Integrated Photonics Devices and Circuits
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Lecture - 20
Optical Waveguides: Theory and Design: Coupled Mode Theory Contd.,

Hello everyone, today lecture will continue coupled mode theory specifically, I would like to establish a generic form of couple mode equations, when there is some kind of small perturbations or they are in the waveguide.

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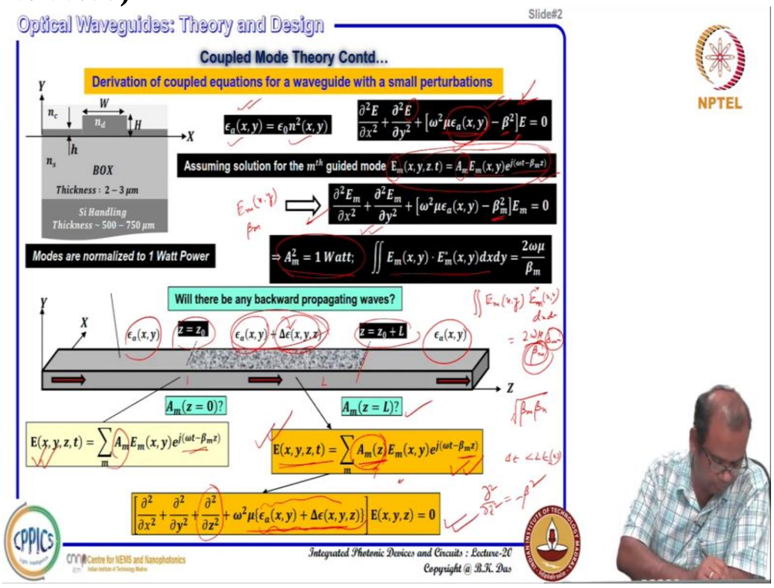

So, we discussed also already that if there is no perturbation that means, simple waveguide cross section is defined by dielectric constant epsilon a x, y and it can be represented in terms of refractive index profile x, y cross x and it is just representative waveguides shown here silicon on insulator waveguide and then for this scalar wave equation where we have used E is the dominant electric field component and omega square mu epsilon a this is comes here and beta square for example, it is either Eigen value or the propagation constant.

That has to be solved from the Eigen mode solver. So, if you just see this differential equation, you have this form of solutions you can assume that means, electric field is a scalar I am not considering vector because it is only oscillating in a certain direction. So, dominant electric field I am considering x, y, z, t so you will have some amplitude A m and profile E m for the mth mode the electrical field distribution x, y.

And the phase part omega t - beta m z, if you just insert this equation here E then you get the differential equation for mth Eigen mode. So, you can solve this equation that is how you can get from here E m x, y series of distributions, you will be find it series of solutions you will be getting beta m that is what is supposed to have and each of these mode if you are just getting you can normalise that 1 with a normalisation constant A m square to 1 watt.

And then according to the orthogonality condition, we have E m star then it will be 2 omega mu by beta m in principle it should be something like that, we know that E m x, y and E m star E n star x, y dx dy it should be equal to 2 omega mu by beta m delta m n so, that is sometimes instead of beta m we can just write square root of beta m beta n because where m = n.

So, ultimately it will become beta m otherwise m not equal n that should be equal to 0 that is the Kronecker delta function let us consider that this is the waveguide 3 dimensional view and standard input output side this is a uniform waveguide defined by epsilon a x y x, y distribution is there entire along the Z direction it has no change in refractive index even in the cladding or in the core.

And we assume that there is a perturbation started at z = z 0, 2 z = z 0 + L, L length from this to this it is L length. So, in this L length you have along with that you have some dielectric perturbation which can be function of x, y and z. Now, we know that if it is the waveguide can support a number of modes solutions are there. So, total field can be just we can establish we can just find total field in the waveguide by summing all the field along with their phase.

So, that is if there are 10 modes so 10 modes super position that will be the total field in the waveguide we can just consider. Now if it is in the perturb if there is a perturbation is there. So, in that case how your waveguide modes will be modified this perturbation obviously, this delta epsilon, whenever I am writing that should be very, very less than original refractive index profile of the waveguide.

That is a very small perturbation so, for this small perturbation, what you could do you can say that you are just breaking the orthogonality condition that means, indeed the modes propagating inside the waveguide they are no more orthogonal they can start interacting each other and they can exchange energy also depending on their some kind of certain conditions.

So, if that is happening, so, what we could say that in this part of the region the modified total electric field.

The total electric field can be written as somehow it is amplitude here this amplitude when there was no perturbation that is you know z dependent amplitude is there, now, you say every mode mth mode method will have z evolution as you function of z it will have some kind of evolution that means the field amplitude that can vary so, if you all the modes if it is starting with 1 watt and in the part of the region they will exchange power.

And they can be different after travelling when the z = z 0 + L reaching that time you can see some different value it will not be no more it will not be any more 1 watt. So, to deal with this type of equation, this is the assumption we have considered that the amplitude will be varying they will be interacting energy will be exchanging each other. So, this is the thing happening, we can assume that this is the solution of the same differential equation.

Whatever we have considered for Eigen mode solution only thing is that the epsilon a will be replaced by epsilon a + delta epsilon x. So, that is what we have replaced here in this equation, I could write this thing and I do not have any z dependent well defined function like beta m z instead of that I have beta m z is there as well as an additional function in A m z is there that is the reason I cannot write del del z = - beta square.

We cannot write instead what we can write this is the epsilon modified epsilon a and instead of beta square I just retain del z 2 along with the del del 2, x 2, del del 2, del y 2. So, that means, I can assume that this when the dielectric constant is modified then I cannot write just simplify beta square instead of that that del 2, del del z 2 operator I include and I can assume that the solutions will be modified electric field solutions which will be superposition of all the modes.

Where the amplitudes will be evolving as a function object that is the assumption, this is the background. So, if we have a perturbation then the mathematical framework all the governing equation are no set now, we need to solve this equation how to solve that?

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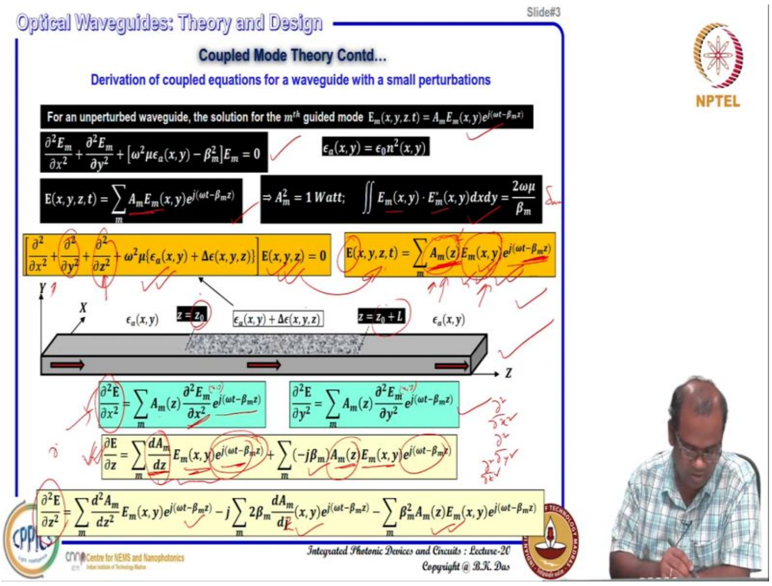

So, I just retain whatever I have discussed so far in this black marked and white fonts all this thing we have whatever discussed in the previous slide that I have retained here. So, here this m and this m if it is different, it would be written a delta m n something like that otherwise they are say mode that means that is how we normalise and we retain that part of that equation.

Where equation and part of solution z dependent function additional component in the solutions we have assumed without that we have simple solution earlier what is there. So, this is the perturbation these things I written this figure also I written z = z 0 to z = z 0 + L, L length something is there. So, to solve this equation, I have this solution. So, what I have to do I have to find the del del 2 del x 2 of E what is the value this one of E.

What is the value this one of these this means, this is the solution I have considered and this 1 whatever the value then that value I can main thing is that to solve this differential equation, this solution you assumed that solution you just insert there and try to see what is the outcome if you can get some simplification or some constant like A m z if you can solve or not our intention is that if we can solve A m z.

Then we know how mth mode is evolved in a part of the region. So, ultimate goal is to find out the value of A m z as a function of z how the amplitude is evolving, whether it is reducing or it is increasing, if mth mode it is actually gaining some power energy from other modes, then it will be increasing and if it is losing, then it will be reducing as a function of time function of z and it will be losing to other modes.

So, that is the whole story so, if I just consider E this one and then try to find out del 2 E del x 2 so, del E 2 del x 2 if you do x y dependent function only this 1 otherwise A m z that will be retained. So, I write A m z del 2 E m meaning this is actually x y, then del x 2 and phase part dependent function time dependent function I am taking partial derivative with respect to x. So similarly this is the only y dependent function is there.

So del 2 E del y 2 I can write same way so only thing is that instead of del 2 the E m x y by del y 2 here instead of that it was the del x 2. Now, we have to find this factor this pattern del 2 del z 2. So, if I try to do del 2 del z 2 on this if I just operate on this thing, then we have 2 z dependent function A m z is there that is what we have to solve and here also exponents delta z is there.

So 2 variable is the z dependent function is there so, we have to do for del derivative for this 2 z dependent function. So, first of all I cannot just directly derive like del 2 E del x 2 similar way we cannot do we just try to make del del z first. So, del del z for first means I just make a derivative first term written, written this second function and then second term I will be making derivative with respect to that and other thing will be written A m z this one written and with respect to this one.

We have a partial derivative z - A beta z to retain and then now, you have 2 terms, this is also z dependent term in each term there is 2 variables z dependent function here, this also the z dependent function, this is the function. So, here also this once more if I del 2 d del 2 z is here, then I will be getting 2 terms here, 2 terms here you can do by yourself, I am just writing directly the exact formulation here.

So, here if you see, this is the value if you are just making a derivative once more derivative d 2 A m by d z 2 A m x y this factor and then this factor z 2 eta dm so, this would be corrected this would be z not d z. And then one more function will be like this beta m square like this. So, I have now this del 2 del x 2 term whatever the value I have now is there and I have also del 2 del y 2 term they are also and del 2 del z 2 term also I have all this term.

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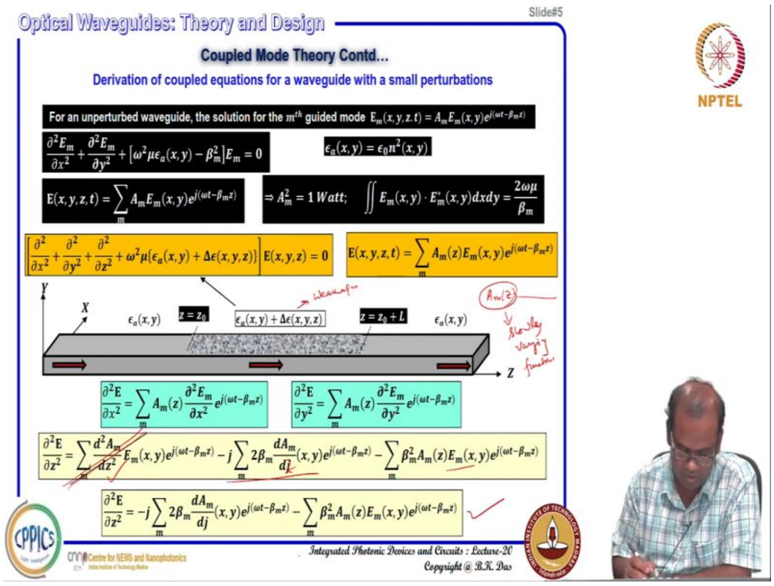

So what I will do next I will be just inserting this into this function here. So, whatever the things I just written down all these 3 terms in clean way 3 this term whatever the thing here and this term whatever the thing here and del del z 2 term I operated on this then again here 3 terms first term second term and other 2 terms will be similar that is why it is added by 2 beta m - A beta 2 beta m and this z also.

This is should be corrected that del del z. So, now what is the next thing? Next thing is that this is one thing I have approximated, we have considered this A m z since it is a weak perturbation this is a weak perturbation A m z it is slowly varying function as a function of z it is not that suddenly check energy will be changing. So, perturbation is very small so, if any energy exchange is happening that is in a slow process as a function of z it just slowly varying things. So in this term, you have d A m d z as well as this one is there.

So, in that case second order derivative with respect to this one I can just ignore. So, this is the approximation, we did that we can assume that the second order variation can be ignored, if any abrupt change is there that has to be kept outside the scope of this process this derivations. So, I just consider slowly varying function, if it is slowly heading varying function then this can be cancelled. So, ultimately you will have this one and this one del 2 del z like this.

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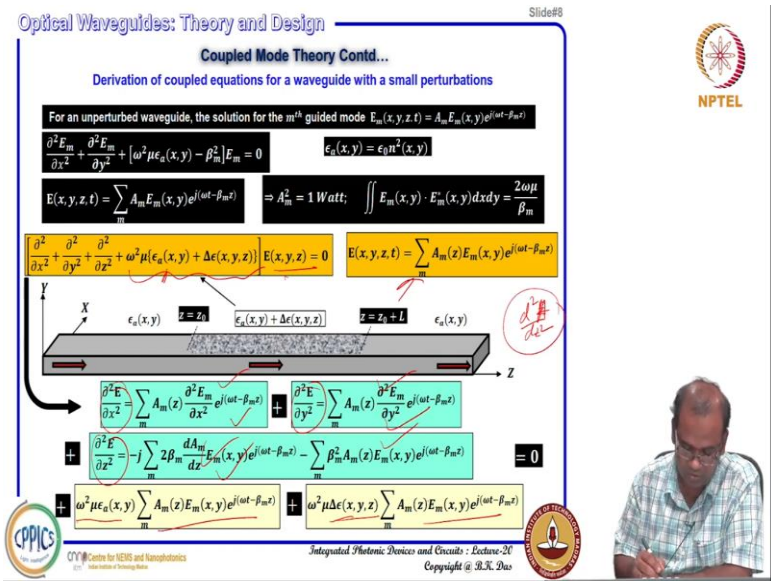

So, we can just now clean up that, this is del 2 E del x 2, del 2 E del y 2, del 2 E z 2 this has a 2 terms. So, what is next, next is you just insert all these 3 terms here, here, here, multiplied by E here, you just inserting in this equation. So, once you insert, then what you return back you get first term whatever you are getting that should be added with these should be added with this.

So this one means you have basically this term adding and this term adding and this term you adding, and then rest of the part this thing, this thing, we have just omega squared mu epsilon A times this one, this one means this pool function I have just added and plus delta u omega squared mu delta epsilon x y z full term I am just writing down. So, that is good. So far, I have to use this term this term and this term as well as this term.

Because this is the second order derivative with respect to z, 2 terms are there after ignoring d 2 E by z 2 d 2 A dz 2 this time we ignore, then we get this thing. So, after simplification after inserting and we; are doing a little bit of simplification.

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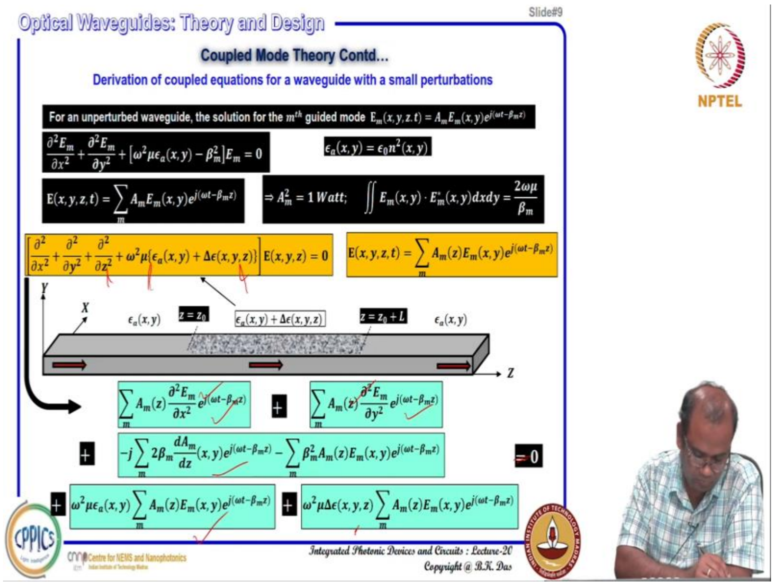

So, what you get in the first term you are getting this one and this one and from the set part term first term, second term, third term is this one. And this is 4th term and this is your 5th term. So, all these you are writing that you are adding and that should be equal to 0 here we are writing.

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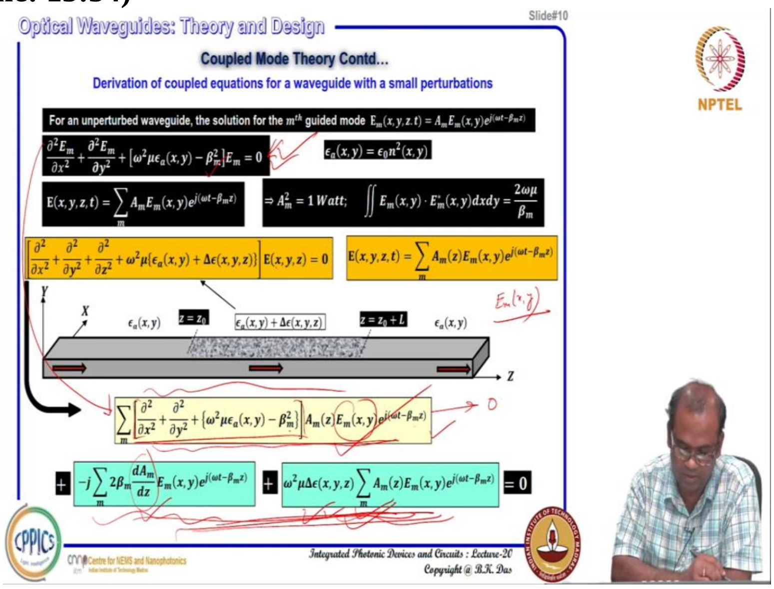

So, now, we are almost done now, you see we can simplify here the first term I can write this is one term is there. So, if you just simplify like this, then A m z E m x y this one and sum over but this one, one term, and another term will be coming with this one and this one is the third term I can rearrange to write down this way. One term and the second term and third term we can add so second term and third term if you see second term you have this evolution of z A m a mth mode how it will be evolved.

And third term you have this perturbation and first term if you look carefully, it is appearing like this equation. So when the waveguide is unperturbed, that because we consider unperturbed waveguide when it is unperturbed waveguide the mode field distribution solution is this one in perturbed waveguide also we consider same mode field distribution apart from that you have some kind of slowly evolution A m z is there. So, if I just use this one here, here you see this one E m x y.

If we just multiply here that means you are getting this expression that means this one equal to 0 this term, I can directly put equal to 0 exactly I just try to use this expression instead of E m this one is the function E m x y E to the z omega by t it will be there so, I can just multiply this one and this one that will be equal to 0 according to this expression. So, now, I have left over that this term is 0.

Then now 2 terms are there in the left hand side. So in this equation, finally, we left over with these 2 terms. So, now, you see this is the equation we get, because in this equation, if you see a minus sign is there and then a plus sign and this one I take to the right hand side.

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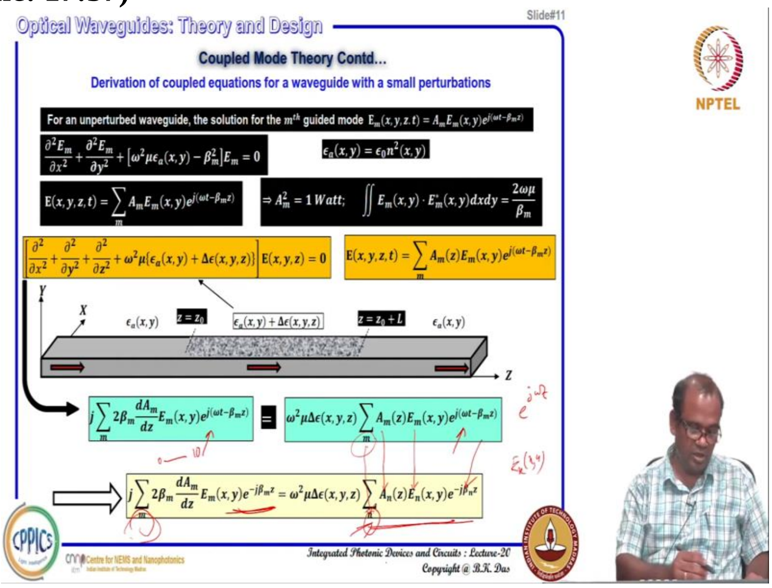

So, if I take to the right hand side what we get, now you see we get this equation, this is equal to this and what we did here, because both side you have j omega t it is also j omega t both side that e to the power j omega t term I factor. So, what I get left hand side I get these one e to the power - j beta e m z and right hand side also m and I am writing a different index here for the right hand side.

It does not matter m also here it is varying from say if it is 10 modes are there m you are varying 0 to 10 here also you can put independently you can put in m 0 to 10 then it is also it will give same value, because left side and right hand side function is different, only thing is that this index runs for all the modes, here also this index runs for all the modes, because these 2 terms are identical we could write m both side that is fine.

But if we create them independently, then we can see any other certain mode for example, say e = k x y if I operate, if I try to see with respect to u k x y, what is happening in both terms to compare that, I just use the separate index here, this is m and this is n I think without any loss of generality, this is actually correct mathematically both sides instead of writing m here I just converted n similarly m is converted into m E m is converted into E m and beta n converted into beta n. So, that is different more different types of solutions I can just think of.

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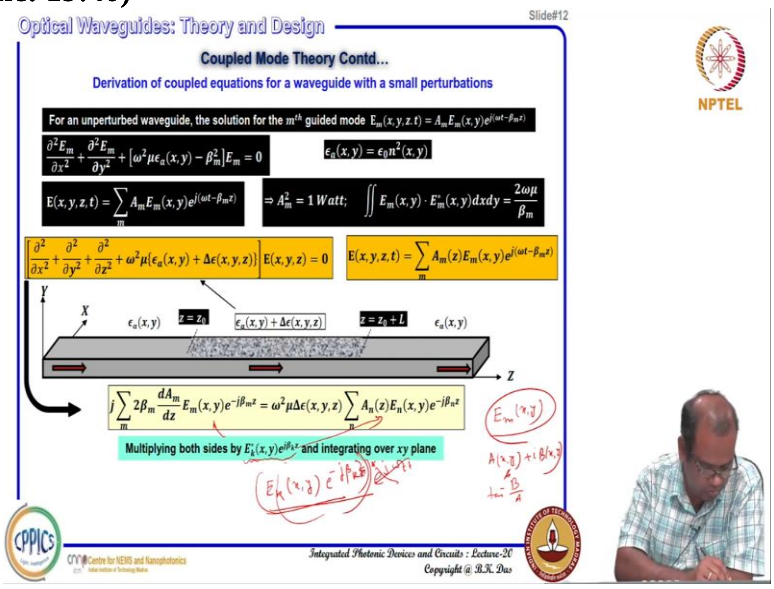

So, far so good now, what do we do, here you see this is the expression we have what we are trying to do, we just multiply E k star it basically what you are multiplying E k x y E to the power - j beta k z. Obviously, you can also multiply it to the j omega t also, that is fine, because I am not used e to the power j omega t terms both side. So, there is also I do not need to consider time dependent but only space dependent part is there.

So, I want to use this one complex conjugate, I want to multiply both sides, I multiply this one this side, I multiply this one and this side and then integrating your x y plane. So, the region will be clear on why I am doing so, if I just multiply E k x y to the power star, that means E k star x y obviously, you should keep in mind that this field can be a complex value

E m x y, any field distribution that can be a complex that can be written as something like that A x y + I B x y something like that.

You can express also so that you can say that at any position, position to position phase can be also different that type of complex thing you can think of that particular coordinate system you can see whatever the angle tan B by A another position they are phase can be different. They can be different oscillating in the different ways that is that always happens also. We know that sometimes first order mode higher order mode, we can see that 1 point one phase another point maybe it is oscillating in with another phase. So that mathematically it is correct. So now if I do so what we get.

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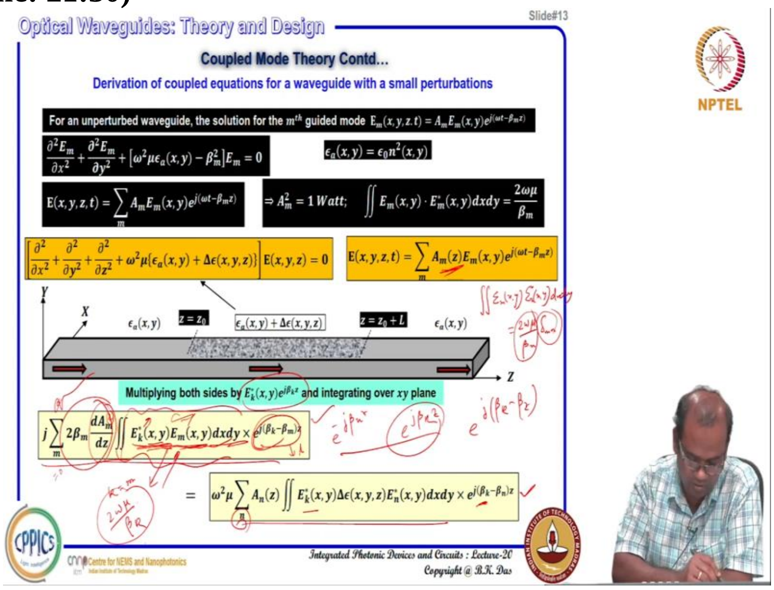

So, left hand side and right hand side I have written into 2 box, let us see here. So, first E k star x y E m x y dx dy and I have multiplied E to the power j beta k z, and originally it was e to the power - j beta m j z was there. So, if you multiply then you are getting e to the power j beta k - beta z and E k star I multiplied here E m this one and dx dy where you are integrating this one.

And obviously, this is slowly varying amplitude of every mode that is what we are writing here that derivative of that one we are having and both sides you have summation, this is summation here and here also, what we do we have here you have this term, you are multiplying E k E k star this side you are multiplying here E k star x y and you have right hand side delta epsilon xy is the perturbation term is there in the right hand side.

So, this equation if you try to interpret specifically that left hand side you see the evolution of a mth field and right hand side that is actually because of this perturbation, all other modes are disturbed and all other modes, how much it is contributing to the mth mode that is the left hand side. So, this side is the source and left hand side is the actually it is gaining at an mth mode how much it is evolving gaining that is actually the interpretation here.

So, by the way what I was talking I was talking that we are multiplying both sides k star and integrating and here I have used index n it was used. So, beta k multiplied e to power j beta k z multiplied and minus beta n we have written here now, look carefully. So, left hand side if you see, you have kth mode, that is fixed one particular mode and you are trying to see all the mode this m is running from 0 to some n whatever number of modes.

So, that means I am just running this field and taking multiplying that and integrating every mode what is the value we are getting that is I am summing along with that corresponding things will be there, but if you just concentrate only on this integration, you know what is this value, when k = m when k exactly equal to m, then only this factor will survive otherwise, because of the orthogonality condition.

We know that earlier we have discussed that E m x y E n star x y dx dy that should be equal to 2 omega mu by beta m delta mn. So, only when m = n then only this value will be 2 omega mu by beta if m not equal to m that will become 0. So, that means when I am running n from 0 to all modes. And this is the kth mode when m is reaching to k mode than only that particular term will be surviving all of the term because of the orthogonality condition.

This Kronecker delta function that will be 0. So, left hand side only I will be left over by only 1 term, not all the m terms you need to consider. That is the reason we multiply this one multiply this one and integrate so, that left hand side will become easier, it is simpler. So, now, what is that next? So, left hand side if you just do this one will be 2 omega mu by beta k for example. And this exponential term you know beta k = m means this should be 0 that will be 1 this factor will be 1.

So, this value will be 2 omega mu by beta k. So, this thing multiplied by 2 omega beta k and m will no more be summation because it is only one term it will be surviving.

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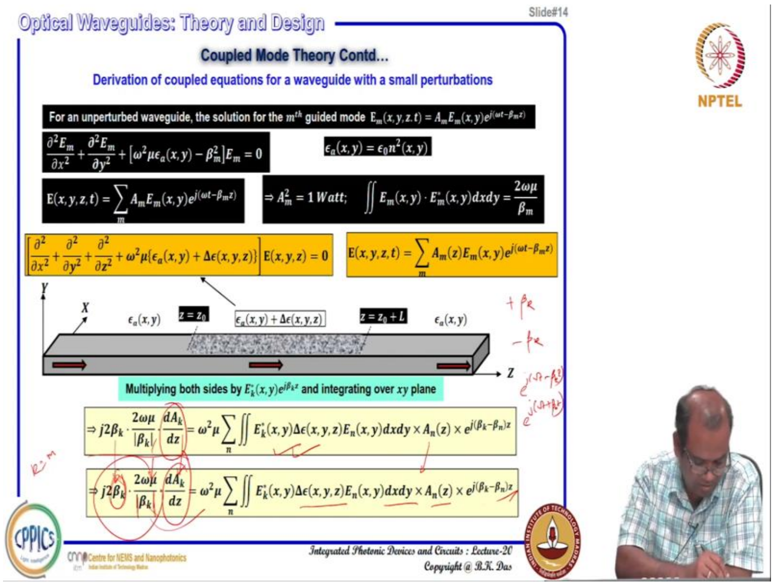

So, we have left hand side if you see in the this is the left hand side j to beta k to omega mu beta k that is coming out of the orthogonality condition power multiplied by 1 watt power of course, and dm by dz because k = m only that is surviving. So, I have just written m = k mth plus I am just putting k value and right hand side if you see what was the right hand side. So, right hand side if you see here also you have E k star E n star.

But this one is there in between you have a x, y, z function is there and n you are running over. So, in this case if you see because delta epsilon x, y, z is involved. So, it is not so, easy like here we could write to omega mu by beta k this integration that is orthogonality condition, but here orthogonality condition is breaking at the presence of delta epsilon here. So, you cannot just simply write like this whatever value comes you have to use that one.

But you have to run from into because it looks like that normally, if delta epsilon is 1 then it would be 2 omega mu by beta k beta n or how about delta k m in something like this, but once it is there as if you are considering because of this perturbation, you are operating this perturbation to the all modes and bit by bit how much overlapping it is coming here that means, you are just operating on this in its nth electric field.

And then you are taking projects how much it is project after operating how much projection is there on the kth mode, it is something like that, in vector mode you can write something like that, if it is a vector E k star dot delta epsilon E n star x y you can think of this is actually operating on this whatever value is coming, you are taking dot product with the projection with that that is what we are overlap with this meaning actually.

So if it was just 1 it would be something like that it make no sense no perturbation that means it is left hand side right hand side easily you could relate so since we cannot do that right hand side, we just keep as it is n summation we keep left hand side it is like this, then what do you see these expression these expression left hand side I just only keep this side this one. And beta k I can take right hand side so, you see this is j to beta k beta k.

So, numerator beta k is there because I cannot cancel this one here, because beta k can be considered as a which direction it is propagating depending on the direction actually plus beta k and minus beta k we can consider - beta k is the forward propagating wave representing when it is e to the power j omega t - beta k z it is forward propagating wave. And if it is e to the j omega t + beta k j it that means, it is backward propagating wave.

So, that is why depending on that sign we can use forward propagating wave or backward propagating wave, but here since it is coming out of power, so in case of power there is no meaning for power plus sign or plus sign. So, in this case 2 omega mu and beta k I think mod I have taken this I want to take to the right hand side. So that I can get the d A k by dz.

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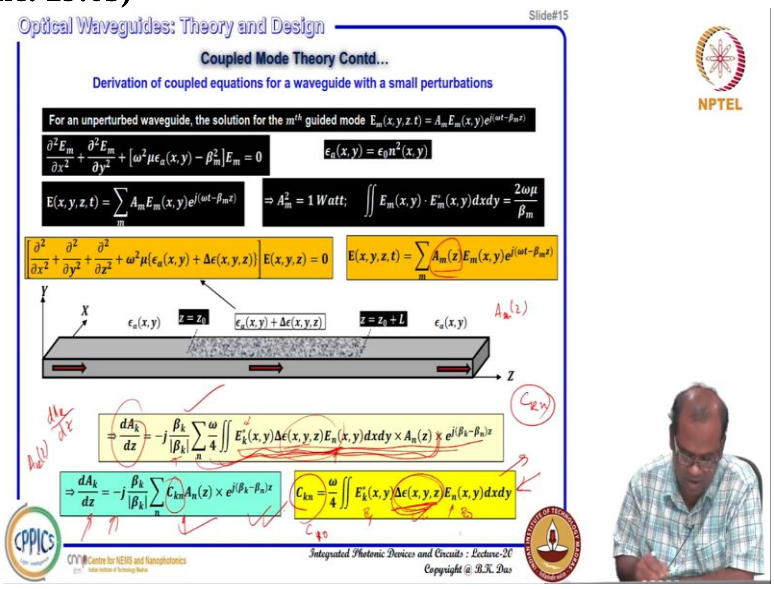

So, d A k by dz this one in the left hand side and right hand side if I put we will be getting like this - j beta k by beta k this has no sign this is absolute value taking, but top one I will be considering whether I am considering positive propagating mode or negative propagating mode. And this one will be there omega by 4 will be there I have taken omega by 4 within the sum.

But it could be omega by 4 can be outside because n we are just running with this thing that means kth mode, you are seeing what contribution coming to the kth mode because of the delta epsilon to all other modes n is running delta epsilon you are just n is running from 0 to 10 or something like that, how many modes are there that you are just operating and then looking into the kth mode.

What is the projection and that will be actually you can read like that, that is the contribution accumulating for kth evolution d A k dz means, how it is actually evolving as a function of dz what is the slope of A k z this one this A kth z dependent mz is constant mth mode considering a kth mode any kth mode arbitrary kth mode what about how it will be evolving we can write like this.

So what we have done we have just a little bit contracted this one just represented it C kn in that is the overlap part that you are explain omega by 4 this particular term up to here we write C kn just if I run if I change n different mode if I say n = 0, 1, 2, 3, 4 so on then I will be writing C k suppose kth mode is 1 then I will be writing C k C 1 0 then I will be writing here E 0 this would be E 1.

So, first order mode and fundamental mode along with the delta epsilon whatever the perturbation is there then you are integrating over entire cross section multiplying omega by 4 that actually we are just we can calculate numerically all these we can calculate by mode solver once we know the mode solver and once we know field distribution through mode solver.

And once we know the perturbation, then we will be able to calculate what are the values of C kn. So, since it has to be estimated numerically and we can insert in this equation so, we cannot consider this solution entirely semi analytical some part is numerically you need to compute. So, whatever you are computing that value we can consider then you can just try to see once you know that value.

Then you can see how to simplify this equation our ultimate goal is that I have to solve A k z kth mode how it will be evaluating k can be any mode first mode or second mode or third mode and so on, that how it will be evolving this is the final equation. So, keep in mind that this is the expression you have to always keep carry forward whenever any particular perturbation things are known, then you will be able to calculate C kn very easily and you can put down here then you can actually estimate what is the evolution for the kth mode.

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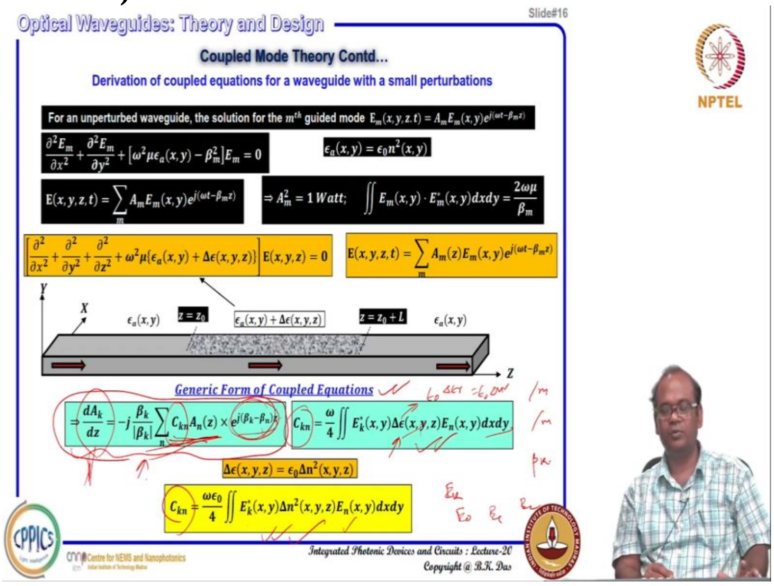

So, this is called actually generic form of coupled equation. So, this is the equation it is very, very important is the base of couple mode theory of course, it is a weekly guided coupled mode equation, why it is coupled mode, you see it is coupling kth mode to all other mode via these terms this is the sum so, you are for kth evolution, you see the contribution coming from all the modes and how that contribution coming because of the perturbation.

If there was no perturbation no more so, will interrupt each other, they will not exchange any energy and because of the presence of delta epsilon, this one is there. And depending on that value, you can find out how much contribution coming from each of these modes and that you sum together then you will find this value at the same time if you see here, 1 additional term is there.

That is very important, we will discuss that that is beta k - beta n z you are just changing you have a particular mode that is fine and you have a particular propagation constant fine. Now, I want to calculate C kn this one for E 0, E 1, E 2 all the fields I want to see here and corresponding beta value also will be coming in the exponential. So, whatever the value it is there that also is responsible for the evolution of kth mode.

So, this is how you need to know so, this is a very important equation for integrated optics or photonic integrated circuit. So, anything coupling any device any deformation any modification, any engineering you want to do, you can just that engineering you can consider as a delta epsilon. And that delta epsilon actually quantify C kn how much it is contributing energy transfer from one to another.

Obviously, whenever you see this one, the dimension will be coming as a per meter. So, per metre travelling length how much it will be contributing if you just put already in per second electrical volt per metre volt per metre, sorry dielectric constant here also you have epsilon 0 delta epsilon r or epsilon 0 delta n square that means, it is Farad per metre is there and a volt per metre volt per metre Farad per metre and metre square.

So, all these radian per second if you just combine all this together then it will be per metre. So, that means, that is the strength per metre contributing for nth mode. So, when I say C kn that means contribution to kth mode because of the nth mode. So nth mode is contributing to kth mode that is how you have to read C kn that C kn will come here n can be 0 whatever the value you can calculate here n can be 1 whatever the value you can have.

And K you are just targeting only one particular you are focusing on only 1 particular mode kth mode kth mode, how much it is evolving because of the all other nth mode. So, that is what we can calculate and put here all the terms we have to include then we can see that how much kth mode is being evolved that is the basic thing of the coupled mode equation this is called actually generic form of coupled equation. So lot of devices will be explaining starting from this one.

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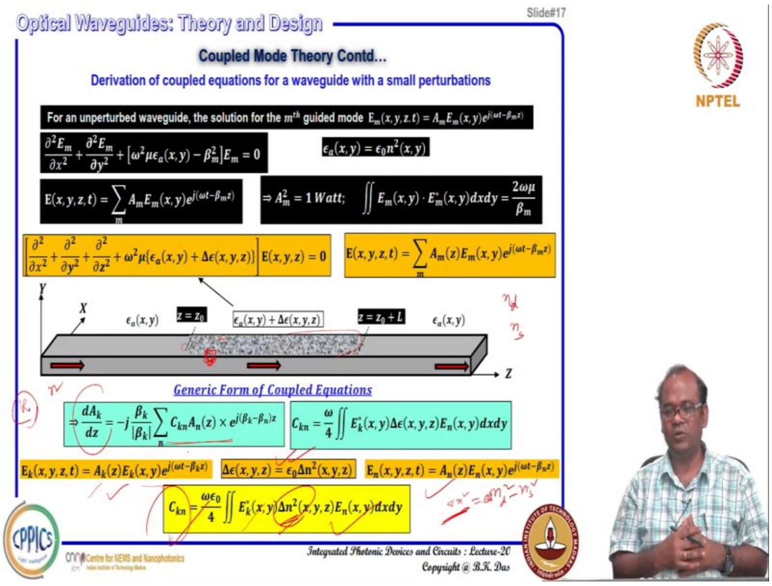

Now, next we clean up so, now we see that kth mode, this is your field distribution A k z and E k x y E to the power j omega t and this is the perturbation and in terms of refractive index we can write suppose some perturbation for example, in this case in this region somewhere you have a perturbation that some silicon is removed completely silicon core some silicon is removed and then silicon dioxide is involved.

So incorporated there some little bit particular point some perturbation you have created maybe some edging is there and then oxide is there that means, that region what is the perturbation that means epsilon 0 delta n square because silicon reflective index was n b n d silicon and silicon substrate is n s. Now delta n squared you can write delta n squared that particular region delta n squared will be equal to delta n d square.

That means, device layer refractive index minus that means, it is n d square - n s square that is actually delta n square that means, instead of device layer you have silicon that means, how much dielectric constant you have changed or delta n changed n d square - n s square so, that much you have to changed and then you have to see how much area that is that type of defect is there that particular area delta epsilon is there.

So, you have to integrate only that area to find out C kn. So, perturbation is there that entire cross section you do not have any perturbation probably maybe only a particular region some this region to this region perturbation is there that particular region you have modification of perturbation change of refractive index that refractive index and then that particular region this delta n is non 0 and obviously, field strength is there kth mode field is there in nth mode field you have solved already numerically.

That you can use and integrate only that perturbation region then you will be getting C kn that is easy to calculate once you know mode profile and you will know where is the perturbation then you can calculate C kn. So, now, this is your E n x A n this is kth mode and nth mode that they are actual field distribution we have written both field will be there. So, I can say that I am just getting what is the evolution of the kth mode.

Similarly, I can concentrate instead of k i can write some other mode k can be 1 initially I was considering k = 1. You can consider k = 2 what is the evolution then you can also get similar type of equation also we will discuss that in course of time suppose, normally in waveguide you can have only few modes will be there you will be designing single model you have got only 1 particular mode will be there or maybe 2 modes maximum you can handle or 3 modes.

And maybe also single mode I have got you got you are handling one more this positive direction another mode reverse direction. So, that means most of the time you will be dealing with only coupling between 2 modes. So, coupling between 2 modes whenever you are considering then you need to have only 1 term because suppose you are considering only kth mode and only nth mode n is not running from 0 to all the modes just only one specific mode n and one specific mode k.

Then in that case right hand side you will have only 1 term. That means I can see the kth mode evolution, how much kth mode is increasing as a function of z decreasing as a function of j because of the nth mode. So, I can restrict our discussion between 2 modes, kth mode and nth mode and kth mode can be fundamental mode in the forward direction and nth mode can be fundamental mode in the reverse direction.

That can be also 2 different modes so, they are coupling when they will be coupling in the they will be propagating in the opposite direction, what will be the evolution when they are propagating in the same direction what will be the evolution that will be our subject for the next lecture. Thank you very much.