Integrated Photonic Devices and Circuits
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Lecture - 26
Integrated Optical Components: DC Based MZI and Microring Resonator (MRR)

Hello everyone, in this lecture today, we will continue integrated optical components and in the previous lecture we discussed directional coupler, its transfer function, transfer matrix. Now, we will be discussing DC based MZI Mach-Zehnder interferometer you know we have discussed earlier Mach-Zehnder interferometer, but that is actually Y junction based you have input waveguide and you have splitted into 2 waveguides.

And then another combiner and that is how Mach-Zehnder interferometer one by one Mach-Zehnder interferometer we have discussed earlier. Now, we want to discuss DC based Mach-Zehnder interferometer which will be 2 input ports which will have 2 input ports and 2 output ports and along with that, we will discuss about microring resonator that is also designed with the help of directional coupler whatever we have learned so far.

So, that is why we will be concentrating on this MZI and microring resonator specifically the transfer function transfer function how it will look like a Mach-Zehnder interferometer and transfer function for unbalanced Mach-Zehnder interferometer we will learn what is balanced and unbalanced and transfer function of a microring resonator. So, that is these 3 things we will be deriving the transfer function of these 3 important components heavily used for photonic integrated circuits.

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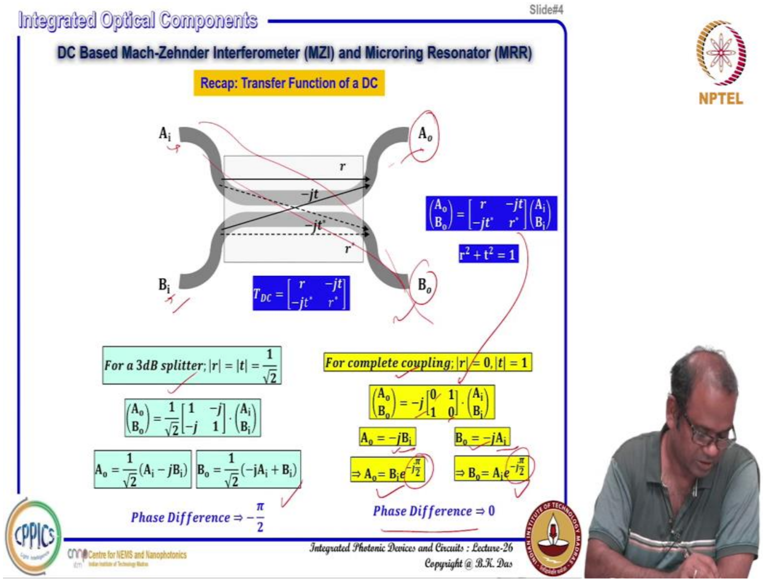

So, I want to give a bit of a recap for transfer function of a DC directional coupler you have 2 input, input 1, input 2 and input 1 the field you are amplitude you are giving A i and B i so AB are the inputs i stand for inputs and 2 output ports you will have A naught B naught and we know that this transfer matrix for a directional coupler this coupled waveguide we can summarized using this matrix.

So, r r star and -jt jt star that we have derived earlier and if we have a input output A naught B naught, we can use this transfer matrix to find out if we know the input and output then we can find out what is the output amplitude with respect to their corresponding input values this A i, B i can be complex and they can have a different phase initial phase that does not matter that will be included in here we can use some complex form so that we can introduce whatever their phase difference or the input.

So, now, we would like to emphasize that this directional coupler is lossless, we assumed there will be some kind of losses, but in this case, we will be considering r square + t square = 1. So, unitary operation takes place. However, there will be in practical cases there will be some losses. So, in that case, whatever outputs you are getting here in, we just consider some kind of loss factor alpha into some kind of directional coupler length.

You can consider then you can find whatever the loss is coming out by at the end whatever you are getting, you can consider that some losses are there that can be easily introduced. So, now, we know that if we want to use this directional coupler for a known r t you know that r equal to simply we can say cos times kappa l dc basically. So, that is actually r and t also we have defined sine function of kappa l so, to have a 3dB splitting this r and t the mode of that.

That is the absolute value of reflection copies and so, called whatever directly going and whatever crossing they can be considered 1 by 1 over root 2. So, 1 over root 2 if you are just considering then we can say that it will be 3dB power splitter. So, if you are just considered r = 1 over root 2 t = 1 over root 2 and then 1 over root 2 this this can be written as this matrix can be written as like this, 1 - j - j and 1 over root 2 is there here, here, here everyone so, I have taken common factor.

Now, if you see A naught if I just write down like this that will be this output that is a superposition of whatever coming out of A input and whatever coming out of B input that is a superposition you will see so, whatever contributing to from B input that will have some kind of a way to phase shift here like this. Similarly, for B output here I can express from this matrix directly we can get.

So, if you just check carefully in this case A naught B naught whatever the original value was their A i B i along with that just additionally this directional coupler alone actually introducing some kind of phase difference that is actually - phi / 2 phase difference if you see if you take this one for example, if you take this one A naught you can write something like this - j 1 over root 2 - j A i + B i.

So, that means it is nothing but - j equivalent to whatever you are getting like B naught so, A and B naught also they are related but with a phase difference of – j, - j means if you just put in phase upon that will be actually j pi / 2. So, - j pi / 2 so, that is how you get pi / 2 phase difference. So, the directional coupler if you have input output whatever phase initially difference there can be but because of these the 2 outputs I am getting with a phase difference of - pi / 2.

So, that thing because any integrated photonic devices or photonic integrated circuits you have to handle superposition principle. So, in that case amplitude as well as phase both are important. So, now, this is for 3dB splitting 3dB splitting that is nothing but if you are just considering this directional coupler for complete coupling that means, if you launch here input here.

So, everything should come down to here that means r should be equal to 0 and if you launch here everything should be coupled here that means, they are also r stars should be 0. So, we consider for complete coupling r = 0 t = 1 that means transmission completely happens. So, in that case your this transfer matrix can be written as so, - j, 0, 1, 1, 0. So, we know that that will be actually only up diagonal matrix will be there t = 1 we are putting t square = 1 r = 0 r star = 0.

So, that is 0 0 shifting here and since plus - j - j is there - j can factor. So, - j if I factor here then I can get what is A naught A naught will be equal to - jB I and B naught = - jA i. So, that means, now you see this - j here also their output A output B also - j that means, I can write down in phase form A naught = B i e to the power - j pi / 2. So, whatever it is they are going there along with that it is introducing pi / 2 phase difference - pi / 2 adding pi / 2 and here also whatever coming here it is adding pi / 2 phase difference.

But as a whole if you just compare A naught B naught if you compare their phase difference because of the directional coupler, because, you see that there is no phase difference because this is also -j pi / 2, -j pi / 2. So, they do not have additional phase difference blocks because of the directional coupler, but both are having some kind of phase they are identical phase. So, that is why the directional coupler in this type of situation does not introduce any phase difference between these 2 output ports.

Because we are considering both input and both outputs. So, this is the directional coupler all we need to know to use to construct more advanced complex photonic integrator circuit or integrated optical circuits.

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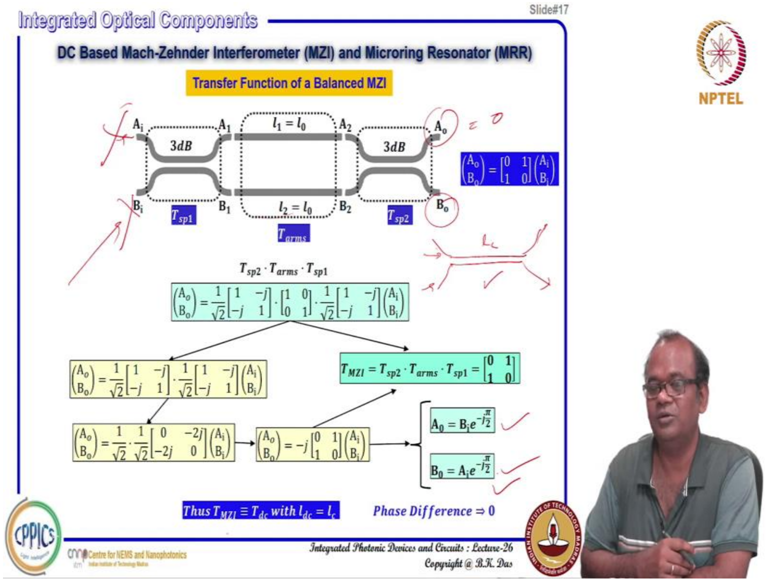

So, let us try to see this type of configuration. You just cannot imagine Mach-Zehnder interferometer constructed by help of 2 identical directional couplers acting as a as 3dB power splitters. So, this is also 3dB power splitter this is also 3dB power splitter and 2 exactly identical directional coupler meaning this waveguide and this waveguide they are also identical this waveguide, this waveguide also identical their gap also identical, their length also identical and it is adjusted such that both at 3dB power splitter.

We know what is the transfer function of 3dB power splitter just a few moments before we discussed that you just connect them using 2 straight waveguide identical straight waveguide l 1 = l naught and l 2 = l naught and these wave guides they are identical you can assume that they are propagation constant they support only single mode fundamental mode and the propagation constants are meet at a same identical.

So, if this type of structure if you can construct, they are called balanced Mach-Zehnder interferometer 3dB power splitter, 3dB power splitter and both the arms connected here they are like a Mach-Zehnder interferometer arms. So, you can imagine when we consider Mach-Zehnder interferometer using Y junction there also we have 1 input splitted into 2 again combined using Y junctions.

Here also you can if you want to use 1 as the input suppose you are launching here then 3dB power splitter gives here gives here 50%, 50% here along with their factors and then this arm and this arm they propagate with the same identical path and then you can combine with another directional coupler the process you can use input here or here. So, you have 2 options

2 input ports 1 of them you can use both of them you can use also whenever necessary. So, similarly here also both outputs are there we can see.

So, you remember that whenever we consider 1 is to 1 Mach-Zehnder interferometer using Y junction. So, their power if their phase difference between 2 arms is pi then the power enter power you are launching that will be completely lost to the substrate, but in this case, it will not be lost to the substance rather that power lost power can be appeared in the other waveguides other output ports that we will see.

Now, our intention is that if you have inputs at both the arms, they can be complex they can have some kind of initial phase difference and with the amplitude known amplitude and you are launching there and now I would like to know what is the things you suppose to get at the output if you can derive some kind of transfer matrix for this entire structure that means A naught B naught.

So, 2 by 2 matrix multiplied by A i and B i then you can use some linear algebra for constructing more advanced photonic integrated circuits using this type of structures. This is very important structure useful for large scale integration of programmable photonic integrated circuits as well. So, our intention is to fill these matrix elements. So, what will be here a 11 a 12 and a 21 a 22 these 4 elements if we know if we can just construct then it would be very good that is what our intention.

So, now, let us annotate this entire structure into 3 zones zone 1, zone 2 and zone 3. So, this is basically we can consider directional coupler where the inputs are same A i B i and just output section if you see that will be A 1 and B 1 annotated and these A 1 and B 1 we can consider that is the input for the arms Mach-Zehnder interferometer arms you are launching here B 1 and you are launching here a 1 and then they will propagate l naught length.

It will propagate also l naught length then you have add the output after traveling it can have some kind of loss it can acquire some phases we are considering lossless case at the end we can just calculate estimate the last output. So, here this will be output A 2 and B 2 only we can just consider now, what is the phase related information it is being carried as it propagates through the device or components.

Now, you have thought 3dB third section that is also 3dB power splitter we can say that in this 3dB power splitter we have A 2 input B 2 input and 2 input ports and you are getting output ports. So, I know how to construct transfer matrix for this. I know how to construct transfer matrix for this and accordingly I can say that this is splitter 1 can be written as 3dB power splitter 1 / root 2 that is just a while ago we have discussed 3dB power splitter. So, your inputs are A i, B i then outputs will be A 1, B 1.

So, we can write simply splitter 1 transfer function. Similarly, splitter 2 your inputs are A 2 and B 2 outputs are A naught and B naught. So, we can write A naught, B naught in terms of A 2, B 2 use another transfer matrix for 3dB power splitter so step forward. Now, let us consider what happens in between here. So, after reaching here, A 1 becomes A 2 how they became A 1 and since it is a propagation constant of the waveguide B and length is l naught.

So, you will acquire some phase e to the power - j beta I not similarly B 1 convert in to B 2 how with a phase additional phase and remember that it is like when it propagates e to the power j omega t - beta z that same thing we are considering and we are not considering the field mode profile distribution instead we consider that is actual normalized only the amplitude that is just to keep you in mind must not forget that what is actual we consider for waveguide and for analysing different type of components.

So, I can say that A 1 will become A 2 and B 1 will become B 2 and this omega t term because all frequencies are same, we can any point we want to know what is the time dependent variation? You have to just multiply e to the power j omega t. So, we do not consider there that e to the power omega t we are assuming that it is a monochromatic operation to the domain analysis is happening.

So, now in this case if I want to write down these 2 equations in a matrix form how it will be written. So, your A 2 B 2 will be output I have written A 1 B 1 will be the input. So, if you see if you just use this type of matrix then you see A 2 = A 1 e to the power - j beta 0. So, A 2 will you will get this equation and B 2 you will get this equation. So, these 2 equations you can write down in a 2 by 2 matrix transfer function here it will remain as a beta l naught it will remain a beta l naught beta l naught, they are identical balanced arms similar.

So, now what we can do we can say that this we can little bit modify because 0 0 half diagonal elements we can write 1 e to the power - j beta l naught it is a kind of factor I can take. So, you know any system 2 by 2 system if you see that some common factor is there in phase factor e to the power j beta l naught that not so, important for analysing the entire circuit because both the output will have same amplitude same phase A 1 will also acquire same phase and become A 2 B 1 also acquire same phase and becomes B 2.

So, if they are phase identical phase that is not so important for further interference. So, that is why this one we can drop in principle that should carry by the both arms, but we can drop because of simplification. So, ultimately the T arms I can say that the transfer matrix for the Mach-Zehnder interferometer arms that was both the arms in short can be written like this. So, this is your 1 1 0 0 you can write A 2 B 2 you can write that one it was 1 - j beta not common so we are ignoring you must keep in mind that.

So, as long as we are restricting your discussion with these 2 fields only and their phase factor common phase factor you can always eliminate that is useless because always that phase factor a common phase factor if you can eliminate only their time dependent similarly whenever we are just considering this phase factor also, we are eliminating the both arms because they are same in the same phase in time.

So, we constructed another matrix we knew that T sp1 is this one this is the matrix and this is the matrix for 3dB power splitter. Now the central region also we have the matrix 2 by 2 matrix. So, what you can do central region, you see, I have this one T sp2 relates A naught B naught that means output in terms of A 2 B 2. So, this A 2 B 2, instead of A 2 B 2, I can write this one, instead of A 2 B 2, I have just written this one.

So that means this one T sp2 and T arms if you multiply, that means I can relate A 1, B 1, if we know A 1, B 1, then I know what is A naught B naught. However, A 1 B 1 also, we can relate with A i and B i with this. So, if I use this one and if I just bring this one to here, write A 1 instead of A 1, B 1, I will be writing these one, then what we get, we get this one. So, I use this one, this one and then I say first this one was there, this one is there instead of A 1, B 1, I am just writing this one.

So basically, this expression comes here. So, I have now 3 matrices. So, this matrix corresponding to T sp1, that means this one, this is T sp1 and then middle one, the arm and then this one, the final one. So, you start like this if I just follow this one it was just total thing, I am writing A naught B naught that means output in terms of A i B i and I have 3 matrices you see it is or whenever you are just considering A i B i you are writing A i B i and then T sp1 the matrix comes here T sp1 and then A i in the left side.

You just add the matrix of the Mach-Zehnder arms this one and then next you get this matrix output splitter T sp2. So, this is T sp2. So that means if I want to relate input to output and if we are using the matrices for individual components, what is the transfer matrix then normally you should keep in mind that whenever you are writing you want to know A naught B naught output in terms of A i B i we are writing and then starting from A i B i. Next matrix I will be just entering here T sp1 and then immediately after A i B i you have.

These matrices and these matrices, these matrix is there then this matrix these matrix will be there and then left hand side you will be getting the so now if we just multiply this one if you know you have different matrix to be multiplied, you have to again multiply them from right to left. So, you have to multiply this 2 this first and whatever you get result that will be again multiplied by this to this so that is simple matrix algebra.

So, if we do that, so first if we just multiply this one these to this 1 over root 2 1 over root 2 will be there this one if you multiply here basically nothing happens 1 1 minus because it is diagonal unitary matrix diagonally 1 1 is there. So basically, we are getting like this and then if you multiply these 2 then you get this one. So, if you just multiply 1 / 1 over root 2 1 / over root 2 it is there and this one if you multiply 1 - j 1 - j 1 - j - 1 - j 1 - j, so simple matrix multiplication you do then you get this one.

So - 2j you can factor here because both the element all the elements are having - 2 j factor so we take and the root 2 root 2 cancel then what we will get - j common and ultimately, we are getting half diagonal elements are there 1 and 1 and - j will be common factor. So now, I know this is going to be the transfer matrix for the entire structure because all these transfer matrices together it is coming like that.

So, what we do we can drop this again -j the common phase e to the power - j pi / 2 that type of things will be their phase, so that I can drop that will be appearing in both the output ports A naught and B naught that I can drop then I will be getting A naught B naught can be related to only the central portion -j common factor removed and then we can just put that this one will be 0 and this one will be 1 and this one will be 1 this will be 0. So, I now know what is the transfer matrix for the balanced Mach-Zehnder interferometer so far, so good.

So that is very useful and then here we would like to see one more interesting thing that if you just see A naught equal to A naught will be 0 1 multiplied by that will be B i input pi / 2 that means A naught what you will be getting whatever you are launching here entire thing will be appearing here, but along with that, pi / 2 phase shift phase difference will be there. Similarly, if I launch here something like that entire thing will be coming here with phase shift of - j pi / 2.

So, they will be crossing each other this system actually giving the entire structure gives you that the power they actually switch their path. So, A i can be I can just route this type of structure can be useful to just routing from one branch to another branch. This is one waveguide is there now, I get another waveguide. So, that entire power can be routed this site and whatever you were launching here, we can just cross them. So, this can be good crossing in this integrated circuit.

So, if you want to just cross the path from 1 structure to another structure, you can use this type of structure also just to cross the path and of course, you have to be or what about the insertion loss you are supposed to get there, fine. So, now, what is the conclusion here again, we say that, it is something like that; it is 3dB power splitter instead of 3dB power splitter, directional coupler.

If you have l c length, then you know if you are launching in one side it will completely coming back this side like this and if you are launching here, it will be completed coming back. So instead of 3dB power splitter as if these middle arms are not present. So, this 3dB this 3dB connect back-to-back. Then you get normally a 3dB power splitter doubled the length. So, that means it is just crossing.

So, the presence of presence of the Mach-Zehnder arms in between that is actually. So, that is actually we can say that this is something structure we get just connecting the directional coupler though you call it as Mach-Zehnder interferometer ultimately it is nothing but a directional coupler of length l c cross coupling completely happens and in that case 2 output ports will not be will not have any phase difference.

But if you just consider this one cancelled only this one you are launching then this one you would not get anything this will be called to 0, but here whatever it will appear here pi by 2 phase differences similarly, if you are launching this one this is cancelled. So, everything will come here, but additional phase whatever path related phase will be there that is there, but in addition to that, because of this all this crossing happening. So, that will create additional pi / 2 phase difference individually. So, that thing you should keep in mind whenever you are using this type of structure this is straight forward, we are analysing easy to use.

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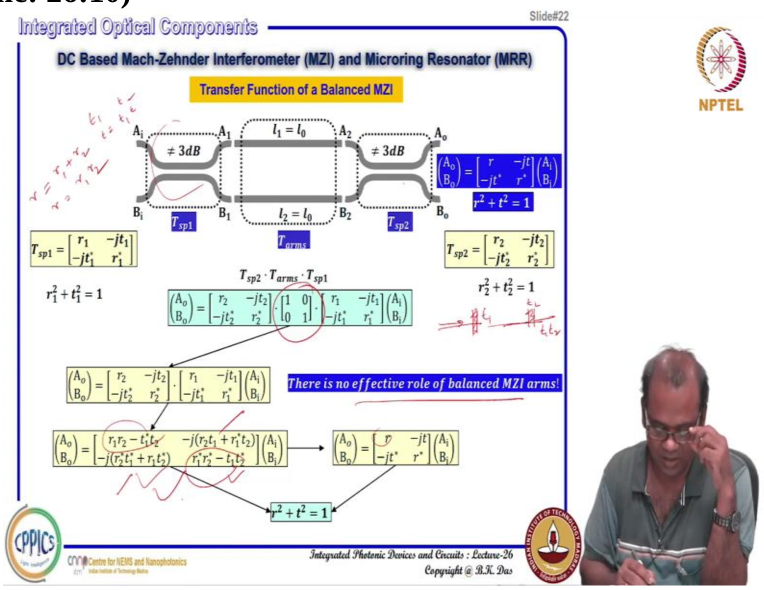

Now, so, we have discussed over transfer function of the balanced MZI. So, we continue that, but same thing we continue but this directional coupler in the both end they are not identical balance when we make mean balanced Mach-Zehnder, when we turn mostly most of the time, we refer that the arms are identical. But we do not bother about what is happening this directional coupler we assume that they are typically 3dB power splitters.

But if it if they are not 3dB power splitter one of them may be 3dB power splitter another may not be 3dB power splitter you can get different types of functions. So, let us see how it will be going like that. So, as usual, normally if it is not 3dB power splitter, you have this first directional coupler splitter 1 we consider and that can be defined by r 1 and t 1 matrix I can consider that through coupling is r 1 cross coupling is t 1.

So, we can define the transfer matrix like this such that it is lossless r 1 square + t 1 square = 1. Similarly, T sp2 I can write here, this is not 3d power splitter. So, I cannot write simply 1 / root 2 1 - j - j 1 something like that instead of that r 2 t 2 I have just written such that it is also lossless r 2 square t 2 square and middle arm as usual we can write middle arm will be that is 0 10 01 that is earlier we have derived that is same identical.

So, if I want to see output to input if I want to relate then this is T sp1, this is Mach-Zehnder arms and this is T sp2 Mach-Zehnder arm. So, we start from here to here, A i, B i and then T sp1 and then T arms then transfer arms for the T sp2 write that one and then again, we do matrix multiplication as usual I have done before. So, I just repeat that one, we just multiply this one this one then we get this one and then whatever the result will be getting we multiply with that one then we will be getting this one.

So, matrix multiplication one to one we just multiply them just inspect carefully this matrix you will see again this one this r 1 r 2 t 1 star t 2 and this one if you see r 1 star r 2 star - t 1 t 2 star that means whatever you get this element, this one is just complex conjugate of that one. r 1 r 2 r 1 star r 2 star t 1 star complex conjugate t 1 t 2 complex conjugate t 2 star. Similarly, these and these you can consider you get - j t and - j t star whatever - j whatever the multiplication is here, -j complex conjugate multiplied here.

So, it is as if you can write similarly, since simply that entire structure can be written like this, that this is as a whole we are writing as r, this one this element I can write like a t, this element I can write a t star, this element I can write a r star and considering these 2 and these 2 are as your that means lossless r 1 square plus t 1 square equal to 1 r 2 square t 2 square = 1 they are not 3dB power splitter.

But what about reflection coefficient transmission coefficient whatever goes their reflection coefficient we call transmission coefficient we call this one and they are different and we assumed this one that has to be no loss is there. Here also if I try to see the determinant of this one that means this one multiplied by this one and this one multiplied by this one that means r square and t square that also will become 1 r square plus t square = 1.

So, arms are lossless we are considering 3dB power splitter a lossless 3dB power splitter db. So, whatever this thing we are getting that is a simple like a 2 by 2 transfer matrix as if r - jt jt -r square. So, that simply we are getting. So, that means, if we are not using 3dB power splitter. We also can find out a 2 by 2 matrix here what are the elements will be there and we can find out if we know r 1 r 2 and r 2, r 1, t 1, r 2, t 2.

Then we can find out what would be value of these elements this thing this factor that we can get we can get the all the corresponding elements. So, that is a straightforward as long as you know the transfer matrix, I think this is just a simple multiplication but, the interesting point is that there is no effective role of balanced MZI arms. So, if you see again since they are identical. So, ultimately this factor whether it is there or not does not matter.

So, as if it is like a one directional coupler, another directional coupler back-to-back if you can connect. So, that is the interesting point one directional coupler and another directional coupler back-to-back if you connect then the r 1 cannot be considered like r 1 = r 2 r cannot be considered like this simply reflection coefficient or simply cannot just consider r 1 r 2 because you know reflection coefficient or transmission coefficient if you are considering t 1 t 2 if you are considering t cannot be considered as simple t 1 t 2.

Just simple multiplication rather you have to find out this type of element this will be your r 1 r and this will be your r star and this will be your t and this will be your t star. So, 2 directional coupler of non-identical coupling coefficients; if you will just connect back to back then you can find out the final transfer matrix in this form. So, the r can be just simply r 1 r 2 - t 1 star t 2 it is not simply r 1 + r 2 or r 1 times r 2 similarly, t = t 1 times t 2.

It is not like that suppose you normally why I am saying that suppose you have a some element is there here and then another element is here some wave is coming then it is something transmitting t 1 and then it is going there and this is t 2 then it will be ultimately the effective transmission coefficient is t 1 t 2 simply we can say that whatever coming here or whatever coming here it will be t 1 t 2.

But in this case, you cannot consider something like that because of the 2 ports because whatever coming here and coming here and the nature of the directional coupler transfer matrix you cannot just simply like simply you cannot use r 1 t 1 and r 2 t 1 simply multiplications rather we have to use this one this is the take home message.

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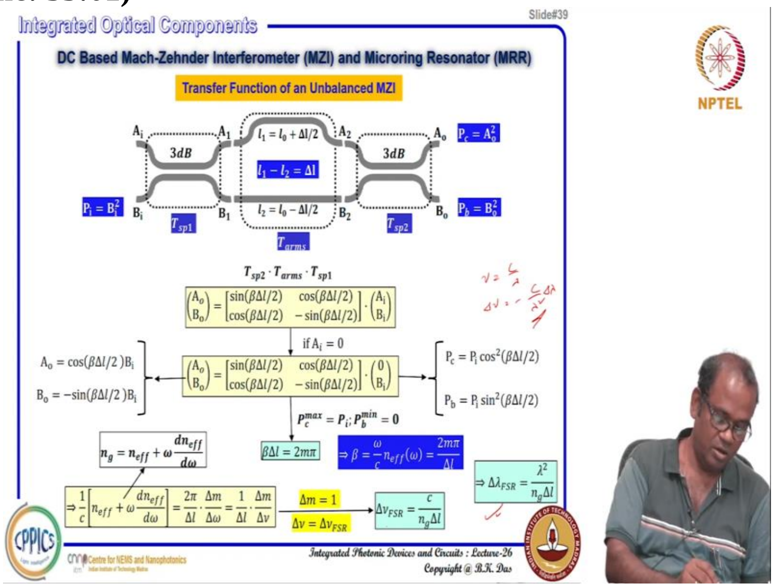

So, now, now we move on to discuss a bit another important integrated optical component that is called unbalanced Mach-Zehnder interferometer. So, why it is unbalanced we will be using 3dB power splitter instead of balanced arm you were just considering their arm lengths are different l 1 not equal to l 2. So, l 1 I consider a fixed value l naught plus delta l / 2 and another arm this is - delta l / 2 such that l 1 - l 2 equals to delta. So, delta l is the path length difference between these 2 arms.

So, just introducing path length you will get a very interesting output if there is no path length difference, we see that whether they are arm, Mach-Zehnder interpretive arms are there not output it is not going to change anything it is like a simple one directional combined directional coupler, but you can find out based on the r 1 r 2 and r 1 t 1 and r 2 t 2 you can find out but if unbalanced how to process it. So, this one the transfer matrix for the splitter 1 standard 3dB power splitter.

So, I have noted same way A 1 B 1 output whatever coming here A 2 B 2 that will be input to the second directional coupler. So, second directional coupler 3dB power splitter I can write depending on the annotation here A 2 B 2 is the input A naught B naught is the output A naught B naught you can represent in terms of A 2 B 2. But in between you know I can say that A 1 A 2 that is actually you have A 2 = A 1 e to the power -j beta l naught + delta l / 2 that is the A 1 A 2 relationship and B 1 B 2 relationship is the B 2 = B 1 e to the power -j beta l naught - delta l / 2.

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So, if I use this one same way that phase part is there, I just consider that the diagonal elements will be like that earlier we abused just e to the power -j beta l naught in case of balanced arm, but in this case this l naught is changed l naught + delta l / 2, l naught -delta l / 2 in the lower arm. So, I just use this one. So, if you process it a bit more so, this one if you see I can take it e to the power -j beta l naught common factor then I will be getting e to the power -j beta delta l / 2 and here e to the power j beta + delta + j beta delta l / 2.

So, this is the thing so, these are the common I can drop out anything common factor in the matrix because that will give you 2 by 2 matrix that will eventually give you same phase in both the arms so, that face factor I can drop. So, I can use this is this thing as a Mach-Zehnder arms Mach-Zehnder interferometer unbalanced arms transfer matrix I can write down that.

So, note down that it is minus it is plus because it is plus is there it is minus is there accordingly you have written now you follow that you know now that this also represented as a matrix form this also you have represented as a matrix form. So, A 2 B 2 value this entire thing I can represent here and once we represent here this A 1 B 1 I can put here then I can get the final value, so I just multiply this one and then using this thing ultimately I am getting these 3 matrices.

How they will be sequentially the input A i B i then input splitter 1 then Mach-Zehnder arms matrices and then splitter 2 I can use and then we can get just simple straightforward that starting from there how I am getting just 3 multiplication I am doing 3 matrices I have just written here these 2 are multiplied I get this one 1 / root 2 this one and then whatever result you are getting you multiply here.

1 over root 2, 1 over root 2 that will become 1 / 2 and this one will be there if you just move on you just simplify this one after simplification what you get this - j - j you take a common factor out so if you just take -j common factor out then you are getting a very nice transfer matrix and which is actually A naught B naught related to the input and output A i B i so now we know that what will be the element is this one.

This one will be sine beta delta l / 2 and this one will be caused beta delta l / 2 and so on. 4 elements it is there we can just enter here and we can get this is the ultimate transfer matrix. So, if it is an unbalanced case obviously if you see that if delta l = 0 then what will be you will be getting this matrix you will be getting 0 1 and then this will be again 1 and this will be called 0. So, that is actually balanced it will convert into balanced one.

So, this is a general form of transfer matrix for Mach-Zehnder arms. So, now unbalanced Mach-Zehnder arm. Now if you just consider one thing suppose this B i this value we are considering 0 nothing is there this is cancelled, the special case I am considering and power I am launching at the input here as A i, A i square equal to B i that is the normalised part that can be considered 1 watt if not 1watt whatever power you can consider that one as if A i square. So, if that is the case, they are this value I will be putting 0.

So, if I put this one 0 then this comes like this and little bit phase shift then what we get, we get one thing the A naught will be sin beta delta l / 2 A i and this one will be 0 and B naught will be this one. So, you see A naught will be sin function of delta L and B naught will be cosine function delta l beta is the propagation constant again if you just put delta l = 0 that means balanced then this one would become 0 A naught will become 0 and b not will become A i. That means entire thing would come here if it is balanced one. So, this is the most general case.

So, you can consider this one. Now suppose nothing is launched here and whatever the output, output here at this point we can consider with respect to this input whatever appearing here we consider this is a bar port and with respect to this one this will be considered as a cross port. So, power at the bar port P b power at the cross port we consider the P c. So, P b will be A naught square and P c will be B naught square.

So, A naught I have already got from this transformer function transfer matrix A naught is this one so, P b will be this one square A i square, A i square = P i. So, P i times sin square beta delta l so, once you know amplitude, we can find out what is the power you will be achieving at the bar port and what are the power at the cross port. So, if you see P b + P c they will be equal to P i because sin square function cos square function.

So, lossless case so, total power is distributed that means, just if you change this delta l it can tune the delta l value then depending on the delta l one arm will be keep on increasing the power sin function. When this bar port when it is maximum cross port you will be getting 0 that time and when you are getting cross port maximum bar port you will be getting 0. So, this delta l actually gives us a flexibility that if I design a unbalanced Mach-Zehnder Interferometer.

If I want to get inter power maybe here then delta l I can choose 0 and if I want the power to be distributed 50, 50 I can just add just delta l so, that I can get 5050. So, these delta l variations you can do actively you are inactively or passively that we will discuss later for the moment you see that the output ports for a single input ports bar port and cross port they are actually complimentary to each other one is a sin square function another digit cosine square function and total power remain constant they are distributed among 2 output ports.

So, now reverse case we consider suppose this is A i = 0 I am launching enter power input power at this port B i then this transfer matrix will convert like this. So, 0 A i = 0 B i 0. So, in that case output port A naught A naught i will be getting here that is cosine function and B naught I will be getting minus sin function. So, now again with respect to this one I can consider this is cross port and this is bar port.

So, in that case cross port I am getting here cosine function and bar port I am getting sin function. So previous case it is sin square cosine bar and cross now cross. Same thing it will be given I am just writing cross first cosine function sin function. So, they are actually simple straight forward. So, once you know the transform matrix you know what is the power you will be getting and depending on the delta l I can find out what is the value now, let us concentrated here suppose, I want to get cross port maximum P i and bar port 0.

So, that beta delta l / beta delta l if I just consider 2 m pi. Then to cancel that means integer multiples of pi that means, integer multiples of pi m = m can be here m can be 0, 1, 2, 3, 4 so, on so, if beta delta l equal to 2 m pi then I am ensure that cross port will be maximum and bar port will be minimum 0 because beta delta l 2 m pi means sin pi / 2. So, you will be getting 0 here and this function will be getting either pi 2 pi 3 pi 4 pi and so on if you can continue then you will be getting maximum here.

And at the same time, you will be getting minimum here. So, if this is the condition come if somehow this delta l times whatever the path length the difference if you introduce in the 2 arms multiplied by beta delta l if that is exactly matching to 2 pi times integer values then you can answer that power entire power whatever you are considering here that will be going to your cross port and other power will be in the bar port.

If it is somehow beta delta suppose beta delta l equal to you, we are considering something 2 n + 1 pi / 2. So, if this type of condition is fulfilled, then you are ensuring that power will be always in the bar port not in the cross port. So, if you fulfil this one then powering the bar port if you fulfil this one then you get the power or the cross port. This is the 2 extreme cases.

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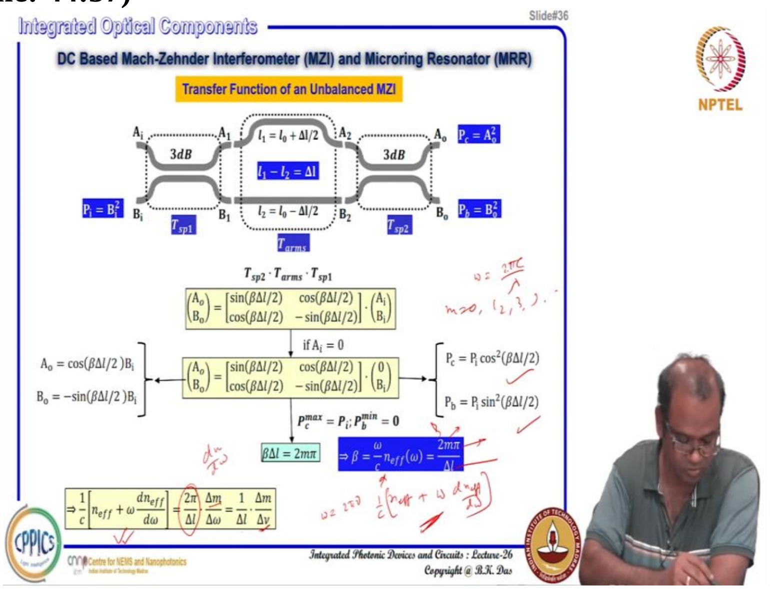

Now you see this beta you go back to our waveguide analysis, beta propagation constant beta you know beta can be defined as omega / c a n effective, effective index of the guided mode and that can be written also to pi / lambda times n effective and this n effective can be a function omega n effective can be a function of lambda for a given wave structure show I have just written this one beta we know this one.

So, beta equal to this one and now, here are beta equal to I can say 2 m pi / delta l. So, now I have omega by I can write that omega / c any n effective omega = 2 m pi / delta l m is integer value I know constant and n effective is a function of frequency I know. So, now what we do we do this one derivative with respect to omega because I want to see what happens to other frequency.

This is the transfer matrix I am getting for a certain frequency beta is a function of a frequency; if I tune my lambda because I know I want to use this type of device per communication for purpose optical interconnect purpose. So, per optical interconnect I may have to use the different wavelengths I may have to use different channels WDM wavelength division multiplex systems etcetera.

So, if you just tune omega what lambda omega and lambda relationship is 2 pi c / lambda c is the velocity of light. So, if I just do derivative what do you get 1 / c if I do omega derivative then it will be n effective and then if I want to put omega is a constant and d n effective / d omega I am just making derivative with respect to omega both side. So, if I determine omega then right side what is going to change delta l is fixed pi it is fixed 2 is fixed.

Still if you want to consider that means, this m or r must be changed that means m equal to we know that m equal to 0, 1, 2, 3, 4 so on. So, m must be changed if I just vary the frequency or make a derivative here in the left-hand side right hand side variation variable is m and other things are constant. So, just m is the variation but it is a quantized 1, 2, 3, 4 so on. So, in spite of that if I just do derivative left-hand side gives me this one that is what I derived here left-hand side.

And right-hand side 2 pi / delta l i just constant fixed and I just m I consider delta m because it is a step type change can happen so, I put instead of d n d omega I consider delta m delta omega now, you see the right-hand side you concentrate 2 pi delta omega delta omega you know 2 pi nu. So, I if I put delta mu 2 pi 2 pi cancel so, one over delta l delta m / delta nu.

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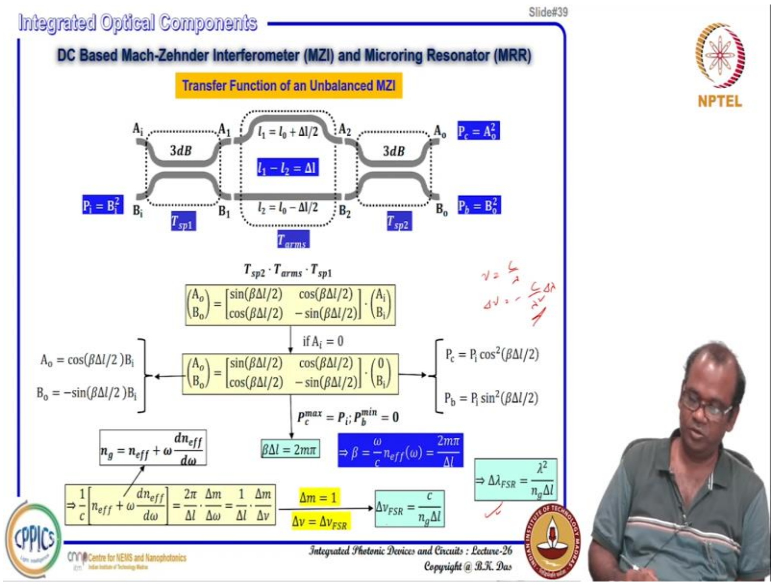

Now if I one to do one more interesting stuff this one this one if you just check carefully, we have derived earlier that is nothing but the group index n effective omega d n effective by d omega is the group index if you are considering different wavelength different frequencies then group index comes into picture. So, I just put here this one then what I consider if i put delta = 1 and corresponding delta nu value.

I define as a FSR I will come to that point delta nu i just defined delta nu FSR. So, 1 / delta nu efficient then I will be I can find out delta nu FSR = c by n g delta l that means, if I just change the order this m by m + 1 if I change then my frequency change to satisfy this condition that will be delta nu. So, that will be that can be can be calculated as a delta nu FSR is equal to this one we are considering delta nu FSR.

And this is in n g by c and delta l I have taken that side or delta nu FSR I take that side. So, I can just simplify it delta nu FSR = c by n g delta l. So, what is this FSR a FSR is actually very important term frequently used in photonics particularly integrated optics and photonic integrated circuit it is called free spectral range. So, why this free spectral range because it is the if I consider beta omega c / n effective = 2 m pi that means this is omega m.

This is satisfying for a particular integer value. Now, if I go for another integer value n + 1 I will get another solution for omega n + 1 that will be 2 times n + 1 pi. So, I will get another because m is integer I can run from 0, 1, 2, 3, 4 so on so, that is why I can get a certain frequency one test frequency where you can get P c maximum here. So, alternately if I just keep on changing frequency depending on the n value certain frequency.

Here you will be getting maximum and at the same time once you get maximum here P b will be minimum here. So, if you launch here and if you to new a lambda value then a particular lambda it matters it lambda or omega that satisfy this equation you will get maximum power here and nothing will be here now it is a same device it will change the frequency change the wavelength you can arrive to another particular wavelength in the next order when m = 1 n + 1 then that wavelength will appear here.

So, this type of device in principle you can actually demultiplex if you have a 2 lambda 1 2 lambda value coming lambda 1 and lambda 2 you are just delta l such that 1 lambda can appear it is satisfying this one that means, say well omega / c n effective omega 1 c n effective = 2 m pi and another one I can write omega 2 / c and n effective that will be 2 n plus that is actual 2 n + 1 pi / 2.

So, this thing if you just consider so, omega 1 and omega 2 if we are adjusted such that this value it is coming all integer multiple of 2 pi / 2 and first frequency give integer multiple of 2 pi then omega 1 will appear here and omega 2 will be appear here omega n means 2 pi c / lambda 1 will appear here and 2 pi c / lambda 2 so, 2 wavelength you can separate them. So, this unbalanced Mech-Zehnder interferometer it is a kind of device which you can actually design for wavelength d multiplexing.

I will discuss that in the next lecture how to design that D multiplexers and sometimes it is called interleaver structures that means every alternate wavelength in a WD communication system you can actually separate into 2 branches that is actually interleaver. But the thing is that I can design that free spectral range so, called delta nu FSR that if one frequency is now it is appearing in cross port then what will be the next frequency, I can say that if corresponding nu 1 = c / lambda 1.

For example, you are considering and the next wavelength will be nu next wavelength next frequency may be nu 1 + delta nu FSR. So, that frequency again it will appear in the cross port. So, that is actually free spectral range. So, where you are getting maximum successive values frequency maximum if you just successive maximum value you are getting corresponding wavelength you just subtract that is actually called free spectral range for the balanced Mach-zehnder interferometer.

Now, in terms of this one if delta nu = c / lambda so, delta nu = - c / lambda square delta lambda. So, delta nu and if you just replace this one you can find out in terms of delta lambda FSR respect or wavelength in terms of wavelength we can just derive.

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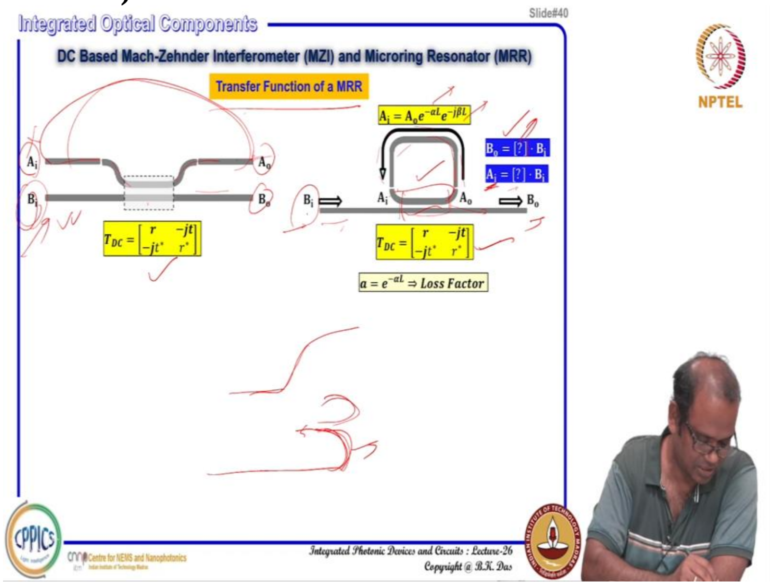

Now, now, the last section that is actually transfer function of a Microring resonator because all this balanced Mach-Zehnder Interferometer and unbalanced Mach-Zehnder Interferometer and they are actually designed with the help of directional coupler, same directional coupler if you just a little bit engineer. You can actually get a very nice different new device that is called Microring resonator.

We would like to introduce this Microring resonator now and we will try to find out what is your transfer function. So, you know this is a directional coupler simple directional coupler just 2 input A i B i and A naught B naught and this first waveguide we just bring closure to the second waveguide to interact to exchange power of a certain length and we can again separate decouple them and this transfer matrix we can just define like this.

r minus j t, j t r and t all they are depending on the what is the coupling strength overlap of the face on their length and their gap all that type of thing we discussed in the previous lecture we know that. Now what do you do you see I have just structured our directional coupler in this form what do you do whatever output you launch B 1 here for example, here you are launching and nothing you give here.

Instead what you do you know that something is going here instead of taking output here you give a feedback directly here? So, the output of the cross port you bring back to give a feedback as the input or something like that positive feedback you are giving in a system in electronic circuit you know positive feedback is just to give output a tap fraction of output and give it to the input.

So, that positive feedback normally you know in electronics that is the kind of on oscillator design you do normally electronic oscillator. So, here also we can think of an optical oscillator by giving a positive feedback. So, this type of structure I can just represent in this form I have this B i input here and then you know this is a directional coupler transfer matrices and whatever coming here you get positive feedback.

And then it causes here and while crossing also a little bit it can come back here and again it can circulate here. So, this type of structure is called microring resonator why it is resonator it will be clear in course of time, but this is actually you see this a simple modification of directional coupler output of the one of the output port you are just giving feedback positive feedback to the one of the input ports.

So, basically directional coupler it is constructed in this form called is microring resonator and we know this one and now, we have to consider one more thing so, far we have been considering that the resonator coupler Mach Zehnder interferometer they are lossless but ultimately waveguide will have some certain kinds of losses as they propagate. So, while propagation some losses will be there that loss we say that you just get derived everything lossless.

And at the end you can just estimate what is the link budget loss budget loss for the entire structure that is good enough, but in this type of so, when we are given positive feedback how much positive feedback you are giving that you must know first because there will be some losses is coming there while propagating because whenever you are bending the waveguide structure you know there will be some kind of if it is if you have a waveguide state waveguide and if you just slowly it is going like this smoothly then almost no loss.

But if you just want to make a quick sharpening, quick bend of a structure. So, in the bend region normally you have you can assume that there is a strong perturbation because of the bending this perturbation and that perturbation actually causes some kind of loss of orthogonality condition. So, power accidents can take place guided mode can radiate power into leaky modes.

So, some losses will be there. So, if it fundamental mode up launching here and then you sharply bend so, after that whatever you are getting here a little bit in this region it can happen that something will be lost apart from your defects simulated loss just bend in this loss will be there that cannot be ignored whenever you are making a very tight bending, just binding ring regenerator type.

So, you can assume that you are annotated this is B i and this is A i getting a feedback and B and B naught A naught when coming back to the back to the directional coupler I say input A i so, you know that you have a loss factor e to the power - alpha l and phase this l is consider as a round trip total length of the perimeter of the ring. So, that is going like that. So, A i and A naught I can get any relationships or certain kinds of relationship.

So, in this type of structure, if you just carefully notice that we have one input and one output in a B i is the input and B naught is the output. So, our intention is that what are the value in terms of B i. Can I find a transfer matrix here like this which relates B i to B naught and similarly, I would like to know in this case, what is insight A i. What is their insight in steady state? You are launching light after a certain time what is the A i there in terms of B i if I know B i.

Can I estimate what is the value of A i can I make another transformative like this. So, that is our objective it is very straightforward if you know directional coupler transfer matrices straight forward you can just derive very quickly, I will just discuss that.

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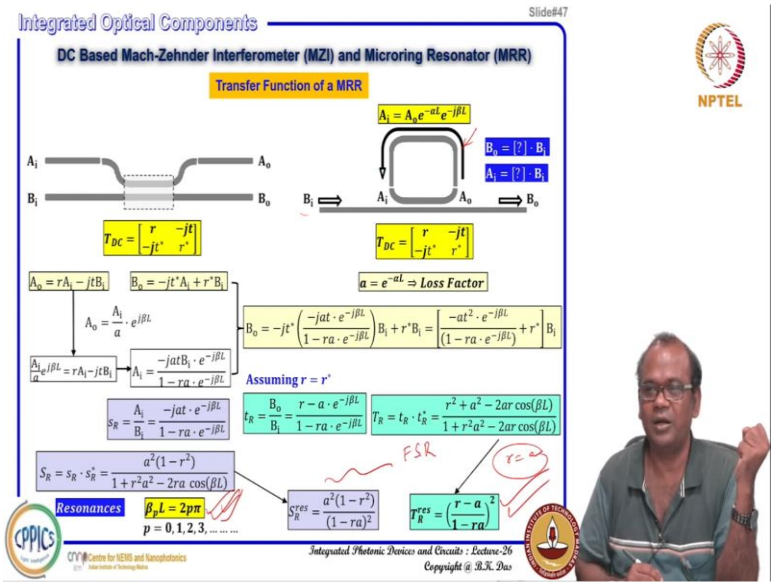

Let us consider this thing. Now, you have this A naught and B naught right now simple standard directional coupler I can write A naught and B naught how is that relationship from here A naught is equal to A naught will be r A i this will contribute r A i and something is contributing here that will be – j t B i. So, that is A naught i I have written here. Similarly, B naught will be r star B i and whatever coming here - j t star A i study that comes from this transfer matrix.

Straight forward 2 equations I am getting and I am assuming this e to the power - alpha l as a that is called loss factor we will call that as loss factors have been what if I take a ratio a not A i this is the loss factor is there other factor is phase. Phase factor that normally do not contribute it does not contribute to the power or intensity. So, I get these 2 equations, let us move on.

Now you will see A naught = A i e to the power j beta l from here if you see A naught = A i e to the power j beta l - j beta l that side will go A naught is equal to and e to the power - alpha l we consider A. So, A I by a e to the power j beta l that is what we have written just here I put a A o in terms of A i if I just substitute, I put this one here. So, this equation comes like this r A i – j t B i. Now, this one from here if you see these 2 this equation relates A i to B i.

A i to B i that is what our intense on A i to B i. So, I can just write A i = I take this one I represent this one just eliminates A i in terms of B i and A beta l are all these are known, we consider that r, t, r star, t star and beta they are known from the waveguide characteristics they have some coupler characteristic. So, I relate A i to B i here from this equation that is true straightforward.

Now, next thing is that use this equation this equation and this equation if an element of A i if I substitute A i here then I can represent B naught and B i, I can relate B naught to B i that is what B naught to B i that is our intention whatever I am getting B naught to B i, so just comparing this 2 I can relate B naught to B i just eliminate A i substitute this A i value here. So, by substituting this one I get this one, this is the ultimate results.

So, B naught equal to this one maybe so, this is that within bracket whatever you were I was looking for that is nothing but you are getting this expression. Now, next thing is that I think without any loss of generality, I consider r = r star normally you know r, r star actually most of the directional coupler we can design we get that that reflection coefficient normally does not have any phase it is can be real.

If we consider then this r star I can put r here right and then we can say that t R = B naught B i B i B naught / B i is this one. So, this one if I put r is there already so, you just simplify you are getting this expression. So, this is your, you are looking for this structure, this what is the transfer. So, one to one device and this is your transfer function amplitude transfer function by the way, because so far, we are discussing amplitude transfer.

So, if you want to know if you know just the reflection coefficient reflective we can find out just take a complex conjugate transfer function we consider transfer function of the ring originator R stands for ring regenerator ring and the complex conjugate multiply you get this one that means, this is actually the power transfer characteristics. So, if you have if you know this value beta value l value r value A value.

Then if you are just launching 1 watt you just put down all these values then you know how much power will be transmitted to the output here. So, this is amplitude transfer function that will be useful for simulating photonic integrated circuits using amplitude transfer function and if you want to use that standalone ring resonator if I want to know what is the input power and what is the output power what is the change as a function of beta, beta means is a function of a lambda.

You can find out what is the output as a function of lambda straight forward you can get similarly if I just pass it from this one, I can have A i / B i that relates how much it and in the steady state how much you are getting all the time power here inside the cavity A i or A naught you can do that also I am just considering A i so, in steady state all the time how much field amplitude will be there just be the directional coupler here.

So, that relation I get this one let us take A i divided by B i and I say that transfer matrix amplitude transfer matrix is S R, S R means 2 storage purpose I am just considering stored inside the cavity how much inside the ring S R and here transmission t R t stands for transmission A stands for storage inside the cavity amplitude storage factor we can say transmission function like this.

And this one again we can consider the energy to storage enhancement inside the cavity. So, you take complex conjugate multiply then you get this one. So, this is interesting you just look this one carefully. A is the last factor that means one round tip one round tip how much it is losing to the outside world R is the directional coupler property how much it is coupling self-coupling.

And you know that r square + t square basically this one also he was just considered lossless directional coupler will consider anything additional losses there that you can consider along with this e to the power - alpha l file that what A you can consider. So, A is the round-trip loss factor we call A is the round trip loss factor all these unknown now if we inspect this is it only variable beta L beta L means omega by c n effective L.

So, this omega means, we can consider 2 pi c by lambda c n effective L c c cancel to the 2 pi lambda and n effective L something. So, either omega or lambda if I tune I can change this beta L value or I can change any 50 waveguide effective index or I can change L let us consider that waveguide length the length perimeter and nature of the waveguide that is fixed. Now in our hand if I tune the frequency lambda then this beta L value will be changing.

So, once it is changing then this value inside the cavity the power storage inside the cavity energy storage inside the cavity that is going to be changed as a function of beta L. So, if this sensitive A cos B L as a function on a beta it is cosine concern. So, cosine function you have

+ maximum is + 1 minimum is - 1. So, when it is + 1 that means denominator will be minimum when cost beta L is + 1.

Then denominator is minimum that means storage power will be more inside that inside the cavity A side will be higher. So, when beta L = beta L = 2 n pi for example, n is the integer if beta you can tune because of the frequency tuner lambda tuning. So, for if you can change your wavelength or frequency then you can see that this cos beta L terms as a function of frequency or wavelength it is changing as a cosine function.

It can vary + 1 to - 1 so that means when it is + 1 you can say that that time inside that particular frequency that particular wavelength inside the energy storage will be more maximum because denominator is a - sign here that particular condition is called resonance condition. So, we call it as a resonance when beta p L = 2 p pi p is the integer instead of m we are writing p p can be 0, 1, 2, 3, 4 and so on.

So, if beta L pi = 2 p pi then this value will become 1 1 means denominator is minimum denominator minimum means ASR will be maximum. So, that is the resonance condition and that resonance condition this will be one right then what we will be getting A square 1 - r square and below denominator it will be a square 1 - r square and this will be 1 + r square a square – 2 a that is a maximum value. That is the resonance condition.

So at resonance so, you can find a wavelength if you have a structure you can find a wavelength or frequency then that time your energy will be highest inside the cavity and that is the resonance condition and that at that resonance condition whatever the value will be, inside that can be decided using this expression. If this re use is a lossless that means A will be equal to 1.

If a = 1 then what we were getting 1 - r square divided by 1 - r whole square that will be the fraction of energy will be stored inside the cavity and that is the maximum possible energy because we have satisfied this equation if this equation is not satisfied then inside energy will be started dropping. So, when it is started dropping that will be actually appearing here because we are considering lossless case know where it is lasting.

If some loss is there that will be lost also that will has to be counted, but other than that, entire power will be coming out there. So, this is the resonance condition where how we define the resonance condition, the resonance condition is that condition where the particular input wavelength or light frequency that is in the transmission that will be minimum. So, I know that transmission characteristics this one is defined by this one.

Earlier we have the right so, I know that this is the resonance condition beta L = 2 m pi I can put here also beta L = 2 p pi that means, this will become 1 this will become 1 so, cosine function at resonance function I can say that r square denominator will be r square + a square – 2ar a numerator will be 1 + r square a square – 2ar here. So, denominator numerator and denominator we know what to do the value at resonance.

I just write down that one. So, at resonance how much fraction of power will be stored and how much fraction of power will be transmitted I know that I can actually transmit some if you can design this ring resonator such that r = a that means rounded loss is exactly equal to self-coupling coefficient r = a. So, in that case the transmission will be completely 0 and here you will be getting r = a.

That means you will be getting a square so, that means whatever in the transmission you would get 0 whatever power you are launching here something will be lost and something will be store. So, that particular situation is called critical coupling if it is not critical coupling your transmission will be minimum but your storage will be maximum when you will fulfil this condition that is resonance condition.

So, now we know that a ring originator can be useful to store energy inside you can use energy storage for a certain wavelength certain frequency and that storage can be available also in according to your demand that will decide that discuss that later. But still important device and again you know this expression it is similar like that, if you change frequency and another wavelength also because p = 1 you will get one concept p =2 another condition p.

So, p is the integer so, it is not only one frequency a series of frequency discrete frequency that will be resonant to this cavity resonator. So, I can find out the separation between 2 successive resonant frequencies that will be again called a free spectral range with what we have discussed while discussing the unbalanced mentioned the entire perimeter so, we can process it same way.

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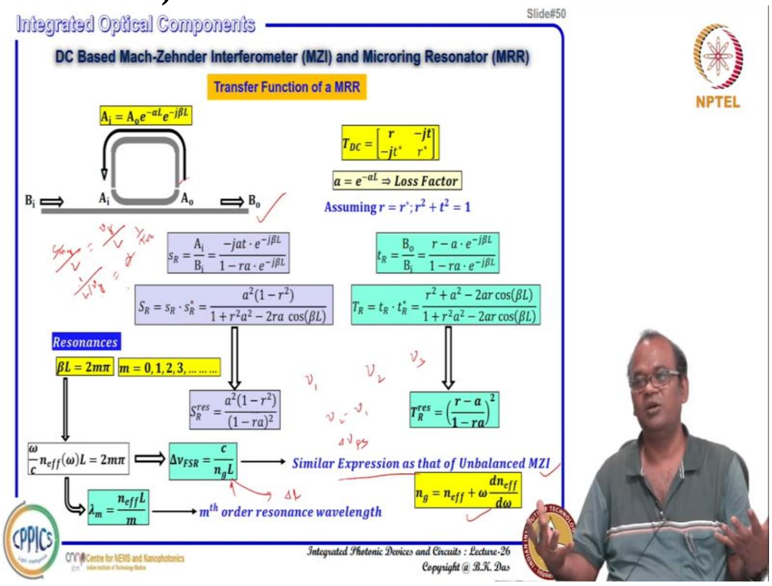

I have just written down everything in summary. So, this is the transfer matrix for the directional coupler loss factor and we have assumed r = r star r squared + t squared = 1 then amplitude storage fraction that is defined by this one S R that is A i by B i A i that ratio with respect to B i how much it is being enhanced inside the ring and how much you are getting the output B naught B i Bi whatever B naught you are getting whatever you were launched.

And whatever you were getting that ratio I am getting and then if you just consider how much energy or power stored inside the cavity and how much power being transfer on a power being transmitted that can be derived like that and at resonance that is the condition beta L equal to here I have just again written m it can be p or m that should be fine an integer as long as you are putting integer that is fine.

So, you are getting this expression at resonance at resonance transmission will be this one that is so far we have derived now if you see one thing you can consider this beta again I am writing omega by c n effective L 2 m pi now omega 2 pi c by lambda if you consider then I can find out lambda m. So, these are the resonance mth rth our resonance wavelength. So, if if I say that lambda A m j n ineffective or just research that I get lambda = 1550 nanometer.

And maybe m = 100 m will be 100 and effectively if you put support L = 100 micron n effective equal to say 2.8 and L = 100 micron into the 10 to the power - 6 and m I can find out lambda m putting 1550 nanometer 10 to the power - 9 metre. So, I can find out what is the order of resonance but this one I will be getting then this is mth rth resonance. So, now if it is little bit f i change the 1550 to something else.

I can find another integer which will be again resonance. So, that easily you can just find out. So, same way whatever I have just considered to find the FSR free spectral range there actually in case of unbalanced Mach Zehnder Interferometer that there is no region resonance there rather we are trying where is the maximum I am getting at output. But here resonance means the wavelength where the energy storage will be maximum inside the cavity inside the ring.

And those frequency that means, difference between 2 successive frequency if one frequency resonant and adding nu 1 another one is nu 2 this m is equal to certain order and another is nu 3. So, it is actually nu 2 - nu 1 that that will be considered as a delta nu called free spectral range free spectral range that can be considered also c by n g L. So, here L is a perimeter length and in case of unbalanced Mach Zehnder Interferometer.

This L we are writing like a delta L that how much difference between the arms that we are using. So, it is basically a similar expression as that of an unbalanced Mach Zehnder Interferometer perimeter, but unbalanced Mach Zehnder Interferometer the power you are getting maximum in one port the other port is the minimum that time when you are getting minimum in one port other port is maximum.

But in this case when you are getting the one particular frequency maximum that means your output is port is this one that maximum value is actually stored inside the ring it is not taken out it is not given out with other ports because other ports actually in principle that is given as a feedback. So, this is a very very important device and again this n g group index depending on the group index i can find out and if you carefully look this expression c by n g, c by n g times L c by n g may be considered as a group velocity by L.

And you know, this can be written as 1 over L by v g L is a perimeter length and v g is the group velocity. So, perimeter length group velocity perimeter length divided by group velocity that means, round trip travel time. So, that round trip travel time we can consider one over tau t round trip travel time of a of an information of any masses inside the ring. So, if you know that round trip travel time if it is known.

Then you can find out free spectral range or if you know the free spectral range, you know any information on how much time it takes to travel one round trip inside the ring. So, this information will be using for further advanced device DJI will just give some kind of hint here. So, all these background things whatever we have discussed here that is very important for optical signal processor design.

Because directional coupler Mach-Zehnder Interferometer microring resonator, these are the 3 passive components we use with different variants in their design for demonstrating different type of functionalities in photonic integrated circuits. I stopped here today.