inherently generated inside the silicon waveguide.

Or silicon crystal when the guided electromagnetic wave has very high power intense then this can happen, so it should not confuse so this is the thing. So, Kerr effect is there in silicon but it is very poor. S35o it is not also very good for modulation purpose.

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So, what is left another thing is that Franz-Keldysh effect this is also electro optic effect but not electrorefraction rather it is called electroabsorption. How that is done suppose you have material, this is your material normally silicon you know it is a semiconductor gallium arsenide also semiconductor you can have a gallium arsenide waveguide as well silicon waveguide as well germanium waveguide as well as long as you are $h \nu$ that photon energy is less than bandgap energy.

So that electromagnetic wave photon associated with the electromagnetic wave under consideration of the light wave that is photon energy lower than bandgap then that is actually

transparent that is how if you see that 1550 nanometre corresponding $h\nu$ in the order of 0.8 electron volt also but silicon bandgap E_g is equal to about 1.12 electron volts that means, the photon energy is much lower than the bandgap that is why 1550 nanometre is transparent in silicon crystal.

So, if you see here is a model to explain the Franz-Keldysh effect suppose this is the conduction band you know this is a picture see as a function of any position along certain direction if you try to see the solve using solid state physics you can say that the electron energy allowed energy that actually comes as a band of energy is allowed and then a band is not allowed again another band is coming.

So, normally in silicon you know normally most of the valence band whatever all the energy levels they will be occupied and then there will be a bandgap and then it will be another allowed band that is actually will be called conduction band. If any electron goes to conduction band that electron in the conduction band start conducting contributes to the current and it is empty called volt that also contributes to current.

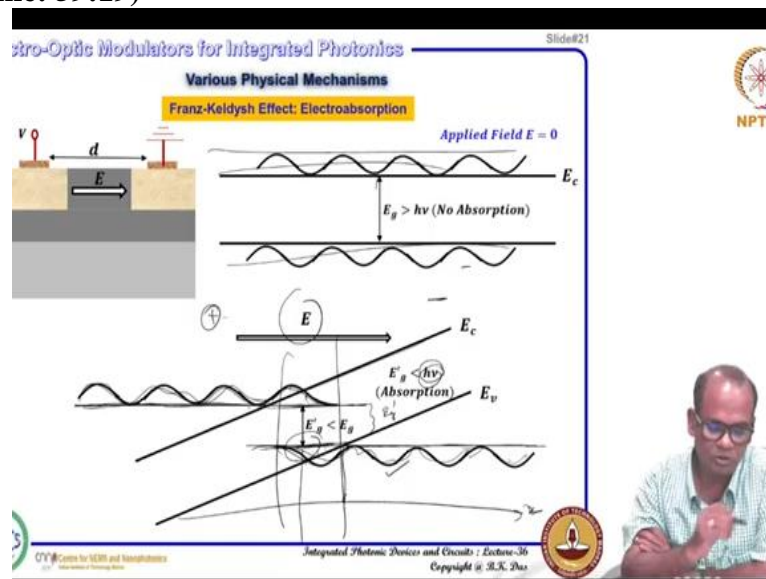
So, normally the electron wave quantum mechanical you can consider that is some kind of wave packet we just represented by this type of wave it is travelling. And here also it is traveling electron wave if electron is there in this band, it will be something like that some traveling wave function we represent like that. And same electron jumps to here it will have certain kind of wave allowed wave it will be like that corresponding to all the energies everything.

So, as long as you are photon energy small, you are not disturbing the electrons in the valence band or in the conduction band normally the photon will be transmitting without any absorption. However, this is the E_c means in this here this axis is energy suppose this axis I am showing the energy and position dependent at any position if you see the valence band, the upper edge, we call it as E_v and bottom edge of the conduction band.

All the energies are there, energy levels are there that may be electrons or there may be occupied may not be occupied if it is not occupied, so it is absolutely paying that it is transmitting, but you know as the room temperature you can see some electrons also can jumps into the conduction band some kind of free carriers are there that is why it is called

semiconductor. So in that case, I said that any energy for exam 1550 nanometre it will not guide.

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However, if we apply field then this band is band entire band energy band that will be like this, because you know, you are applying electric field that means it is a positive potential it is a negative potential. So that in this air electron will see it lowered since energy and in this region electron actually gears up its energy is raised this some energy raised that is why your band it actually conduction band and valence band, they will be tilted like that.

The amount of field you are giving amount of potential difference actually we are giving depending on that, if it is uniform for example, then you can see some kind of band. So, if this type of bending is there, then you can see that at this point if you see certain energy of electron and this is your wave. So, in this energy if any electron is there in the conduction band or in this energy in this region, there are some positions this is your position some electrons are there with this energy, they are like a wave nature.

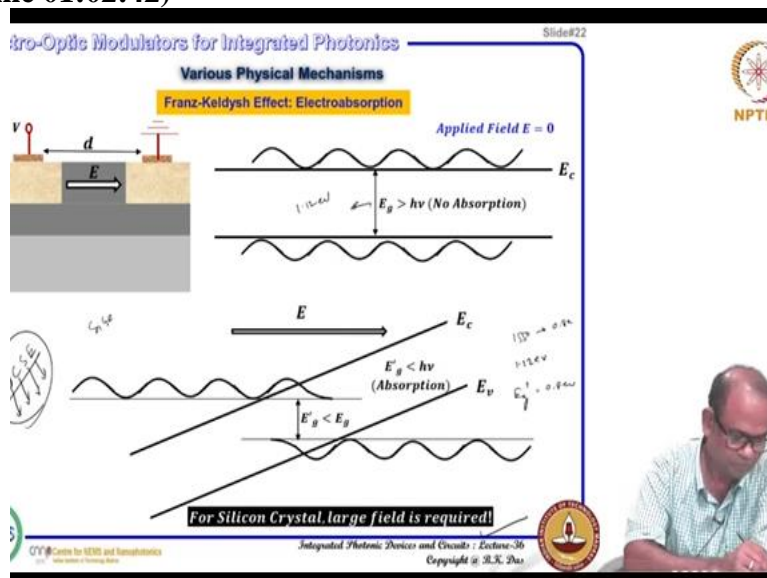
You know wave nature at this energy if it is coming to this position for example, this is your x then you see it sees some kind of band gap for example, so that was forbidden band is starting. So, you know it is same like your electromagnetic wave when you are trying to confine a light, then guided mode will have some kind of evenness tail electric field tail will be there outside the core region cladding region extended here also the electron wave function can explain something like that as a even tunnelling things will be there.

So that means as this energy level it is forbidden typically but some because of the field, you can see some probability of electrons also available at this position with this energy, some probability will be there because of this mean. Similarly, if I see a wave function of electron solutions here, you can expect some kind of probability of that wave function is here so for this energy level, if I just considered in this particular region.

For example, if your electromagnetic wave is propagating with energy edging and here you will see this energy gap here it is reduced now E_g prime, it is no more E_g , normal E_g is this one when it is bend then you are considering this energy level here at this point, because of some tunnelling electron wave function in the conduction band, electron wave function in the valence band is there. So, there may be some kind of overlap will be there. So, and this photon energy $h\nu$ which is actually maybe now coming greater than your E_g prime.

So, in that case that photon can get absorbed by the electron in the valence band at this point and that electron can go to the conduction band. So, in this way that light can get absorbed. So, this is actually Franz Keldysh effect, it can actually influence some kind of absorption so when you are applying electric field absorption is high and withdrawing electric field absorption is low, so you can get intensity modulation.

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But, if you just use actually silicon this Franz Keldysh effect for silicone crystal large electric field is required to get a band for example 1550 nanometre wavelength it is actually 0.8 electron volt. So, you have to bend in such a way that 1.12 electron volt bandgap for silicon bandgap is 1.12 electron volt and you need to reduce it to about less than 0.8 electron volt.

So, that you need a very large electrical to bend such that your E_g prime can come at least 0.8 electron volt, so that much larger electric field is required so it is not useful.

However, later on very recently people are actually creating some kind of quantum confined stark effect that means you are making some kind of quantum structures, nano structures quantum well structure using silicon germanium and so on. And they put demonstrate that naturally this Franz Keldysh effect actually can possible even in the smaller electric field and that type of device structure is called quantum confined stark effect.

Quantum confined stark effect using that effect very good quality modulators have been already demonstrated and but in this course, I will not be discussing if you have some more interest on this type of modulator quantum confined stark effect there are a lot plenty of literature available online you can study.

(Refer Slide Time 01:04:22)

The slide, titled "Various Physical Mechanisms" under the heading "Electro-Optic Modulators for Integrated Photonics", illustrates the Free Carrier Concentration Effect (FCCE) for Electrorefraction and Electroabsorption. It features a diagram of a p-n junction with regions labeled p⁺, p, n, and n⁺, and a distance d. The Drude-Lorentz Theoretical Model is presented with the following equations:

$$\Delta n = \frac{-e^2 \lambda_0^2}{8\pi^2 c^2 \epsilon_0 n} \left(\frac{N_p}{m_p} + \frac{N_n}{m_n} \right)$$

$$\Delta \alpha = \frac{e^2 \lambda_0^2}{4\pi^2 c^2 \epsilon_0 n} \left(\frac{N_p}{\mu_p m_p^2} + \frac{N_n}{\mu_n m_n^2} \right)$$

Handwritten notes include $\mu \propto E$, $\mu = \frac{qE}{m}$, and $\mu = \frac{q}{m} E$. The slide also includes logos for NPTEL and the Center for SEM and Nanotechnology, and is identified as "Integrated Photonic Devices and Circuits : Lecture-36 Copyright © B.K. Das".

Now, the final thing the most important part as I mentioned before that free carrier concentration effect which actually can produce both electrorefraction as well as electroabsorption effect. How is that you know, normally any material it can be semiconductor it can be metal, it can be dielectric whatever you can think of that there is a certain kind of free carrier can be generated in the conduction band because of the impurities mostly parts metal and semiconductors.

Especially for semiconductor if you use a very classical model called Drude Lorentz Theoretical Model it is a very easy any textbooks are listed textbook if you see normally

people demonstrate with the harmonic oscillator model, I am not going into the detail, but I am showing here if you have a free carrier in the crystal which you have a dielectric material, dielectric material means, you have a atom.

And then electron will be tight surrounding it certainly electron maybe outer band electron would be tightly confined into the close to the atom it is not free for a dielectric material, but for semiconductor or metal you have actually you have some chances that the carrier can be movable can be free from the atoms it can say are many more atoms it can move from one item to another item under influence of some electric field proper doping conditions etcetera it is possible. Doping concentration proper doping with a semi conductor can become a conductor.

So in that case, if you use this Drude Lorentz model you can see that this refractive index of any semiconductor particularly the real part and imaginary part κ_n and κ_{normal} is there κ actually results into α we have shown basically $4 / \lambda$ times κ that is actually your α . So, you could just use that that certain carrier concentration of N_e electron carrier concentration and hole carrier concentration.

So, hole effective mass electron effective mass then the real part of the refractive index change can be represented like that, theoretically it can be shown and it is easy to be shown. So, I have not derived here if you want you can derive by yourself also you can consult some textbook also standard textbook and following that thing once you know the refractive index change of the real part using chromo chronic relationship or imaginary part of the dielectric constant.

You can solve and you can find out your κ as well as $\Delta \alpha$ how much loss will be there again concentration dependent. So, here actually mobility is there that means it semiconductor you know mobility is very important how fast it can move, because of the how good it can be mobile because of the applied electric field normal you know drift velocity is proportional to electric field and when we just remove the proportionality constant we use the that proportionality constant as a μ that is called mobility.

Drift v_d is that drift velocity of the electron in the semiconductor under the influence of electric field so that is our mobility. So, basically mobility means nothing but drift velocity

divided by E so that mobility can be different for electron and hole. So, depending on the hole mobility and electron mobility one can see that there will be some kind of attenuation as far as your free carrier concentration is remember that if $N_e = 0$ $N_h = 0$ free carrier, free electron, free hole then entire thing will be 0.

If this one is 0, this one is 0 that means a dielectric so delta n will not be changing that no result will be coming as a refractive index change. So that is actually the theoretical model.

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The slide, titled "Various Physical Mechanisms" under the heading "Electro-Optic Modulators for Integrated Photonics", discusses the "Free Carrier Concentration Effect (FCCE): Electrorefraction & Electroabsorption". It features a diagram of a p-n junction with an applied voltage V and distance d. The Drude-Lorentz Theoretical Model provides the following equations:

$$\Delta n = \frac{-e^2 \lambda_0^2}{8\pi^2 c^2 \epsilon_0 n} \left(\frac{N_e}{m_e} + \frac{N_h}{m_h} \right)$$

$$\Delta \alpha = \frac{e^2 \lambda_0^2}{4\pi^2 c^3 \epsilon_0 n} \left(\frac{N_e}{\mu_e m_e^3} + \frac{N_h}{\mu_h m_h^3} \right)$$

Two empirical models by Soref and Bennett are also presented:

- Soref - Bennett Empirical Model (@ $\lambda = 1550$ nm)**

$$\Delta n = \Delta n_e + \Delta n_h = -8.8 \times 10^{-22} \Delta N_e - 8.5 \times 10^{-18} (\Delta N_e)^{0.8}$$

$$\Delta \alpha = \Delta \alpha_e + \Delta \alpha_h = 8.5 \times 10^{-18} \Delta N_e + 6.0 \times 10^{-19} \Delta N_h$$
- Soref - Bennett Empirical Model (@ $\lambda = 1310$ nm)**

$$\Delta n = \Delta n_e + \Delta n_h = -6.2 \times 10^{-22} \Delta N_e - 6.0 \times 10^{-18} (\Delta N_e)^{0.8}$$

$$\Delta \alpha = \Delta \alpha_e + \Delta \alpha_h = 6.0 \times 10^{-18} \Delta N_e + 4.0 \times 10^{-19} \Delta N_h$$

Handwritten notes on the slide include $\frac{\partial n}{\partial N_e} \rightarrow \frac{\partial n}{\partial N_h}$ and $\frac{\partial \alpha}{\partial N_e} \rightarrow \frac{\partial \alpha}{\partial N_h}$. A note at the bottom states: " ΔN_e and ΔN_h are in cm^{-3} ".

But later on Soref and Bennett in 1987 they have actually done a lot of experimental work and they actually demonstrated a device where they could control the carrier but controlling the carrier they found that what is the contribution of refractive index for electron change and contribution for the refractive index change due to hole for effective carrier density change of this one, this is the numerical model, they are just standard paper is there 1987 it is published by Soref and Bennett you can find this as the numerical expression.

And normally this is actually Drude Lorentz model classical model it is there may be a lot of approximation is there that is why if you just plot directly it would not satisfy your experimental results, that is the reason because there may be many other effects this is a very simplified model harmonic oscillator model, but we get appealing that using that model the carrier concentration if you change delta in amount then you can see refractive index is going to change also.

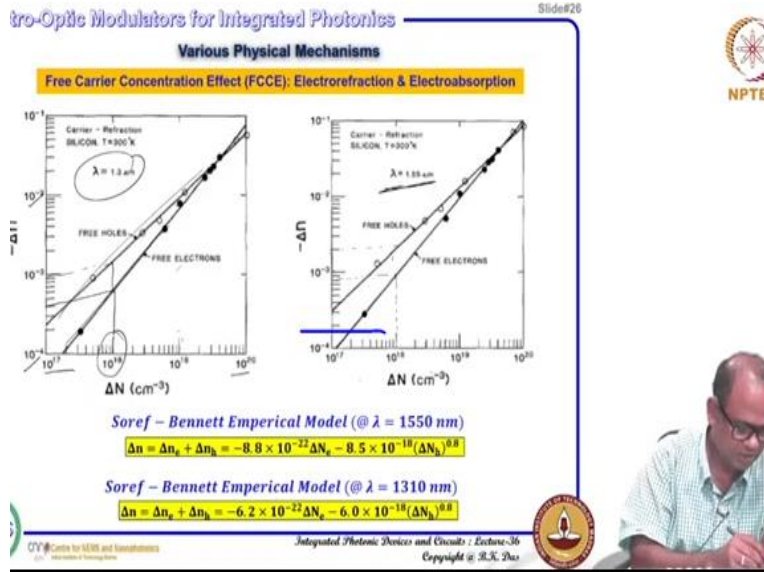
So that is how we experimentally verified and it is model just you do not need to consider what is the mobility, what is the effective mass and what is the refractive index, what is the other constant etcetera. So, you just use this formula for 1550 nanometre then you will know what is the refractive index change happening because of the this is contribution coming from your free carrier electron, this is coming from free carrier hole.

Similarly, $\Delta\alpha$ also can be empirically model for 1550 nanometre. They are also empirically model for 1310 nanometre that is the nowadays actually till yesterday people were more interested on 1310 nanometre because that is the wavelength used per fiber optic communication with this percent is minimum. So, they are also they have modelled empirically and slightly print this 8.8 it is becoming 6.2, 8.5 it is becoming 6.0 here and for $\Delta\alpha$ is almost similar.

And this ΔN_h that means hole carrier concentration change and electron carrier concentration change is represented in per centimetre cube whenever you are inserting here you should make sure that concentration you are inserting per centimeter cube that means cc concentration per centimetre cube, concentration per cube centimeter that is why cc it is mean. So, if you just use then you can say that even if you use.

For example, in the order of refractive index change in the order of carrier concentration changing the order of 10^{-17} to 10^{-18} then the refractive index change can be possible in the order of 10^{-4} which is actually very good your modulation purpose or phase modulation purpose.

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So, now the final slide actually they have using this empirical model actually you see that carrier concentration is 10 to the power -17 to 10 to the power -18 for free electrons FCCE at 1300 nanometer free electrons refractive index change is happening you see 10 to the power -4 it is starting at 10 to the power -7 and you can go up to 10 to the power 18 you can even get an order of 10 to the order 7 into 10 to the -2 and for holes refractive index is slightly higher same amount of care for example, if you just consider 10 to the power 18.

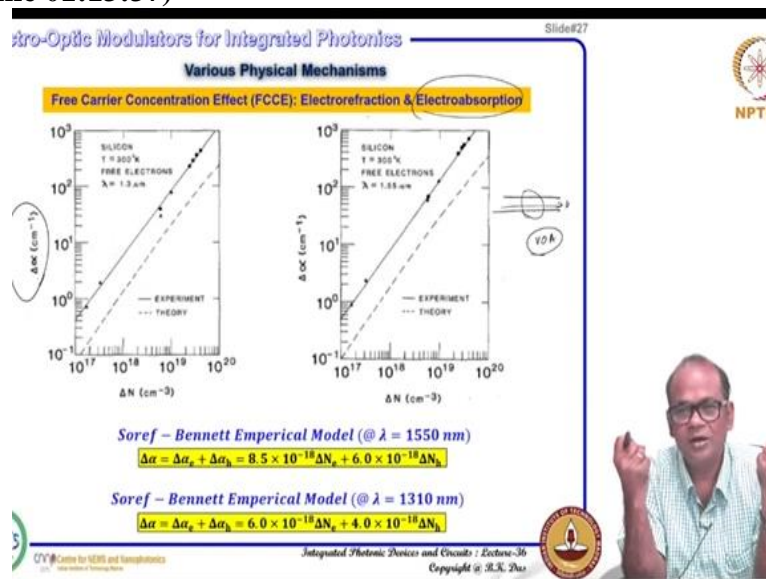
Then electron refractive index changes contributes up to this one so for example, it will be say 1, 2, 3 into 10 to the power -4, but for hole you will see almost in the order of 10 to the power -7 same amount of concentration change for the hole you get different refractive change. These are the experimental results these dotted these things are experimental results, and these were the empirical fields for hole and for electrons.

So similarly, this is for your 1550 nanometre this is slightly it is different you can say for example, 10 to the power 8 if you see here you will be getting more refractive index change for example, around say 7 into 10 to the power -4, but for holes you will be getting more than 10 to maybe 1 into 10 to the power -3 and so on. So that means it is a very good methods because it is silicon, it is a semiconductor, you can control the carrier concentration by doping and biasing and then you can change your refractive index.

And if you can change your refractive index, you can do phase modulator design, if you can do phase modulator design by means of electrorefraction method then you can go for

amplitude modulation or intensity modulation using Mach-Zehnder interferometer or microring resonator.

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And at the same time, they have also calculated delta alpha also because once you change your refractive index real part simultaneously, you are actually contributing your imaginary part also changing that imaginary part changing actually will cause some kind of loss. So, you have to design very carefully tactfully that your phase shifter length should not be sufficient, should not be long enough that it will actually the electromagnetic wave or light wave or guided mode is completely attenuated.

Because of the presence of the imaginary part or because of the kappa or extinction or you can say that, because of the electroabsorption method. So, you have to say that electroabsorption should not dominate over electrorefraction in designing a good modulator because electroabsorption in that case it will be basically loss factor you want electrorefraction that is what you should concentrate.

Alone electroabsorption you can also think simple waveguide and you can just control a carrier in the certain area then you can say that electrorefraction happening at the same time electroabsorption also happening so that means, you can actually control the later light output here. So, in this way, you can actually design one special type of device called VOA variable optical attenuator.

So, if you want to control the intensity of the guided light that also you can use electro absorption method by controlling carrier concentration in the waveguide and also electroabsorption modulator of course, but you have to keep in mind, what is the speed, how fast you can modulate the carrier concentration in the waveguide region, how fast you can again, populate and how fast you can actually depopulate the carrier in the waveguide region that actually decides that speed of the modulator, that part will actually discuss in the next lecture.