Integrated Photonic Devices and Circuits
Prof. Bijoy Krishna Das
Department of Electrical Engineering
Indian Institute of Technology, Chennai

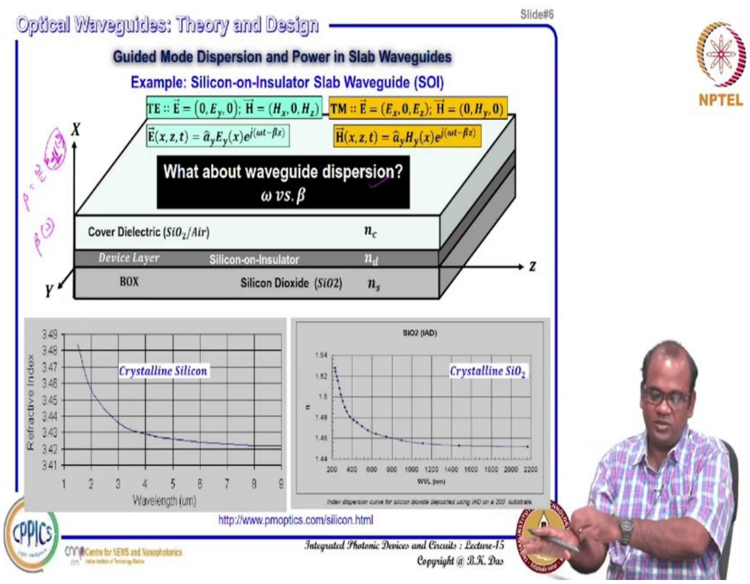
Lecture – 15
Optical Waveguides: Theory and Design
Guided Mode Solutions for Slab Waveguides Contd

Hello welcome, we will continue our discussion on the optical waveguide and we have discussed earlier already how to solve the modes and what will be the field distributions and whatever the phase velocity and what is the cutoff, how many modes will be supported by a given slab waveguide or planner waveguide structures. And today we are going to discuss that if your operating wavelength differs operating frequency omega differs.

You know that omega operating wavelength we consider any mode if you just consider in TE polarization we have discussed something like that, that only y component will be there x and then E to the power this is a y unit vector e to the power j omega t minus beta z and if it is a mth mode I can write m like this. So and also you we know how to find out the TE polarization how to find out also these other components like magnetic field components like say H x and H y and all the components say 0 E y 0.

Electric field and magnetic field component you can consider H x 0 H y and all of them are actually x dependent because x come to an x direction is confined x function, function of x and along with that will be propagating with phase omega t, time dependent phase and propagation direction beta t and beta is equal to nothing but omega / C n effective and if it is different mode all this field distribution will be different and associated mode propagation constant also will be defined like that omega / C n effective m. So, all this we discussed they are actually function of a given frequency omega, omega = 2 pi / lambda C.

(Refer Slide Time: 02:55)



Optical Waveguides: Theory and Design

Slide#6

Guided Mode Dispersion and Power in Slab Waveguides

Example: Silicon-on-Insulator Slab Waveguide (SOI)

TE :: $\vec{E} = (0, E_y, 0)$; $\vec{H} = (H_x, 0, H_z)$

TM :: $\vec{E} = (E_x, 0, E_z)$; $\vec{H} = (0, H_y, 0)$

$\vec{E}(x,z,t) = \hat{a}_y E_y(x) e^{j(\omega t - \beta z)}$

$\vec{H}(x,z,t) = \hat{a}_y H_y(x) e^{j(\omega t - \beta z)}$

What about waveguide dispersion?

ω vs. β

Cover Dielectric (SiO_2/Air) n_c

Device Layer Silicon-on-Insulator n_d

BOX Silicon Dioxide ($SiO2$) n_s

Index dispersion curve for silicon dioxide deposited using IAD on a 200 substrate.

http://www.pmoptics.com/silicon.html

Integrated Photonic Devices and Circuits : Lecture-15

Copyright @ B.K. Das

So, we will be just following that one if frequency changed then what happens? So, for that purpose we just go back to the basics again the device structure, silicon-on-insulator device structure, wafer structure if you see it is a 3 layer medium and of course in the bottom there will be substrate silicon handling wafer will be there handling wafer and then on that buried oxide, silicon dioxide will be there which you consider n s and then your actual device layer.

This is device layer which is called silicon-on-insulator refractive index n d and cover dielectric also can be silicon dioxide or air also one can consider sometimes people use also Si 3 N 4 which is having refractive index also lower than silicon and approximately refractive index is 2 something like that you can also consider here. For our discussion most of the time we will be considering this as air but only refractive index difference you can consider something like that.

So, this is a slab waveguide and you are considering that this is X axis, this is Z axis and Y axis you are considering infinitely extended what is happening in that direction you do not know at least you know that wave is having a wave vector that will have k x component and you will have also z component k z component and k x component actually contributes the standing wave along x direction here in this direction it will be confined.

And k z component actually helps to propagate in the forward direction which is actually nothing but the beta whatever mode you are getting that will be propagating along the beta direction. So, now, if we just think of the same thing whatever we discussed for TE polarization, this is the field components E y, H x, H z, if I just concentrate on E y, E y would be x dependent it will be a j omega t beta z, that is what we learned so far.

Orange color for TM polarizing green color is for TE polarization or T mode and for TM polarization you have electric field will be in the xz plane that means, it is actually it has a normal component along the X direction the interface and magnetic field H y is the tangential component and if we just concentrate since only one component is y component magnetic field, that also I can solve using your wave equation scalar wave equation x dependencies and the propagation constant we can find out.

So, where I solve using these components, I will be able to solve say so, called beta TM and here I will be solving beta TE if it is frequency same, we can write that beta TE is equal to omega / C n effective m for TE and here this beta TM will be equal to omega / C same frequency n effective m TM. So, this m integer different and this m integer different. So, same waveguide it can support TE polarization and TM polarization also with a different mode index m. So, that is fine next.

So, I have defined n c cover refractive index n d device layer refractive index n s substantive refractive index that is fine. But, so far if I want to change omega I want to operate which omega frequency we should use of course, we are interested around omega corresponding to omega if it is defined by 2piC / lambda, lambda = 1550 nanometer is prepared that is the third generation communication window that is fine we can use this thing.

But we have to see that any frequency if we want to vary if I want to find out the relationship beta = omega / C n effective if I want to find out at different frequency what is about value of beta omega it may not be just proportional effective index can be also function of omega. So, I need to know at least that operating window whatever material system we are using that must be transparent that frequency should see electromagnetic waves should see it is a dielectric material medium that means, it can be characterized only with the effective index no sigma value.

No conductivity will be there, no losses will be there, no band transitions will be their photon absorption will be there that type of things should be considered whenever you are considering frequency of the electromagnetic wave. So, in this system silicon-on-insulator. So, it substrate you have actually light will be confined in the device layer here light to be confined but you know the evanescent tail will be in the oxide and evanescent tail also will be in the cladding top cladding that can be oxide here silicon nitride etcetera, will be there.

So, we need to operate the frequency range which is actually transparent to top cladding, transparent to bottom cladding, transparent to core device layer. Core device layer silicon in this case in this example, so, crystalline silicon if you see in this picture I have taken from somewhere and authentic one. So, if you see that the transmission of a crystalline silicon starts from 1 point about 1.1 and 1.1 micrometer, this is actually microns wavelength in micrometer.

So, 1.1 micrometer, below that wavelength is lower and photon energy for that corresponding photon energy is cut only there that is very high and that actually absorbs electromagnetic will get absorbed and electron hole pair will be generated electron from the valence band, it will acquire energy from the photon incoming photons associated with the electromagnetic wave at that frequency and it will be released to conduction band.

So, almost everything will be absorbed any intensity if we just fall in it most of the time, depending on the thickness everything will be absorbed, that is why any wavelength lower than these that means very high frequency operations. So, you will see that will not be transmitted that is not transparent and that is opaque. So, it is not transparent. So, you cannot consider as a dielectric medium for those frequencies but 1.1 micron to almost 9 micron 10 micron region 9 wavelength region.

So, this is a broad band transmission. So, starting from 1 micron to almost a 9 micron 10 micron wavelength silicon is transparent. So, this is a good part. So, that means the core layer as far as transparency point transparency is concerned, the electromagnetic wave has a broad transmission spectrum. So, you can use anywhere from 1 micron to 1.1 micron to 10 micron within that remember that we have the third generation communication window as well as second generation communication windows 1310 also it is falling within that.

So, they are transparent but, if you just think of oxide, oxide is a different silicon dioxide crystalline silicon dioxide if you see transparency actually starts from 0.2 micron. So, 0.2 micron about 200 nanometer wavelength and it can transmit only up to 4 micron around 4 micron here transparency is almost 100% here a little bit there because there will be some reflection etcetera will be there that is it 60% less than 60% showing because final reflection will be there from the interface that has not been extracted and glass normally silicon dioxide.

Their refractive index is low so final reflection and will be much lower that is why it is almost 100% it will be transmitting and you will see transmission like this up to 4 micron. So, that means, if I use a wavelength lambda, which is beyond 4 micron, that may be transparent in the silicon, but that will be absorbed in the oxide. So, I can use up to 4 micron which is actually from lambda 1 to 4 micron I can say that, So, lambda can be less than 4 and greater than 1.1 micron.

So, within that range, it is transparent in both in the box region as well as in that core region. So, that is what it is very important whenever you are using silicon on insulator what is the wavelength range of operation or frequency range of operation you should know we know that this n c, n d, n s what about the refractive index that refractive index whether it is a wavelength dependent or not or frequency dependent or not we need to know that, that means whether the material dispersion present or not.

If so, while designing your optical waveguide even though it is 1D or 2D whatever you need to know the when the bulk what is their refractive index, whenever I am just sweeping my frequency, I am solving the effective indices for that I need to know all the every frequency range what is the refractive index. So, if you see crystalline silicon it is shown, because from starting from 1.1 micron it is there this is the wavelength micrometer up to 9 micrometers is shown.

So, refractive index, this is the refractive index it is dropping starting from about 3.48 around 1.1, this is no, this is about 1.5. So, it is shown about 1.3 or 1.5 something like that and it is going like that. So, initially at shorter wavelength, you see refractive index is somewhat it is falling, falling means it is 3.48, 3.43 and then in longer wavelengths, it is almost stagnated or cogent was 3.45 or something like that, this is the refractive index happening.

So, if we operate at 1550 nanometer is about 3.4778 we have mentioned earlier 3.4778 at 1550 nanometer below that it is about bandgap absorption is there so, they could measure they could calculate from here up to here and it is showing a longer wavelength that is flat, but NaCl if you just change wavelength or frequency if we are changing. So, you see some kind of material dispersion is their frequency dependent refractive index dispersion means, if any material when a material is called dispersive.

That means, it is wavelength dependent refractive index is there that means wavelength dependent page velocity is there in a homogeneous medium. So, you see this type of dispersion is there material dispersion is there, but if you just consider if you are concentrating only maybe 1500 nanometers to 1600 nanometers very small range 0.1 micrometer range in the scale within that 0.1 micrometer refractive index change is not that much big for silicon.

Similarly, for silicon dioxide if you see, it started from 200 nanometer up to 2.2 micrometers it has shown in this graph this wavelength it is shown. So, if you see initially up to visible region 800 nanometer 1000 nanometer it is dropping and beyond that it is dropping means it is dropping from 1.53 to 1.45 or so and then after that it is almost nearly constant around our operating wavelength region it is actually 1.465 something like that this is thing we are considering here 1.45 % like that refractive index.

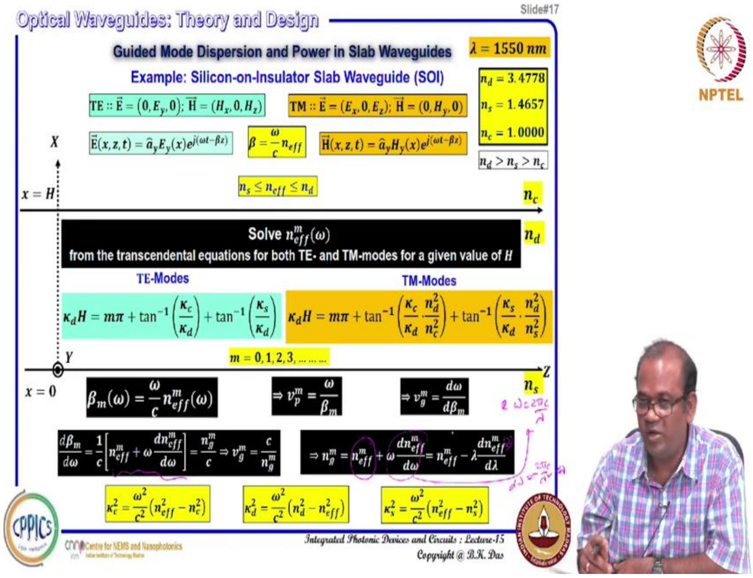
But here if you see if I just consider putting 100 to 1600 nanometer refractive index is not changing much. So, I can consider it is non dispersive around this operating wavelength range. So, whenever you are designing your waveguide moves everything if there is a material dispersion is there or not that has to be counted in your calculations that much I just wanted to mention here.

So, this is a material dispersion and we have seen that how the material dispersion can be ignore or can be considered seriously that I have seen which operating or wavelength region of operation you are considering that has to be kept in mind, but, apart from that, what about waveguide dispersion that is the major thing discussion point today that means omega versus beta.

That means, this beta must be a function of omega but whether that is a linear like this beta equal to omega / C n effective if this n effective is not a frequency dependent then it is a constant then I would get beta versus omega just straight line, but if n effective is frequency dependent that means, for a given mode this n effective or beta then they will become nonlinear and we have to take care about that if it is that is nonlinear.

If it is linear that means, normally we know that the group velocity and phase velocity will be identical we will discuss that in course of time in this case we have to find out if waveguide dispersion other than this material dispersion suppose I am considering that this n s and n d and n c. So, called refractive indices for the device layer, bottom oxide, top cladding, they are fixed for example, they are not changing much as it functions of frequency that is fine. Does it mean they guided modes that mean n effective is also frequency independent or not? So, that is most important thing that is what we want to see today.

(Refer Slide Time: 16:55)



Optical Waveguides: Theory and Design

Slide#17

Guided Mode Dispersion and Power in Slab Waveguides

$\lambda = 1550\ nm$

Example: Silicon-on-Insulator Slab Waveguide (SOI)

$n_d = 3.4778$
$n_s = 1.4657$
$n_c = 1.0000$

$n_d > n_s > n_c$

$TE :: \vec{E} = (0, E_y, 0); \vec{H} = (H_x, 0, H_z)$

$TM :: \vec{E} = (E_x, 0, E_z); \vec{H} = (0, H_y, 0)$

$\vec{E}(x,z,t) = \hat{a}_y E_y(x) e^{j(\omega t - \beta z)}$

$\beta = \frac{\omega}{c} n_{eff}$

$\vec{H}(x,z,t) = \hat{a}_y H_y(x) e^{j(\omega t - \beta z)}$

$n_s \le n_{eff} \le n_d$

Solve $n_{eff}^m(\omega)$

from the transcendental equations for both TE- and TM-modes for a given value of H

TE-Modes

$$\kappa_d H = m\pi + \tan^{-1}\left(\frac{\kappa_c}{\kappa_d}\right) + \tan^{-1}\left(\frac{\kappa_s}{\kappa_d}\right)$$

TM-Modes

$$\kappa_d H = m\pi + \tan^{-1}\left(\frac{\kappa_c}{\kappa_d} \cdot \frac{n_d^2}{n_c^2}\right) + \tan^{-1}\left(\frac{\kappa_s}{\kappa_d} \cdot \frac{n_d^2}{n_s^2}\right)$$

$m = 0, 1, 2, 3, \dots\dots\dots$

$$\beta_m(\omega) = \frac{\omega}{c} n_{eff}^m(\omega) \qquad \Rightarrow v_p^m = \frac{\omega}{\beta_m} \qquad \Rightarrow v_g^m = \frac{d\omega}{d\beta_m}$$

$$\frac{d\beta_m}{d\omega} = \frac{1}{c}\left[n_{eff}^m + \omega \frac{dn_{eff}^m}{d\omega}\right] = \frac{n_g^m}{c} \Rightarrow v_g^m = \frac{c}{n_g^m}$$

$$\Rightarrow n_g^m = n_{eff}^m + \omega \frac{dn_{eff}^m}{d\omega} = n_{eff}^m - \lambda \frac{dn_{eff}^m}{d\lambda}$$

$\& \ \omega = \frac{2\pi c}{\lambda}$

$d\omega = -\frac{2\pi c}{\lambda^2} d\lambda$

$$\kappa_c^2 = \frac{\omega^2}{c^2}\left(n_{eff}^2 - n_c^2\right) \qquad \kappa_d^2 = \frac{\omega^2}{c^2}\left(n_d^2 - n_{eff}^2\right) \qquad \kappa_s^2 = \frac{\omega^2}{c^2}\left(n_{eff}^2 - n_s^2\right)$$

Integrated Photonic Devices and Circuits : Lecture-15

Copyright @ B.K. Das

We take same reference again that the silicon on, insulator slab waveguide. So, cover refractive index n c, n d, n s we can always consider their fixed the material dispersion which is shown, but for our calculation we can consider that within the small window the material dispersion can be ignored, but actual calculation you should not ignore I just take the reference of this one around 1550 nanometer now I just put down the expression typical expression for beta equal to omega / C n effective and we know that this n effective must be greater than n s and less than n d.

n d is the highest refractive index and any guided mode will see something effective index in between n s and n d anything beyond n d never possible. So, if core refractive index is n d. So, any guided mode cannot have effective index more than that it should be maximum can be equal or close to equal it cannot be equal it is a near to the n d and but it should be obviously greater than n s that is what we have proved earlier also from the critical angle and Snell’s reflection you know in the interface we have discussed many times that thing.

So, this much you should be keeping in mind. So, what is next and also TE TM polarization what are the components and how X direction confinement Y direction the y component X direction for electrical field also for TE polarization and its magnetic field for TM polarization consider. Now, we just go back again that the solutions differential equation for TE polarization we have to consider scalar wave equation this one and for TM polarization we have this one that is what we have already discussed I just repeated here and kappa c kappa d kappa s.

We have introduced here starting from the vector applications and effective index is the solution we just consider, fine. Now, solutions, same thing I am just repeating here whatever we discussed in the previous lectures, that electrical E y component for the TE polarization in the cover region exponential decay and the core region the sinusoidal co sinusoidal exponentially decaying for magnetic field y component also we have shown like this.

Next thing and we know that how to calculate also the other components for the TE polarization x component that is related to E y component here and H z component also related to E y. So, once we know the E y function, function of x, then we can find a z component as a function of x and H x component also function of x, I can find similar situations also this thing I just write down the expression for understanding purpose.

Here also, you see you can find x component and z component but wherever you are finding the z component you just use it keep in mind that, additional thing is there. So, n i, n i means c, d, s if you are using z component in the box layer substrate layer n s silicon dioxide layer then this n i should be considered actually s n s value you have to consider here and if it is in the core that means, in the refractive index will be n d and whatever value it is coming H y and you have to just scale it with the n d square that directly comes from your Maxwell's equations.

Nothing in our hand. So, that thing also considered here it is n c, so, we know how they are related. So, ultimately after solving one of the component that is the dominant component H y component TM polarization E y component in the TE polarization and then all other components we can find and the only thing is that n effective also we have solved their so, called Eigen equation we have used and we have solved that is again I just write down here Eigen mode solutions for TE polarization this is the expression.

And for TM polarization again if I just consider the boundary continuous here boundary continuous here using these equations, then same way like TE polarization if I solve for TM polarization, I will get a little bit different equation because here we are considering magnetic field, but, that step I am not discussing here you can take as a home assignment directly equation

I have written Eigen mode solution for TM polarization would be looking like that, it is almost same, but along with that K c / K d if you see n d square / n c square comes here.

n d square / n s square comes here kappa is kappa d. So, that is the only difference of the eigenmode solutions, you remember that using this thing actually we could solve because kappa c kappa d all are actually n effective. So, they are basically transcendental equation, left hand side and right-hand side both are n effective will be there here n effective will be there, if I solve this one, I will be solving n effective for TE polarizations and if I solve this one, I will be getting n effective for TM polarization that is it.

So, both the polarizations they can have different they satisfy different solutions that solutions or equations are slightly different because their boundary conditions are different in this case TM polarization you are considering tangential component of the magnetic field is continuous and for TE polarization tangential component of the electric field is continuous that is why they are difference.

These equations are coming from carl H = del d del t all other components will be getting and here carl E = minus del d del t. So, from that we are getting all these components just discussed earlier you just follow that know how to solve n effective frequency dependent n effective for mth mode. So, you have to use this transcendental equation for a given frequency here frequency is given one frequency is given.

So, you give a frequency value this is the expression beta = omega / c and effective and I if I give a particular frequency I will be getting for that frequency what is the effective index for the fundamental mode I can get that frequency what is the effective index of the first order mode I can find what is the effective index of the this one I can find out a effective index of the all of them for a certain frequency I calculate if I change the frequency obviously n effective because everywhere omega is involved, so, I will get different effective index.

So, changing frequency means for the fundamental mode itself, so, one particular frequency can actually excite different modes, but different modes that is a certain frequency will be different effective index will be different, but if you are restricting yourself only one mode and you are changing frequency also you are changing the frequency you will get a different solution for n effective and even for the fundamental mode even for m = 0 you will get different solutions. So, you can get solution by changing omega as well.

So, once you know this, that effective indexes relationship is this one, propagation constant relationship. You know that since your phase factor is this one phase velocity will be V p for mth mode mth effective index will be there beta value once you get once you get calculated beta value then you can get omega / beta m and mth mode, I can find what is the phase velocity. So, if different mode if I am getting different propagation constant they will be traveling with a different phase velocity.

And if I am considering frequency is different this beta m calculation also will be different in that case phase velocity will be different. Next thing is that another important thing is that group velocity. So, group velocity is d omega / d beta m. So, group velocity meaning suppose you know any particular frequency you are using wavelength you are using monochromatic things, let us say traveling wave is going but for our application purpose particularly for communication you have to encode the data time dependent.

Some signal you will be modulating and that is how you can use as your optical interconnect. So, in that case, when you start modulating time dependent various and you are trying to on off or something like that, that means, you were if you input your domain you can see that it is a different frequency will be coming. So, in that case, you need to see what is the how the group of frequency traveling so, that group of frequency that envelope that information how it travels that is decided by the group velocity d omega / d beta m.

If this curve is not a straight line beta versus an n effective carl as a function beta versus omega beta carl omega versus beta carl, if it is not a straight line that means, then phase velocity and group velocity will be different if it is straight line this omega / beta will be equal to d omega / d beta phase velocity group velocity will be similar, but if they are not straight line, linear then your phase velocity and group velocity will be different.

So, group velocity it is very, very important because you need to know how fast your data will be transmitting that actually gives you what is the flight time of the information though it is very high the close to because it is you are dealing with the light at the highest speed, but still you can say that how slow it is your information with moving data engine moving so, group velocity is important that is why it is important to know this dispersion curve beta versus omega curve.

Once you know beta versus omega, then I can suppose this is your omega and this is your beta then you get something some solution like this at a particular frequency you are getting suppose this frequency you are getting here and this is here you are operating in this frequency in this particular frequency you have to find what is the slope d omega / d beta. So, that actually gives you if this omega is your carrier, I can say that if that carrier is modulated with information, how fast that will be moving that is actually group velocity.

So, that is why omega beta curve in your optical waveguide it is very important, what is the group velocity dispersion. So, next what is that you just try to find out what is the group velocity from their little bit to modify, I know this one suppose I know omega dependent n effective omega dependent beta then if I try to find out d beta / d omega from their d beta / d omega that means omega is there this one. So, I can write this expression.

So, if you see beta equal to omega / C n. So, now d beta / d omega. This is also omega dependent that is why you have a 2 variable. So, I can write 1 / C I can keep n first and omega if I just make a derivative with respect to omega that means it will be 1 and then plus I can write omega and dn / d omega. So, this is the expression I will be getting that is what I have written, d beta / d omega and d beta / d omega if you see here that is d omega / d beta is a group velocity.

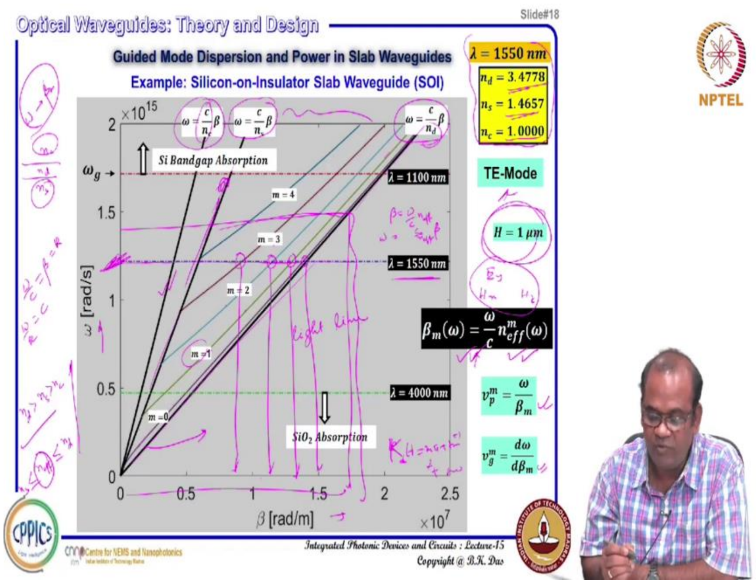
So, d beta / d omega will be inverse of the group velocity here we see 1 / C and these one we represent as n g m what is that and n g m / C. C is the velocity of light. So, that velocity of light in that denominator and this one term parameter which is actually defined by this expression that is in the numerator. So, normally we can call this as a group index of the m th mode, because we know that v g group velocity equal to C / n g group index.

So, this can be written d beta / d omega is called as a group velocity 1 by v g that means 1 / v g = n g / C that means v g will be C / n g m. So, if you know light velocity of light and if you can derive this expression, if you if you know the value of this one at a certain frequency, then I know what is the group index that means, I need to know effective index at a particular frequency and then I need to know that frequency and I need to know what is the slope at that particular effective index slope at that particular frequency.

If this slope is positive then I can find this n g is greater than any effective slope is positive if it is negative n g can be less than n effective. So, typically most of the time n g is greater than in n effective group index is higher than n effective that means, we know that when it is modulated it will be actually moving slower relative phase velocity will be higher typically. So, now, this is what we have written n g m this n g m we are writing like this for the mth mode, if you solve a mth mode effective index and a mth mode.

This one then it would be like that, but sometimes you note omega equals you can write omega = 2 pi C / lambda. So, d omega also you can just write if omega equal to this one d omega = 2 pi / C lambda square -d lambda delta lambda. So, if you just convert omega into lambda then n effective -lambda d n effective / d lambda. So, if you whether you can get effective index in terms of frequency or in terms of lambda that does not matter that group index you can calculate easily.

(Refer Slide Time: 31:27)



Optical Waveguides: Theory and Design

Slide#18

Guided Mode Dispersion and Power in Slab Waveguides

Example: Silicon-on-Insulator Slab Waveguide (SOI)

$\lambda = 1550\ nm$

$n_d = 3.4778$

$n_s = 1.4657$

$n_c = 1.0000$

TE-Mode

$H = 1\ \mu m$

$$\beta_m(\omega) = \frac{\omega}{c} n_{eff}^m(\omega)$$

$$v_p^m = \frac{\omega}{\beta_m}$$

$$v_g^m = \frac{d\omega}{d\beta_m}$$

Integrated Photonic Devices and Circuits : Lecture-15

Copyright @ B.K. Das

So, once you know group index you know the group velocity and you will know the how your information traveling how fast. So, in this example, we have just tried to plot the omega beta curve considering, this is lambda = 1550 nanometer we consider this is the refractive index and we consider these refractive indices is constant for inter frequency span that means, I have assumed that no material dispersion is there and I have considered the device layer thickness is 1 micrometer stretched along the X direction.

And I am considering only TE mode TE mode means I have only E y component and H x component H z component for a mode, if a mode electromagnetic wave propagating as a mode in the waveguide you have these 3 components TE polarization just considering you can consider for TM polarization what is happening also and you have this expression earlier whatever discussed that expression repeated here phase velocity, I have written here group velocity written here.

So, I would like to know I want to plot omega beta, beta is the propagation constant of a guided mode omega beta m for example, I will do my expression is this one I know how to calculate n effective. So, I should be able to plot that one then. So, here this X axis is considered as a beta, Y axis is omega and in this picture if you see, I think so, many things are there step by step will try to understand. So, this curve straight line is corresponding to omega = c / n c beta.

Beta is the propagation constant you know if it is a homogeneous medium with a cladding refractive index n c then any homogeneous medium you know and there is no material dispersion. So, in that case omega beta relationship is straight line slope will be c / n c, c is the velocity of light by n c that is a phase velocity of that particular medium. So, if it is a cladding material only n c = 1. So, that means omega = c beta that means omega / c beta c b = beta, that means normally it is a k vector it is a propagating like the speed of light omega / k = c.

So, that is the thing. So, if we plot that homogeneous medium in a homogeneous medium omega versus beta curve that will be a straight line assume that as a function of frequency the material dispersion is not there. So, this is the straight line corresponding to this one. Similarly, if you plot omega versus beta in the substrate, you have this medium you have a device layer n d and this is your n s this is your n c and I know that n d greater than n s greater than n c, so, this is the case.

So, if you consider instead of cover homogeneous medium silicon dioxide box layer that is continuous, then you get another straight line if it is homogeneous medium I am considering. So, you will be getting this thing and same thing, if you are considering for core if it is a silicon medium. So, this is your beta this is your omega that will be against straight line so such a plot in a homogeneous medium that is a straight line then normally called as a light line.

So, this is the light line this straight line is the light line for silicon as a homogeneous medium if you are allowing electromagnetic wave propagation in a homogeneous silicon medium, then this is the light line any frequency consider for that light line, this is the frequency and if you take this one is a beta. So, omega whatever the omega value this is omega value and this is a beta value if you take omega / beta that will be C / n d.

So, that is a phase velocity of that medium particular medium. So, similarly, this light line you consider for the box layer, so, you can consider your velocity is the omega / beta = c / n s. So, this is again like this for the cover medium pi, now, you know that your n effective is going to be less than or equal to n d greater than equal to n s. So, any solutions you get for any frequency range that will be something like beta if it is a straight omega / C n effective and omega = C / n effective beta.

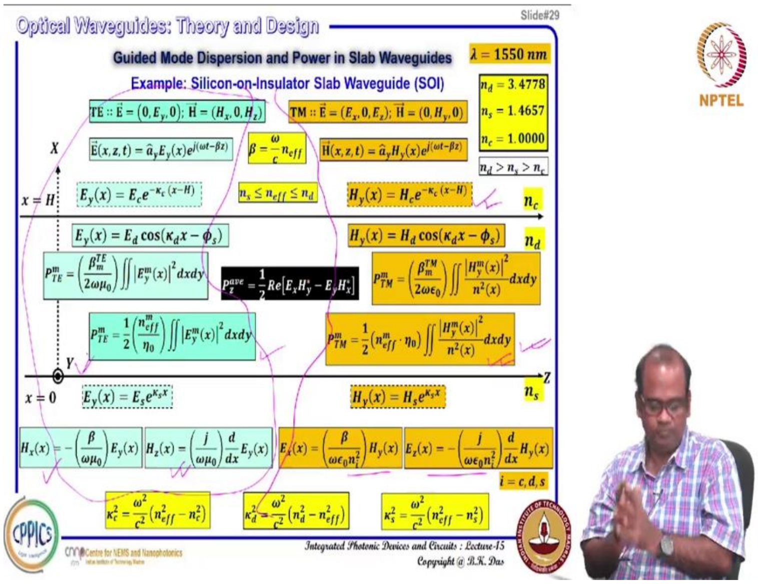
So, since this n effective is less than n d and greater than n s so, n effective solutions will be coming in between these 2 light lines, these 2 light lines, light lines for n d, light lines and for n s, in between your n effective will be there. So, any solutions for n effective that will be in this region in this between these light lines you can have n effective solutions and we have seen that if you are considering m = 0 then you plot those earlier equations whatever the equation you have consider this kappa d H = m pi tan inverse plus tan inverse.

That equation we have explained earlier that Eigen equation for solving effective indices. So, m = 0 if you put you get this m = 0 this curve we were getting this is almost starting a little bit higher and then as you increase the beta value or omega value it is margin towards n d, because the effective index slowly, slowly it will be margin towards as it as frequency increases this way then it will be going like that.

Similarly, for m = 1 you get another relations omega beta relations omega beta these are the curve for this one for different mode m = 1 you are getting this curve m = 2 you are getting this curve m 1, m 2, m 3, m 4 4 curves you are getting. So, you are getting 1, 2, 3, 4, 5, 5 different curves it is plotted here. Now, 5 different curves you can get a solutions from that and you will consider H = 1 micrometer 5 different things.

But, ultimately we know that for H = 1 micrometer, there will be 4 modes, we will try to explain where is the 4 modes coming you know, these value this frequency is corresponding to 1550 nanometer, this is the frequency and if you go in this direction then you see it is actually cutting m = 3, m = 2, m = 1, m = 0 that means, I have 1 beta solution and another beta solution for that frequency another beta solution here another beta solutions here. So, these are the 4 different beta value solutions I will be getting for this frequency.

(Refer Slide Time: 38:53)



Slide#29

Optical Waveguides: Theory and Design

Guided Mode Dispersion and Power in Slab Waveguides

Example: Silicon-on-Insulator Slab Waveguide (SOI)

$\lambda = 1550\ nm$

$n_d = 3.4778$

$n_s = 1.4657$

$n_c = 1.0000$

$n_d > n_s > n_c$

$TE :: \vec{E} = (0, E_y, 0);\ \vec{H} = (H_x, 0, H_z)$

$TM :: \vec{E} = (E_x, 0, E_z);\ \vec{H} = (0, H_y, 0)$

$\vec{E}(x,z,t) = \hat{a}_y E_y(x) e^{j(\omega t - \beta z)}$

$\beta = \frac{\omega}{c} n_{eff}$

$\vec{H}(x,z,t) = \hat{a}_y H_y(x) e^{j(\omega t - \beta z)}$

$n_s \leq n_{eff} \leq n_d$

X

x = H

$E_y(x) = E_c e^{-\kappa_c (x-H)}$

$H_y(x) = H_c e^{-\kappa_c (x-H)}$

n_c

$E_y(x) = E_d \cos(\kappa_d x - \phi_s)$

$H_y(x) = H_d \cos(\kappa_d x - \phi_s)$

n_d

$$P_{TE}^m = \left(\frac{\beta_m^{TE}}{2\omega\mu_0}\right) \iint \left|E_y^m(x)\right|^2 dxdy$$

$$P_z^{ave} = \frac{1}{2} Re\left[E_x H_y^* - E_y H_x^*\right]$$

$$P_{TM}^m = \left(\frac{\beta_m^{TM}}{2\omega\epsilon_0}\right) \iint \frac{\left|H_y^m(x)\right|^2}{n^2(x)} dxdy$$

$$P_{TE}^m = \frac{1}{2}\left(\frac{n_{eff}^m}{\eta_0}\right) \iint \left|E_y^m(x)\right|^2 dxdy$$

$$P_{TM}^m = \frac{1}{2}\left(n_{eff}^m \cdot \eta_0\right) \iint \frac{\left|H_y^m(x)\right|^2}{n^2(x)} dxdy$$

Y

Z

x = 0

$E_y(x) = E_s e^{\kappa_s x}$

$H_y(x) = H_s e^{\kappa_s x}$

n_s

$H_x(x) = -\left(\frac{\beta}{\omega\mu_0}\right) E_y(x)$

$H_z(x) = \left(\frac{j}{\omega\mu_0}\right) \frac{d}{dx} E_y(x)$

$E_x(x) = \left(\frac{\beta}{\omega\epsilon_0 n_i^2}\right) H_y(x)$

$E_z(x) = -\left(\frac{j}{\omega\epsilon_0 n_i^2}\right) \frac{d}{dx} H_y(x)$

$i = c, d, s$

$\kappa_c^2 = \frac{\omega^2}{c^2}\left(n_{eff}^2 - n_c^2\right)$

$\kappa_d^2 = \frac{\omega^2}{c^2}\left(n_d^2 - n_{eff}^2\right)$

$\kappa_s^2 = \frac{\omega^2}{c^2}\left(n_{eff}^2 - n_s^2\right)$

CPPICS

Centre for NEMS and Nanophotonics

Integrated Photonic Devices and Circuits : Lecture-15

Copyright @ B.K. Das

Silicon-on- insulator slab waveguide again example we are talking we want to know a mode suppose a waveguide having H = 1 micron or so, or 2 micron or maybe even less 250 microns to 1 micron that is not standard and just considered here for these calculations or understanding purpose that is this something can be 1 micrometer, it can be less than that as I mentioned earlier, it can be as low as some 100 nanometer also, we will see that later.

So, I would like to know suppose there are several modes, they can different modes will travel with a different phase velocity that is one thing, but their group velocity also can be different because of the dispersion omega beta relationship, but a mode I would like to know how much energy it is carrying by mode. I would like to know for example, how to calculate that you know that pointing vector this is pointing vector means that means pointing vector is defined by power flow per unit area per unit time along the propagation direction energy flow.

This is a power not energy flow I would write energy flow, energy flow per unit area per unit time along the propagation direction that is actually pointing vector and normally along the propagation direction if it is a transverse mode that is why it is E cross H and we know that we have discussed earlier that if you are using E and H complex then to take average time average we have to take E cross H star and then the real part you have to consider.

And you have to multiply by half then only that is actually actual real value power flow per unit area per unit time energy flow I would say and so, energy flow per unit area is a power basically. So, if I just expand this cross one E cross this is a vector a this vector b and their cross product we can just express in this format a x that means, unit vector along the x direction unit vector along Y direction unit vector along Z direction and the first vector x y z and H x star H y star H z star.

Now, since our waveguide is aligned along the Z axis that means, mode is propagating along Z axis, so, that mode will carry energy or power along the Z axis. So, that means, I only need to concentrate what is the power flow along the Z direction Z component This is a vector basically, I need to know this power along Z direction obviously, to just calculate say this is actually a p z if you calculate p x also average x you will be getting also some valuable p x, but p x this direction whatever power goes and back reflect back that actually clears the mode.

So, that will not carry energy along the X direction you know along the x direction when essentially decaying field. So, that direction no energy is being flowing. So, only energy flow that is happening along Z direction. So, in that case Z direction what is that pointing with that I could express like this. So, this is Z direction means this one this one goes that means E x times H y star -E y times H x star and real part half into there.

So, we have written accordingly. So, this is the if we know the components for a given mode, then I know how much power flow or energy flow per unit area per unit time and on the progressive direction you will be able to know. So, let us concentrate for TE polarization again TE polarization these are the components this is the components I just consider E y. E y can be expressed like that X direction some field distribution will be there.

That for TE polarization we have the field directions field components and then y component we express as a function of x and then propagation direction beta is their beta can be expressed by this and the field distribution in the upper cladding and in the core region bottom cladding it is there and we know that for this purpose I need to have some energy flow in the Z direction we need to find out what is the other components magnetic field H x, H z.

I know that from the Maxwell's equation H z, H x and H z component they can be expressed they can be derived once we know the y component or the electric field that you want to written here. So, now, for the magnetic field also likewise, I can define y component of the magnetic field in the cover region core region and lower cladding box region and other components like E x and E z again we can derive from the curl equation.

Maxwell’s curl equation that expression I have shown earlier also this is the straight forward just from the curl equation and we know that this kappa c, kappa d, kappa s they are defined like this that I repeated here. Now, what I will do I know that z component Z direction this is the power flow energy flow per unit area. So, now, what I will do? Let us concentrate only at TE mode TE polarization where I have this component and H x, H z.

So, if I try for TE polarization you do not have H y component H y component is not there or the x component is not there also H y component is also not there. So, this part will not be there for TE polarization. So, for TE polarization I can write this one this is the expression half real part minus sign E y H x. H x can be expressed in this form. So, if I just because I solved E y everywhere now, H x can be solved from the E y just you have to multiply -beta / omega 0.

So, if I just replace H x here this H x I can just put here this one here, then if we just multiply here, then you get x dependent energy flow per unit time per unit area I cannot or I if I want to find out what is the energy flow in the Z direction here in the top cladding whatever the field E y x is here, I have to use this one here coordinate system accordingly, core region E y x is defined differently defined. So, that means E y x is a different region is different.

So, I can find out what is the intensity or energy flow per unit area per unit time here, what is the energy flow per unit area per unit time here we have to use this equation here and if we want to say whatever energy flow in this direction happening, because you have evanescent tail it will not carry in X direction, but this evanescent field can carry energy in the Z direction. So, whatever the field strength is there you just multiply square it and scale it then you can find whatever energy flow in this direction.

That is why your mode your energy can flow not only in within the core it also can carry energy in the cladding region only along the Z direction, but not in the X direction that is why it is evanescent. Now, similar thing for TM polarization, but TM polarization I have this is the actual energy flow pointing coming from pointing vector where for TM polarization we have x, z and you know what is the value of H y that is the expression I have solved H x.

H y x in the top cladding H y x in the core region H y x in the bottom cladding region I know these are the expression we have solved and from the Maxwell's curl equation we can express x and z component for the TM polarization. So, for TM polarization what is missing we do not have E y component that is 0 and we do not have H s component that is 0 for TM polarization I will have the energy flow per unit area per unit time is x H y star.

So, H y star I know a H y expression everywhere I know H y and E x expression I can derive here using that E x I put here I can express the energy flow per unit area per unit time in terms of magnetic field component tangential component because that is the solutions, I have all the position I know what is the energy flow intensity everywhere intensity flow in this direction and Z direction I can just find out, but in terms of H y.

But remember that when I am just calculating for TE depolarization beta / omega mu 0 here is there, but in case of TM polarization this n i square because you have to consider this refractive index whether it is in the core region or top cladding or bottom cladding accordingly, it will be placed n i it will be considered n c square n d square in the core region n s square in substrate regions.

So, this region all this thing 3 different regional energy flow that also depends on what is the dielectric constant there not only field distribution magnetic redistribution here we are considering electric field distribution only E y component here magnetic field distribution you are considering that is the reason we need to consider what is the dielectric constant here that directly comes from your Maxwell's equations, you do not have any other reasons for that.

So, all this expression involved in that. So, once we know that energy flow per unit area per unit time I want to know because your field is distributed throughout evanescent to core region and everything. So, total power how to calculate? What do you have to do you have to integrate. So, that is the reason whenever I am just written down this this expression energy flow 3 directions. So integration, so, this integration I have just considered also along Y direction because waveguide.

You cannot have just 1 dimension and just X direction confinement but Y direction I have just so far considered it is an infinitely extended. So, but actual calculation how much power will flow so you have to consider up to what some extent, though this is basically everything ideal along Y direction, so we can consider dy as the 1 micro meter 1 meter for example, or 1 nanometer 1 unit. So, in that case, I can say that per unit length because I assume that along Y direction this component is not varying.

That is our assumption H y component is not varying along Y direction, along Y direction in also not varying only X direction it is varying. So, n square x, x I have written here. So, if I integrate suppose, this integration will have 3 different functions, this function will be different. So, I can just decompose into 3 parts in the top cladding, I have to use this expression and I have to integrate, so, I can just integrate this one, this one and this one. So, first part is this is -infinity to 0 x = - infinity to 0.

And then second part that will be 0 to H, I will be using this expression E y here and I will be using third part that means H 2 + infinity that expression I will be using and everywhere I should use then only I can find out because evanescent tail if it is it is some penetration depth is there, but it will be extended theoretically exponential time it is excellent up to infinity. That is why for integration purpose I can use this limit then I can find what is the total power that is flowing for the certain mode if I just consider a solution for a certain mode?

I will be writing that particular mode that beta value for TE polarization if it is TM polarization you solve beta m then you can calculate what is the value for the power and a little bit of modifications? So, this expression if you just do this thing, beta equal to you write omega / C n effective m for TE polarization if you just replace that one omega mu 0 and mu 0 epsilon 0 you know that mu 0 epsilon 0 = 1 / C square. So, all these things if you do then you can little bit modify this expression like this in terms of n effective and eta naught.

Eta naught you know that the intention impedance or mu naught / epsilon not this one. So, we know that power carried by your mode to estimate that you need to know field distribution throughout cladding region and core region and you need to know the effective index of the guided mode if you know that then you can find out basically how much power is flowing of course, in this E y m x know E d is there it is the constant where we consider know most of the time one while plotting the electric field distribution.

We will consider 1 that is normalized to 1 volt per meter for example, it can be 5 volt per meter depending on that it will scale up and once you know normalize E d 1 with respect to E d, you will know what is the relationship with the E c you will know what is the relationship with the E s from the boundary condition. So, you will be able to know only thing is that you need to know what is the value of E d.

So, it is just you are considering normalized 1 you can consider 1 volt per meter and dy if you are considering what that means we can say 1 meter then we can say that power flow in by the guided mode it will be watt per meter. Watt per meter means 1 meter we are considering along the Y direction and X direction I am integrating everything. So, I can calculate what is the power flow power carried by the mode.

So, this is very important, I will be discussing for the 2D when we will be discussing for the 2D wave guide that Y direction also some distribution will be at that time why limit also has to be considered this one the waveguide is very much ideal case. So, ideal case though it is ideal, but we are getting into clear picture how the mode will be there how that will be propagated and what would be the relationship.

What is the dispersion relationship what would be the group velocity everything we are getting, but obviously this is ideal situation where Y direction we have considering infinitely extended which is not practical though? So, this is the lesson see we are just writing finally, this is the final expression in press. So, this part all this orange color part is have corresponding to TM mode and all the green color part expression.

Together it is summarized here all the electric field distributions and other components magnetic field corresponding magnetic field and here for TM polarization you have the solutions and corresponding electric fields and power how much you can calculate how much power it is it will be carried, that can be calculated until for TE polarization also how much it will be carried. You are also able to express.

So, this is where this is how we want to close the discussion on 1D optical waveguide and we have taken only for silicon-on-insulator platform that is basically as you buy this commercially available optical grade as you buy, they are giving you just 1D waveguide already, but this 1D waveguide is not so, practical for integrated photonics purpose, because you need 2D confinement photonic where you need for optical interconnect.

So, we need to now discuss this 2D waveguide how to the waveguide can be designed, what is the theory involved on that and what are the other additional things you need to consider that I will be just discussing in the next lecture. Thank you very much.