

Integrated Photonic Devices and Circuits
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Lecture – 14
Optical Waveguides: Theory and Design
Guided Mode Solutions for Slab Waveguides Contd

Hello welcome. We continue our optical waveguides theory and design we have been discussing so far slab waveguides and what are the guided mode solutions different approach. First we discussed the ray optics model and we understand there is a limitation up to certain extent it is useful beyond that we need to go for solve the full vectorial wave equations. And in the last lecture we have established the full vectorial wave equations starting from Maxwell's equations in homogeneous medium particularly in 3 layer structure for 1d waveguide.

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Optical Waveguides: Theory and Design Slide#2

Guided Mode Solutions for Slab Waveguides Contd.
Example: Silicon-on-Insulator Slab Waveguide (SOI)

$\lambda = 1550 \text{ nm}$

$n_d = 3.4778$
 $n_c = 1.4657$
 $n_s = 1.0000$
 $n_d > n_s > n_c$

Upper Cladding $x \geq H$

Waveguide Core $0 \leq x \leq H$

Lower Cladding $x \leq 0$

$n_d > n_s > n_c$

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Particularly considering silicon on insulator substrate, this is the reference frame we consider this is a Z axis and this is so called device layer with having refractive index n_d and cladding refractive index n_c and substrate which is specifically for silicon on insulator it is a box material I mean refractive index n_s such that $n_d > n_s > n_c$ at $\lambda = 1550$ nanometer the values are given here just for keeping in mind I just keep it stagnated in the corner of the screen.

So, the region for the device layer is 1 dimensional between $x = 0$ to H and semi infinitely extended this direction x is greater than H and x less than 0 and in the Y direction it is also assumed in finite extended. And no electric field change everything we are considering in that certain direction that means also I am considering that any wave is propagating that is only $K Z$ component this direction propagation vector is there. And any mode solutions are supposed to be come that will be because of the standing wave in this direction. So, in this direction there will be some standing wave x direction some kind of confinement will be there.

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Optical Waveguides: Theory and Design Slide#3

Guided Mode Solutions for Slab Waveguides Contd. $\lambda = 1550 \text{ nm}$

Example: Wave Equations for SOI Slab Waveguide

Let's focus only on TE-polarization $\vec{E} = (0, E_y, 0); \vec{H} = (H_x, 0, H_z)$

$\vec{E}(x, z, t) = \hat{a}_y E_y(x) e^{i(\omega t - \beta z)}$ $\beta = \frac{\omega}{c} n_{\text{eff}}$ $n_s \leq n_{\text{eff}} \leq n_d$

Regions and Equations:

- For $x \geq H$: $\frac{d^2 E_y}{dx^2} - \kappa_c^2 E_y = 0$; $E_y(x) = E_c e^{-\kappa_c(x-H)}$ (Evanescent Field)
- For $0 \leq x \leq H$: $\frac{d^2 E_y}{dx^2} + \kappa_d^2 E_y = 0$; $E_y(x) = E_d \cos(\kappa_d x - \phi_s)$ (Unknowns! $n_{\text{eff}}, \phi_s, E_c, E_d, E_s$)
- For $x \leq 0$: $\frac{d^2 E_y}{dx^2} - \kappa_s^2 E_y = 0$; $E_y(x) = E_s e^{\kappa_s x}$ (Evanescent Field)

Refractive Indices: $n_d = 3.4778$, $n_1 = 1.4657$, $n_2 = 1.0000$, $n_c = 1.0000$, $n_s > n_c > n_1$

Wave Numbers: $\kappa_c^2 = \frac{\omega^2}{c^2} (n_c^2 - n_{\text{eff}}^2)$, $\kappa_d^2 = \frac{\omega^2}{c^2} (n_d^2 - n_{\text{eff}}^2)$, $\kappa_s^2 = \frac{\omega^2}{c^2} (n_s^2 - n_{\text{eff}}^2)$

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And as I mentioned that we are focusing only on TE polarization, were only E_y component will be there that is the tangential component which is easier to handle E_y for TE polarization in the interfaces to be continuous. So, be continuous here and so we have defined also the homogeneous equation in 3 different media. It is a cladding region we are defining with kappa c. Kappa c is defined like this; assume that $n_{\text{effective}}$ is the propagation constant.

Effective refractive index of the guided mode that is a propagation constant related to beta is this one and obviously any effective will be less than n_d and n_s greater than n_s obviously if it is greater than n_s it should be greater than n_c . And as I mentioned this whenever you were moving from top cladding to insights device layer the kappa d will become kappa d square where a kappa d is defined like this and in that lower cladding kappa s this is defined by this.

So, considering $n_{\text{effective}}$ this condition κ_c^2 , κ_d^2 , κ_s^2 all are positive values because $n_{\text{effective}}$ is greater than n_s , $n_{\text{effective}}$ is lower than n_d , $n_{\text{effective}}$ is greater than n_c . So that is why all these are positive and they are corresponding solutions we have just considered like this in the top cladding covering region and standing wave in the core medium within this range.

And again it is exponentially decaying in the region we are $x < 0$. And we mentioned that these are basically unknowns $n_{\text{effective}}$, ϕ_s , E_c , $n_{\text{effective}}$ are sometimes you can say that beta values are, beta is equal to say $2\pi / \lambda / n_{\text{effective}}$ or this is $\omega / c n_{\text{effective}}$ and E_c we have considered E_c here we have consider E_d here we have considered E_s here just a constant and along with that we have ϕ_s .

So, all these things we unknown to us, we need to understand, we need to know how to evaluate them then I can say that what will be the modal picture how they are confined how the field distribution along this x direction for a E_y . E_y is important for electric field only component for TE polarization. And since it is exponentially κ is positive the solutions coming like that it will be exponential decaying like this direction or this direction field strength that why it is called evanescent field.

So, no energy will be flowing; no energy flow in this direction but energy can propagating this direction travel as a traveling wave and which is actually defined by $e^{-\beta z}$ the β is the actual figure of merit actual value corresponding to different modes, we will be discussing that in course of time

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Guided Mode Solutions for Slab Waveguides Contd..

Example: Wave Equations for SOI Slab Waveguide

Let's focus only on TE-polarization $\mathbf{TE} :: \vec{E} = (0, E_y, 0); \vec{H} = (H_x, 0, H_z)$

$\vec{E}(x, z, t) = \hat{a}_y E_y(x) e^{i(\omega t - \beta z)}$ $\beta = \frac{\omega}{c} n_{eff}$ $n_s \leq n_{eff} \leq n_d$

$n_d = 3.4778$
 $n_s = 1.4657$
 $n_c = 1.0000$

$n_d > n_s > n_c$


for $x \geq H$
 $E_y(x) = E_d e^{-\kappa_c(x-H)}$ $E_y(H) = E_c$ **Continuity at $x = H$**
 $E_c = E_d \cos(\kappa_d H - \phi_s)$

for $0 \leq x \leq H$
 $E_y(x) = E_d \cos(\kappa_d x - \phi_s)$ $E_y(H) = E_d \cos(\kappa_d H - \phi_s)$

for $x \leq 0$
 $E_y(x) = E_s e^{\kappa_s x}$ $E_y(0) = E_s$ **Continuity at $x = 0$**
 $E_s = E_d \cos \phi_s$

Unknowns!
 $n_{eff}, \phi_s, E_c, E_d, E_s$

$\kappa_c^2 = \frac{\omega^2}{c^2} (n_c^2 - n_{eff}^2)$ $\kappa_d^2 = \frac{\omega^2}{c^2} (n_d^2 - n_{eff}^2)$ $\kappa_s^2 = \frac{\omega^2}{c^2} (n_s^2 - n_{eff}^2)$



So, far so good so what we do? We have the solution this one this is valid from x greater than equal to 0 to x less than or equal to H that means this is valid obviously at x = 0 and this is x less than or equal to 0. So, this is also valid at x = 0 that means if I put x = 0 here and x = 0 here whatever the value comes they must be equal. So, this is at x = 0 from this expression what we get we get x 0 it is actually cosine E d cosine phi s.

And here x = 0 it is actually E s so this thing must be 0 must be equal that is what I have written here. So, similarly if I go to the other interface, if I put this solutions here x = H whatever value will be there I am putting x = H then it the exponential term x - H 0 so it is E c and from here if I just put x = H that is device layer thickness. So, E y, E d cos k d H - phi s there must be equal at x = 0 they are continuous tangential component we are talking only E y component.

So that is the reason they must be equal again so I get another equation and the interface. So, that means this thing helps us to get some relationship between E c and E d here we could find and here also we can find E s E d. So, suppose if I know the value of E d so, you can consider E d may be here whatever considering maybe normalized to 1 then I can find out E s and I can find out E c provided if I know what is the value of kappa d and what is the value of phi s.

So, now next thing is that E c, E d and E s they are immaterial they can be found from these equations obviously but next thing is that kappa d and phi s and kappa d and phi s is how to find

that. So, you need to derive one more equations couple more equations so that not one more couple more equations you need to solve kappa d phi s of course we need to know not only kappa d phi s you need to know also kappa c, kappa s square all they will be known.

Once you know the n effective that is you should keep in mind effective index solution any solutions any guided mode solution if you want to be should have a real visible solution along z direction that is related to beta and beta is it related to n effective that much must be known to you in advance. So, I have just written those 2 equations this equation I have written here. And this equation I have written here for I just stack it for future and we are just keeping in mind that these are unknowns though we know that E c, E d, E s somewhat related to this equation but in spite of that we should keep in mind there are unknowns I have just written.

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Guided Mode Solutions for Slab Waveguides Contd. $\lambda = 1550 \text{ nm}$

Example: Wave Equations for SOI Slab Waveguide

Let's focus only on TE-polarization $\mathbf{TE} :: \mathbf{E} = (0, E_y, 0); \mathbf{H} = (H_x, 0, H_z)$

$\mathbf{E}(x, z, t) = \hat{a}_y E_y(x) e^{j(\omega t - \beta z)}$ $\beta = \frac{\omega}{c} n_{\text{eff}}$ $n_s \leq n_{\text{eff}} \leq n_d$

$n_d = 3.4778$
 $n_s = 1.4657$
 $n_c = 1.0000$

$n_d > n_s > n_c$

for $x > H$
 $E_y(x) = E_d e^{-\kappa_d(x-H)}$

$E_x = E_d \cos(\kappa_d H - \phi_s)$

for $0 \leq x \leq H$
 $E_y(x) = E_d \cos(\kappa_d x - \phi_s)$

$E_x = E_d \cos \phi_s$

for $x \leq 0$
 $E_y(x) = E_s e^{\kappa_s x}$

Unknowns!
 $n_{\text{eff}}, \phi_s, E_s, E_d, E_c$

$k_c^2 = \frac{\omega^2}{c^2} (n_c^2 - n_{\text{eff}}^2)$ $k_d^2 = \frac{\omega^2}{c^2} (n_d^2 - n_{\text{eff}}^2)$ $k_s^2 = \frac{\omega^2}{c^2} (n_s^2 - n_{\text{eff}}^2)$

$\nabla \times \mathbf{E} = -j\omega\mu_0\mathbf{H}$ $\nabla \times \mathbf{H} = \mathbf{j}\omega\epsilon_0\mathbf{E}$

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Now next thing is that we just put down here to clear up some space are here. So, these 2 equations at the boundary whatever coming I have just repeated here and then we just follow Maxwell's equation now, because we have the solutions for the E y and E y all 3 region I know the field distribution electric distribution but we have some other components H x, H z. Let us try to see concentrate on those components.

And if we can find some more equations so that they may help us to find all these unknown constant values so we start from this curl equation for the electric field in frequency domain

Fourier domain curl $\nabla \times \mathbf{E} = -j\omega\mu_0\mathbf{H}$ this basically curl $\nabla \times \mathbf{E} = -\nabla \mathbf{B} \cdot \nabla t$ that expression $\nabla \cdot \mathbf{t} = j\omega\mu_0\mathbf{H}$ that is what there are true.

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Guided Mode Solutions for Slab Waveguides Contd.
 Example: Wave Equations for SOI Slab Waveguide

Let's focus only on TE-polarization $\mathbf{TE} :: \mathbf{E} = (0, E_y, 0); \mathbf{H} = (H_x, 0, H_z)$

$\vec{E}(x, z, t) = \hat{a}_y E_y(x) e^{j(\omega t - \beta z)}$ $\beta = \frac{\omega}{c} n_{eff}$ $n_s \leq n_{eff} \leq n_d$ $n_c = 1.0000$ $n_d > n_s > n_c$

$\lambda = 1550 \text{ nm}$
 $n_d = 3.4778$
 $n_s = 1.4657$
 $n_c = 1.0000$

for $x > H$
 $E_y(x) = E_d e^{-\kappa_d(x-H)}$
 $E_x = E_d \cos(\kappa_d H - \phi_s)$

for $0 \leq x \leq H$
 $E_y(x) = E_d \cos(\kappa_d x - \phi_s)$
 $E_x = E_d \cos \phi_s$

for $x \leq 0$
 $E_y(x) = E_s e^{\kappa_s x}$

Unknowns!
 $n_{eff}, \phi_s, E_s, E_d, E_x$

$\kappa_s^2 = \frac{\omega^2}{c^2} (n_s^2 - n_{eff}^2)$ $\kappa_d^2 = \frac{\omega^2}{c^2} (n_d^2 - n_{eff}^2)$ $\kappa_c^2 = \frac{\omega^2}{c^2} (n_{eff}^2 - n_c^2)$

$\nabla \times \vec{E} = -j\omega\mu_0 \vec{H}$
 $\frac{\partial}{\partial y} = 0; \frac{\partial}{\partial z} \equiv -j\beta$
 $\begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & 0 & -j\beta \\ 0 & E_y(x) & 0 \end{vmatrix} = -j\omega\mu_0 (\hat{a}_x H_x + \hat{a}_z H_z)$

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Now we know for this 1d waveguide structure we know $\nabla \cdot \nabla y = 0$ that means along y direction field is not changing this is assumption and $\nabla \cdot \nabla z$ if you just consider all the z dependencies e to the power minus beta z is there. So, if you do derivative the partial derivative with respect to z that minus j beta and same thing will be coming. So, we know this thing next thing what we know this curl of a vector we can expand in this matrix form determinant form.

So, this is your unit vector along the x direction, unit vector along y direction, unit vector along z direction and then $\nabla \cdot \nabla x$ and $\nabla \cdot \nabla z$ $\nabla \cdot \nabla y = 0$ I put here and $\nabla \cdot \nabla z = -j\beta$. So, basically in principle you should be writing like this curl a x, a y and a z and it should be $\nabla \cdot \nabla x, \nabla \cdot \nabla y, \nabla \cdot \nabla z$ and then it should be E_x, E_y, E_z that is actually if it is a vector \mathbf{E} c if it is A vector suppose curl of A you want to find out what is that thing then I would be writing A_x, A_y, A_z .

So, it is E vector that is why I have written and $\nabla \cdot \nabla y = 0$ for our waveguide reference and $\nabla \cdot \nabla z = -j\beta$ because of consideration that mode is confined along x direction propagating along z direction. So, y direction can be 0 so $\nabla \cdot \nabla 0$ this thing we write now. And right hand side if you see $j\omega\mu_0$ then I can consider H field is having x component, y

component, z component though we have defined earlier that. This y component would not be there that is not required at all but once we consider E y component and no other component is there for electric field let will automatically find this is H y is 0.

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Guided Mode Solutions for Slab Waveguides Contd..
Example: Wave Equations for SOI Slab Waveguide

Let's focus only on TE-polarization $TE :: \vec{E} = (0, E_y, 0); \vec{H} = (H_x, 0, H_z)$

$\vec{E}(x, z, t) = \hat{a}_y E_y(x) e^{j(\omega t - \beta z)}$ $\beta = \frac{\omega}{c} n_{eff}$ $n_s \leq n_{eff} \leq n_d$ $n_d > n_s > n_c$

$\lambda = 1550 \text{ nm}$
 $n_d = 3.4778$
 $n_s = 1.4657$
 $n_c = 1.0000$

for $x \geq H$
 $E_y(x) = E_d e^{-\kappa_c(x-H)}$

for $0 \leq x \leq H$
 $E_z = E_d \cos(\kappa_c H - \phi_s)$
 $E_y(x) = E_d \cos(\kappa_d x - \phi_s)$
 $E_x = E_d \cos \phi_s$

for $x \leq 0$
 $E_y(x) = E_s e^{\kappa_s x}$

Unknowns!
 $n_{eff}, \phi_s, E_c, E_d, E_s$

$\nabla \times \vec{E} = -j\omega\mu_0 \vec{H}$
 $\frac{\partial}{\partial y} = 0; \frac{\partial}{\partial z} \equiv -j\beta$
 $\hat{a}_x \frac{\partial}{\partial x} \begin{pmatrix} \hat{a}_x \\ \hat{a}_y \\ \hat{a}_z \end{pmatrix} = -j\omega\mu_0 (\hat{a}_x H_x + \hat{a}_y H_y + \hat{a}_z H_z)$
 $\begin{pmatrix} \hat{a}_x \\ \hat{a}_y \\ \hat{a}_z \end{pmatrix} \cdot \begin{pmatrix} \hat{a}_x \\ \hat{a}_y \\ \hat{a}_z \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$
 $H_y(x) = 0$
 $H_x(x) = -\left(\frac{\beta}{\omega\mu_0}\right) E_y(x)$
 $H_z(x) = \left(\frac{j}{\omega\mu_0}\right) \frac{d}{dx} E_y(x)$

$\kappa_c^2 = \frac{\omega^2}{c^2} (n_d^2 - n_{eff}^2)$ $\kappa_d^2 = \frac{\omega^2}{c^2} (n_d^2 - n_{eff}^2)$ $\kappa_s^2 = \frac{\omega^2}{c^2} (n_{eff}^2 - n_s^2)$

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So that is what we will be getting next thing so H y x 0 typically but what about H x because I want to find out this is H x this is H z. So, this side I will find out what is the x component and x component if 2 vectors are identical left hand side and right hand side so individual components must be equal. So, H x obviously that should be also x dependent variations confinement will be there I can say that a x component will be this one you have to multiply this one and you have to subtract by this one.

So, this one meant that means you have J beta E y x = j omega 0 H x, x component x component equate and then you get this relationship. So that means if I know E y somehow E y x is known that expression we have derived earlier. So that expression is actually directly related to magnetic field x component only some scaling factor will be there beta whatever beta value comes we are this is unknown though beta is related to n effective like that unknown though.

But once better known but we can say that it is a kind of beta is fixed for a certain mode that means for that mode E y and H x they are just scaled up this proportional only negative sign there that is it no other difference. And similarly from this expression I can find out z component

z component the left hand side, z component right hand side if I equate then z component of the magnetic field I can say that is a z component means this thing I am just concerned z these 2 things barring then that means this to this and this to this that means $\nabla \times \mathbf{E} = \mathbf{j}$.

So, if you $\nabla \times \mathbf{E} = \mathbf{j}$ if you do and along with that you have $\mathbf{j} = j \omega \mu_0 \mathbf{H}$ this side. So that means $\mathbf{j} / \omega \mu_0$ so I can relate. So, basically if you know E_y , y component of the electric field if you know z component of the magnetic field you can find out by making a derivative and with some scaling $j / \omega \mu_0$. So that is the relationship we are getting from simply from curl equations here.

So, and here also if you see y component simply if you see y component this one and this will bring that means this multiplied by this one 0 minus this by this 0 also so obviously $H_y = 0$ that is clear. So, I can if I know E_y component obviously I have derived for x greater than H within H less than 0 then if I just make a derivative with respect to x then I can get magnetic field one for the z component and if it is just multiplying this one I will be getting y component.

So that means if I concentrate only on E_y component the other component magnetic fields they can be derived using just standard this way and obviously you know all these thing once you find say H_x and H_z also you will be there that is also a function of x that will be also function of x once you make a derivative they will be also will be oscillating with e to the power $j \omega t$ because that is common to both electric field and magnetic field.

So, electric field and magnetic fields this part is common that does not mean that e electric field and magnetic field need to be in phase because we can always consider these fields these solutions E_y whatever it is considered real they can be complex also. So, their solutions can be complex so they can be going out of phase or not that we will see for the moment it is as of now it is looking like this set, anyway you see this a imaginary part is coming.

That means electric field whatever the thing is they are here y component x component they do not have they are in phase but if you just consider this one if you take a derivative also depending on this function and j everything it will see what is their phase relationship etcetera

that would be set. So, we know one more thing that if you solve for E y other components can be easily found using Maxwell's equations.

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Guided Mode Solutions for Slab Waveguides Contd.
Example: Wave Equations for SOI Slab Waveguide

Let's focus only on TE-polarization $TE :: \vec{E} = (0, E_y, 0); \vec{H} = (H_x, 0, H_z)$

$\vec{E}(x, z, t) = \hat{a}_y E_y(x) e^{j(\omega t - \beta z)}$ $\beta = \frac{\omega}{c} n_{eff}$ $n_s \leq n_{eff} \leq n_d$ $n_c = 1.0000$ $n_d > n_s > n_c$

$\lambda = 1550 \text{ nm}$
 $n_d = 3.4778$
 $n_s = 1.4657$
 $n_c = 1.0000$

for $x > H$
 $E_y(x) = E_c e^{-\kappa_c(x-H)}$
 $E_c = E_d \cos(\kappa_d H - \phi_s)$

for $0 \leq x \leq H$
 $E_y(x) = E_d \cos(\kappa_d x - \phi_s)$
 $E_s = E_d \cos \phi_s$

for $x \leq 0$
 $E_y(x) = E_s e^{\kappa_s x}$

Unknowns!
 $n_{eff}, \phi_s, E_c, E_d, E_s$

$\kappa_c^2 = \frac{\omega^2}{c^2} (n_c^2 - n_{eff}^2)$ $\kappa_d^2 = \frac{\omega^2}{c^2} (n_d^2 - n_{eff}^2)$ $\kappa_s^2 = \frac{\omega^2}{c^2} (n_{eff}^2 - n_s^2)$

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So, I have written down that thing that H x, and H z. Now you see I just tried to find out what is the H z component in 3 zones x greater than H that means in this region x within H x less than 0 only H z I am focusing why H z? Because out of 2 components of the magnetic field x component and z component H z component that means it is oscillating along z magnetic field oscillating along z direction that means along propagation direction.

So that propagation direction means that is actually tangential to the interface. So, this is the z component H z component will be oscillating this direction. So, this is a tangential to the interface both the interface that is tangential. So, you know tangential components are continuous at the interface so instead of considering x component and focus on z component of the magnetic field so that at the boundary continuity application I can apply.

So that is why I just derived directly what is that I have just found directly what is the tangential component or longitudinal component of the magnetic field obviously for x component it is just a proportional you do not need to make any derivative or etcetera because the longitudinal component you need to make some derivative. So, if you make a derivative here this one is

coming minus k_c you will be there $j\omega$ not is there this relationship along with that minus k_c κ_c will be there that is written.

So, similarly within the co region that is the equation and this is the lower cladding region this is the equations straightforward it is just coming from Maxwell's equations.

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Optical Waveguides: Theory and Design Slide#13

Guided Mode Solutions for Slab Waveguides Contd.. $\lambda = 1550 \text{ nm}$

Example: Wave Equations for SOI Slab Waveguide

Let's focus only on TE-polarization $\vec{TE} :: \vec{E} = (0, E_y, 0); \vec{H} = (H_x, 0, H_z)$

$n_d = 3.4778$
 $n_s = 1.4657$
 $n_c = 1.0000$
 $n_d > n_s > n_c$

$\vec{E}(x, z, t) = \hat{a}_y E_y(x) e^{j(\omega t - \beta z)}$ $\beta = \frac{\omega}{c} n_{eff}$ $n_s \leq n_{eff} \leq n_d$

for $x \geq H$

$E_y(x) = E_d e^{-\kappa_c(x-H)}$ $H_x(x) = -\left(\frac{j}{\omega\mu_0}\right) \kappa_c E_d e^{-\kappa_c(x-H)}$

$E_z = E_d \cos(\kappa_d H - \phi_s)$

for $0 \leq x \leq H$

$E_y(x) = E_d \cos(\kappa_d x - \phi_s)$ $H_x(x) = -\left(\frac{j}{\omega\mu_0}\right) \kappa_d E_d \sin(\kappa_d x - \phi_s)$

$E_z = E_d \cos \phi_s$

for $x \leq 0$

$E_y(x) = E_s e^{\kappa_s x}$ $H_x(x) = \left(\frac{j}{\omega\mu_0}\right) \kappa_s E_s e^{\kappa_s x}$

Unknowns!
 $n_{eff}, \phi_s, E_c, E_d, E_s$

$\kappa_c^2 = \frac{\omega^2}{c^2} (n_c^2 - n_{eff}^2)$

$\kappa_d^2 = \frac{\omega^2}{c^2} (n_d^2 - n_{eff}^2)$

$\kappa_s^2 = \frac{\omega^2}{c^2} (n_s^2 - n_{eff}^2)$

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I have written down that this thing I have written now good. So, I have now all the components in our hand that means E_y component they are in 3 region and they are magnitude how they are related that one equation this equation this equation also known E_c how it is related to E_d and E_s how it is related to E_d and also magnetic field tangential component of the magnetic field. And normal component of the magnetic field H_x component this direction also is known but I am focusing more on tangential component here. So, I have written down here all this just to duplicate it here.

(Refer Slide Time: 19:08)



Guided Mode Solutions for Slab Waveguides Contd..
Example: Wave Equations for SOI Slab Waveguide

Let's focus only on TE-polarization $TE :: \vec{E} = (0, E_y, 0); \vec{H} = (H_x, 0, H_z)$

$\vec{E}(x, z, t) = \hat{a}_y E_y(x) e^{i(\omega t - \beta z)}$ $\beta = \frac{\omega}{c} n_{eff}$ $n_s \leq n_{eff} \leq n_d$

$\lambda = 1550 \text{ nm}$
 $n_d = 3.4778$
 $n_s = 1.4657$
 $n_c = 1.0000$

$n_d > n_s > n_c$

for $x \geq H$
 $E_y(x) = E_d e^{-\kappa_c(x-H)}$
 $H_x(x) = -\left(\frac{j}{\omega\mu_0}\right) \kappa_c E_d e^{-\kappa_c(x-H)}$

Continuity at $x = H$
 $\kappa_c E_d = \kappa_d E_d \sin(\kappa_d H - \phi_s)$

for $0 \leq x \leq H$
 $E_y(x) = E_d \cos(\kappa_d x - \phi_s)$
 $H_x(x) = -\left(\frac{j}{\omega\mu_0}\right) \kappa_d E_d \sin(\kappa_d x - \phi_s)$

Continuity at $x = 0$
 $\kappa_d E_s = \kappa_d E_d \sin \phi_s$

for $x \leq 0$
 $E_s = E_d \cos \phi_s$
 $E_y(x) = E_s e^{\kappa_s x}$
 $H_x(x) = \left(\frac{j}{\omega\mu_0}\right) \kappa_s E_s e^{\kappa_s x}$

Unknowns!
 $n_{eff}, \phi_s, E_c, E_d, E_s$

$\kappa_c^2 = \frac{\omega^2}{c^2} (n_{eff}^2 - n_c^2)$ $\kappa_d^2 = \frac{\omega^2}{c^2} (n_d^2 - n_{eff}^2)$ $\kappa_s^2 = \frac{\omega^2}{c^2} (n_{eff}^2 - n_s^2)$

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And then I tried to apply continuity at $x = 0$ because tangential component, tangential component $x = 0$ I can write $x = 0$ if I put here this side will become $\kappa_d E_d \sin \phi_s$ times this one and this side will be $j / \omega \mu_0$. Because $x = 0$ if you put this is \sin minus ϕ_s minus sign will come minus, minus plus and here also plus there $x = 0$ you get something so you get it 0 you know if you just equate this thing at $x = 0$ put $x = 0$ and equate these 2 equations.

Then you are get this one. That means I have a relationship one more relationship apart from this one boundary condition here I have one more equation here, similarly I have boundary at $x = H$ I have one more equation here. I just put $x = H$ here, $x = H$ here and then I equate so here $\kappa_d E_d$ and here $\kappa_c E_c$ there how related through this one. So, I have 2 more equations equation number 1, equation number 2, equation number 3, equation number 4 fine.

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Optical Waveguides: Theory and Design Slide#19

Guided Mode Solutions for Slab Waveguides Contd.

Example: Wave Equations for SOI Slab Waveguide

Let's focus only on TE-polarization $TE :: \vec{E} = (0, E_y, 0); \vec{H} = (H_x, 0, H_z)$

$\lambda = 1550 \text{ nm}$
 $n_d = 3.4778$
 $n_s = 1.4657$
 $n_c = 1.0000$
 $n_d > n_s > n_c$

$$\vec{E}(x, z, t) = \hat{a}_y E_y(x) e^{i(\omega t - \beta z)}$$

$$\beta = \frac{\omega}{c} n_{eff} \quad n_s \leq n_{eff} \leq n_d$$

Unknowns!
 $n_{eff}, \phi_s, E_c, E_d, E_s$

$k_c^2 = \frac{\omega^2}{c^2} (n_c^2 - n_{eff}^2)$

$k_d^2 = \frac{\omega^2}{c^2} (n_d^2 - n_{eff}^2)$

$k_s^2 = \frac{\omega^2}{c^2} (n_s^2 - n_{eff}^2)$

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So, I have written down this equation, equation we have written 4 equations. So, our unknowns are this one I should keep in mind, my ultimate goal is that to find out all these values how to find that let us move on to only keeping this thing. Now you relate this to just I have removed the magnetic field because whatever necessary equations I have there so I again concentrating only on electric field and this equation and this equation if you just compare just divide it then what you can eliminate?

You can eliminate E d, you can eliminate E c then that means I can have a relationship between phi s and k c, k d, kappa c, kappa d, phi s, kappa c, kappa d, kappa c, kappa d what is both cases n effective unknown here also n effective unknown that means in this equation I have 2 unknowns. So, if I want to solve n effective and phi s n effective kappa d also phi n effective right so in 2 unknowns are phi s and n effective unknown. So, relating these things one equation I am getting and another thing if I relate this to a question I get another equation.

So, again I have 2 unknowns n effective in this equation n effective and phi s, n effective and phi s. So, I have 2 equations equation number 1, equation number 2, 2 unknowns. So, it is it would be easier for us now to solve that 2 unknowns 2 equations you can just solve since they are not so easy just to eliminate one and to get one solutions Gaussian elimination method that is not so easy. So, we little bit moving a little bit tricky way.

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Optical Waveguides: Theory and Design Slide#24

Guided Mode Solutions for Slab Waveguides Contd..

Example: Wave Equations for SOI Slab Waveguide

Let's focus only on TE-polarization $TE :: \vec{E} = (0, E_y, 0); \vec{H} = (H_x, 0, H_z)$

$\lambda = 1550 \text{ nm}$
 $n_d = 3.4778$
 $n_s = 1.4657$
 $n_c = 1.0000$
 $n_d > n_s > n_c$

$\vec{E}(x, z, t) = \hat{a}_y E_y(x) e^{i(\omega t - \beta z)}$ $\beta = \frac{\omega}{c} n_{eff}$ $n_c \leq n_{eff} \leq n_d$

for $x \geq H$: $E_y(x) = E_c e^{-\kappa_c(x-H)}$

for $0 \leq x \leq H$: $E_y(x) = E_d \cos(\kappa_d x - \phi_s)$

for $x \leq 0$: $E_y(x) = E_s e^{\kappa_s x}$

Assume $\tan \phi_c = \frac{\kappa_c}{\kappa_d}$

$\tan(\kappa_d H - \phi_s) = \frac{\kappa_c}{\kappa_d}$

$\kappa_d H - \phi_s = m\pi + \phi_c$

$\kappa_d H = m\pi + \phi_c + \phi_s$

$m = 0, 1, 2, 3, \dots$

Unknowns! $n_{eff}, \phi_s, E_c, E_d, E_s$

$\kappa_c^2 = \frac{\omega^2}{c^2} (n_c^2 - n_{eff}^2)$ $\kappa_d^2 = \frac{\omega^2}{c^2} (n_d^2 - n_{eff}^2)$ $\kappa_s^2 = \frac{\omega^2}{c^2} (n_s^2 - n_{eff}^2)$

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So, I have written down this equation like this to get some space this side. So, you assume this kappa c, kappa d because you know kappa s kappa d you are getting tan phi s. So, kappa s means this one and kappa d the ratio square root that is actually tan phi s. So, again kappa c that is related to your top cladding cover cladding. So, there also I can just define one more parameter introduce one more constant phi c intentionally put phi c.

That means one more constant I introduced that will help us actually because kappa c, kappa d known that means I know phi c so once n effective known I can know phi c so effectively this constant is not additional thing it is all inclusive. So, now what we get this equation we get kappa d H - phi s and right hand side phi c because if I compare this 2 so this, this I am replacing here tan phi.

So, I can write kappa d H - phi s = m pi + phi c because you will not tan function it is periodic over phi. So that is the reason we can easily write this one I am now getting a generic equation where m = 0, 1, 2, 3, 4, n s so this equation is true for this thing. So, every phi it will be repeating so I am just now similar to our ray optics model I could introduce one more integer here also similar to that that is m.

So that means if I try to solve this one particular 0 entry will give me one set of solutions another set of m = 1 I will get another set of constant solutions m = 2 another set of constant solutions.

So, it is something similar to whatever we solved earlier in ray optics model θ_1 , θ_2 and so in effective n there so we are also almost similar results we are getting. So, from this one I am just simply manipulating left hand side right hand side.

So, you are getting same way left hand side right hand side here also I have $n_{\text{effective}}$ here function of $n_{\text{effective}}$ this one also will be function of $n_{\text{effective}}$, this one also function of $n_{\text{effective}}$ both side is a function of $n_{\text{effective}}$ I have completely eliminated the constant E_c , E_d , E_s all these are not there I have just equation a single equation which is only $n_{\text{effective}}$ involved nothing else.

So that means $\tan \phi_s$ also this is similar to ϕ_c this is also only if you check clearly this $n_{\text{effective}}$ involves k_s / k_d all are $n_{\text{effective}}$ all other things are known ω_c , n_d , n_c , n_s everything known. So, I have the transcendental equation here and that is a some kind of Eigen solutions we can consider for guided mode. So, this thing I am just simply writing $k_d H = m \pi \phi_c = \tan^{-1} k_c k_d$ and ϕ_s equal to this one.

This is so called transcendental equation left hand side and right hand side both are actually $n_{\text{effective}}$ there. So, we can similar to ray optics model we can just plot left hand side as a function of $n_{\text{effective}}$, I can plot left hand side as a function of $n_{\text{effective}}$ some value $n_{\text{effective}}$ can range from n_s to n_d and right hand side also I can plot as a function $n_{\text{effective}}$. So, I will be getting somewhere I will be getting some intersection.

And that intersection will be solution for the $n_{\text{effective}}$ discrete solutions you will be getting 4 different m values. So, instead of solving θ and then $n_{\text{effective}}$ we have the equation now directly we can solve $n_{\text{effective}}$ instead of θ . So, then we do not need to the ultimate goal is that we have shown that the ray optics model the θ it is very difficult to control that which particular angle you can go for you need to couple.

Instead I think normally in practice we can couple light for all modes but Ray optics picture it is somehow it is not giving good understanding how you could excite different modes because θ angle is somewhat Snell's law dominating controlled by Snell's law and according to that

you cannot control in principle but here we do not have any theta we have the only solution for n effective.

And then we can see how coupling is whatever the coupling is coming from the whenever you were signing light from the output side and trying to launch different modes and normally practically they are coupling and how much they will be coupling to different modes that way it is we should find out in a different way at least you do not need to bother about theta all this way that it is in principle it is satisfying or not those things you do not need to concentrate.

(Refer Slide Time: 27:34)

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Guided Mode Solutions for Slab Waveguides Contd.
Example: Wave Equations for SOI Slab Waveguide
 Let's focus only on TE-polarization $\vec{T}E :: \vec{E} = (0, E_y, 0); \vec{H} = (H_x, 0, H_z)$

$\vec{E}(x, z, t) = \hat{a}_y E_y(x) e^{i(\omega t - \beta z)}$ $\beta = \frac{\omega}{c} n_{eff}$ $n_s \leq n_{eff} \leq n_d$

$n_d = 3.4778$
 $n_s = 1.4657$
 $n_c = 1.0000$
 $n_d > n_s > n_c$

$\lambda = 1550 \text{ nm}$

for $x \geq H$
 $E_y(x) = E_d e^{-\kappa_c(x-H)}$

for $0 \leq x \leq H$
 $E_x = E_d \cos(\kappa_d H - \phi_s)$
 $E_y(x) = E_d \cos(\kappa_d x - \phi_s)$
 $E_z = E_d \cos \phi_s$

for $x \leq 0$
 $E_y(x) = E_s e^{\kappa_s x}$

Unknowns!
 $n_{eff}, \phi_s, E_c, E_d, E_s$

$\tan(\kappa_d H - \phi_s) = \frac{\kappa_c}{\kappa_d}$ Assume $\tan \phi_s = \frac{\kappa_c}{\kappa_d}$
 $\kappa_d H - \phi_s = m\pi + \phi_c$
 $\kappa_d H = m\pi + \phi_c + \phi_s$
 $m = 0, 1, 2, 3, \dots$

$\kappa_d H = m\pi + \tan^{-1}\left(\frac{\kappa_c}{\kappa_d}\right) + \tan^{-1}\left(\frac{\kappa_s}{\kappa_d}\right)$ Solve n_{eff}^m

$\kappa_c^2 = \frac{\omega^2}{c^2} (n_c^2 - n_{eff}^2)$
 $\kappa_d^2 = \frac{\omega^2}{c^2} (n_d^2 - n_{eff}^2)$
 $\kappa_s^2 = \frac{\omega^2}{c^2} (n_{eff}^2 - n_s^2)$

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So, next thing is that that from this equation you can solve whatever the n effective m if you just change a m value you give entry m = 1 value n effective 0 and you get m = 1 you get another solution and for that everywhere I am just using this thing for certain value m n effective will be there I will be finding first n effective 0 and then I will be finding n effective 1 per m = 1 and then I will be finding n effective 2 for this thing I can solve.

(Refer Slide Time: 28:06)



Guided Mode Solutions for Slab Waveguides Contd.
Example: Wave Equations for SOI Slab Waveguide

Let's focus only on TE-polarization $\mathbf{TE} :: \vec{E} = (0, E_y, 0); \vec{H} = (H_x, 0, H_z)$

$\vec{E}(x, z, t) = \hat{a}_y E_y(x) e^{i(\omega t - \beta z)}$ $\beta = \frac{\omega}{c} n_{eff}$ $n_s \leq n_{eff} \leq n_d$

$\vec{E}_m(x, z, t) = \hat{a}_y E_m^m(x) e^{i(\omega t - \beta_m z)}$ $\beta = \frac{\omega}{c} n_{eff}^m$ $n_d > n_s > n_{eff}$

for $x \geq H$
 $E_y(x) = E_d e^{-\kappa_d(x-H)}$ $\kappa_d^2 = \frac{2\pi}{\lambda} \sqrt{n_d^2 - n_{eff}^2} = \frac{\omega}{c} \sqrt{n_d^2 - n_{eff}^2}$

for $0 \leq x \leq H$
 $E_y(x) = E_d \cos(\kappa_d x - \phi_s)$ $\kappa_d^2 = \frac{2\pi}{\lambda} \sqrt{n_d^2 - n_{eff}^2} = \frac{\omega}{c} \sqrt{n_d^2 - n_{eff}^2}$

for $x \leq 0$
 $E_y(x) = E_s e^{\kappa_s x}$ $\kappa_s^2 = \frac{2\pi}{\lambda} \sqrt{n_s^2 - n_{eff}^2} = \frac{\omega}{c} \sqrt{n_s^2 - n_{eff}^2}$

Unknowns! $n_{eff}, \phi_s, E_s, E_d, E_x$

$\kappa_d H = m\pi + \tan^{-1} \left(\frac{\kappa_s}{\kappa_d} \right) + \tan^{-1} \left(\frac{\kappa_s}{\kappa_d} \right)$ Solve $\rightarrow n_{eff}^m$

$\kappa_s^2 = \frac{\omega^2}{c^2} (n_s^2 - n_{eff}^2)$ $\kappa_d^2 = \frac{\omega^2}{c^2} (n_d^2 - n_{eff}^2)$ $\kappa_s^2 = \frac{\omega^2}{c^2} (n_s^2 - n_{eff}^2)$

$\lambda = 1550 \text{ nm}$
 $n_d = 3.4778$
 $n_s = 1.4657$
 $n_c = 1.0000$

$m = 0, 1, 2, 3, \dots$

$n_s \leq n_{eff} \leq n_d$

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I know n effective values I know then I can find n effective n right n effective m, m can be 0 then I can find one kappa c value m = 0 I will be putting in if I put here 0 I put j 0 I am finding n effective 0 that solution if I put then I will be getting kappa c 0. And if I put m = 1 that is means n effective 1, n effective 1 I will be putting lambda known here what are the things all other values are known in n c, n c known then I will be getting kappa c 1.

And then I will be getting kappa c 2 corresponding to n effective different solution gives you different value likewise you can get for kappa d, likewise you get for kappa s that means I get a set of kappa values kappa c, kappa d, kappa n. If 0 I can put one kappa value here corresponding kappa value here kappa d corresponding kappa d here, kappa d here. So, I get a one set up solution if corresponding to m = 0 that is the solution for fundamental mode.

Once you are putting 0 here 0 here and getting 0 here and 0 here that means corresponding to m corresponding n effective we are putting whatever the solutions you are getting and you can use their relationship here also and phi s obviously you know phi s value phi s = tan inverse kappa s / kappa d or something like that kappa s kappa d. So, then you can find whatever they profile the all the 3 region you know so that is fantastic.

(Refer Slide Time: 29:46)



Guided Mode Solutions for Slab Waveguides Contd.
Example: Wave Equations for SOI Slab Waveguide

Let's focus only on TE-polarization $TE :: \vec{E} = (0, E_y, 0); \vec{H} = (H_x, 0, H_z)$

$\vec{E}(x, z, t) = \hat{a}_y E_y(x) e^{i(\omega t - \beta z)}$ $\beta = \frac{\omega}{c} n_{eff}$ $n_s \leq n_{eff} \leq n_d$

$x = H$ $E_y^m(x) = E_c^m e^{-\kappa_c^m(x-H)}$ $E_m(x, z, t) = \hat{a}_y E_c^m(x) e^{i(\omega t - \beta_m z)}$ $\beta = \frac{\omega}{c} n_{eff}^m$

$E_c^m = E_d^m \cos(\kappa_d^m H - \phi_s^m)$ $\kappa_c^m = \frac{2\pi}{\lambda} \sqrt{n_{eff}^m{}^2 - n_c^2} = \frac{\omega}{c} \sqrt{n_{eff}^m{}^2 - n_c^2}$

$E_d^m(x) = E_d^m \cos(\kappa_d^m x - \phi_s^m)$ $\kappa_d^m = \frac{2\pi}{\lambda} \sqrt{n_d^2 - n_{eff}^m{}^2} = \frac{\omega}{c} \sqrt{n_d^2 - n_{eff}^m{}^2}$

$E_s^m(x) = E_s^m \cos \phi_s^m$ $\kappa_s^m = \frac{2\pi}{\lambda} \sqrt{n_s^2 - n_{eff}^m{}^2} = \frac{\omega}{c} \sqrt{n_s^2 - n_{eff}^m{}^2}$

$x = 0$ $E_s^m(x) = E_s^m e^{\kappa_s^m x}$ $m = 0, 1, 2, 3, \dots$

Unknowns! $n_{eff}, \phi_s, E_c, E_d, E_s$

$\kappa_d H = m\pi + \tan^{-1} \left(\frac{\kappa_c}{\kappa_d} \right) + \tan^{-1} \left(\frac{\kappa_s}{\kappa_d} \right)$ Solve $\rightarrow n_{eff}^m$

$\kappa_c^2 = \frac{\omega^2}{c^2} (n_{eff}^2 - n_c^2)$ $\kappa_d^2 = \frac{\omega^2}{c^2} (n_d^2 - n_{eff}^2)$ $\kappa_s^2 = \frac{\omega^2}{c^2} (n_s^2 - n_{eff}^2)$

$n_d = 3.4778$
 $n_s = 1.4657$
 $n_c = 1.0000$

$\lambda = 1550 \text{ nm}$

$n_d > n_s > n_c$

$n_s \leq n_{eff} \leq n_d$

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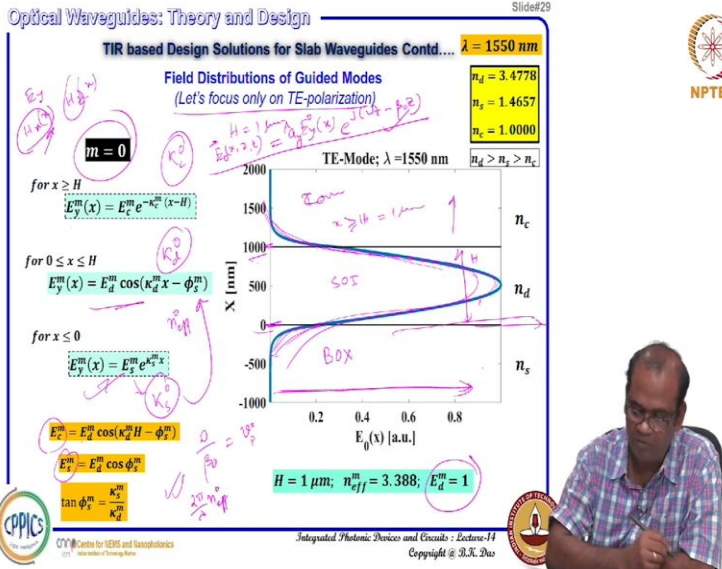


Now you take here that is this whatever I explained that if you just enter here all these values kappa c goes here kappa d comes here kappa s m comes here. And whatever boundary condition also you are getting that will be also no more easy that will be m dependent also E c m, E d m, I have just introduced that corresponding m, m = 0 means everywhere whatever solutions you are getting they will have a different set of relationship.

So, 3 regions, 3 solutions and boundary condition actually relates with the E d m to E c m and E s m here. So, all these things are salt so these unknowns is no more unknowns now all these known only thing is that if we are putting E d = 1 E s I can know from here and E c I can know from here. And phi s also known phi s is related to tan, tan phi is equal to whatever the things tan phi s I mentioned earlier again tan phi s equal to say kappa s / kappa d.

Tan phi s so once you know phi s once you know kappa d once you know E d then you can actually know what is the field distribution here and what is the field distribution here and what is the field distribution here and they will be satisfying actually in the boundary continuing to also be satisfied. So, we will be able to plot also this how these fields will be looking like using these equations and we can derive this equation. So, it is straightforward so we do not need to consider any more theta.

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Now so let us see if I just consider $m = 0$ everywhere we put that means I need to solve $\kappa_c = 0$ I need to solve $\kappa_d = 0$ I need to solve $\kappa_s = 0$ and they can be solved because we could solve $n_{\text{effective}} = 0$ and once we know $n_{\text{effective}} = 0$ and I know this one this one this one and then I can find out what is the value of $\phi_s = m$. And if I consider $E_d = m = 1$ so $E_d = m = 1$ means $\phi_s = m$ known κ_d known I can find E_c and I can find out also E_s all these values will be known.

So, I can actually plot in the co region this is your co region you see I am considering $H = 1$ micron thickness earlier consideration $H = 1$ micron 1 micrometer. So, I start from 0 here so this is your device layer n_d so this is your SOI silicon layer and this is your box layer and this is your top layer, this is for cover x layer. So, this is $x \geq H$ this region $H = 1$ micro meter here it is only nanometer scale and here it is 0 scale.

And then this is cosine function if you just plot using simple MATLAB program then you can find out that field is coming and in the interface you see it is not suddenly it is dropping to 0 because of the evanescent field this field you can plot here they are continuous and similarly you can continue. So that means even though effectively your waveguide thickness is H your field distribution this is again I am just showing this axis is your field strength.

This field strength I am showing though it is actually if you are considering this cover region and co region cladding region this is z axis this is z axis as well as electric field strength time also

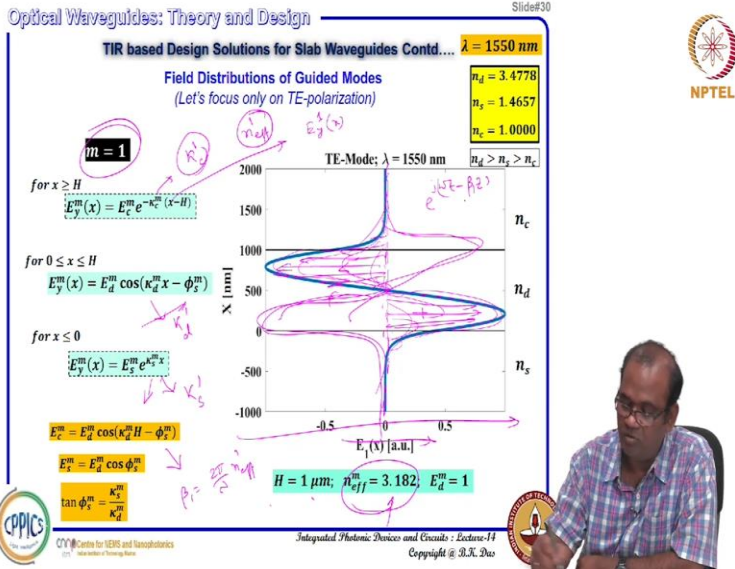
showing, so everything is positive this is a field. And whenever you see this is the field distribution x direction if your x direction waveguide is confined we want to confine our x direction. So, this would be the field distribution for the fundamental mode.

And this distribution is entirely together we will be calling like $E_y(x)$ for 0 field distortion and that is E y component and that will be oscillating along y direction and that will be propagating along z direction with the phase velocity like this $\beta_0 z$. So, this will be your $E_0(x)z(t)$ along y direction I am not considering anything. So, $xz(t)$ that is actually this is the field electric field associated to the fundamental mode.

And you know once you know all this field you have already solution for magnetic field also you can find out for TE polarization we are considering TE polarization you have to consider E y field and H x field H z filled and we know from the Maxwell curl equation how they are related. So, you can also plot how the distribution for H x and H z there will be also x dependent distribution will be there associated with that they will be propagating in this direction.

So that means whatever components you derive that is a field component along the x direction what is the profile and you know which direction they would be oscillating and you know all of them will be traveling along the z direction like a mode. So, particular mode is a particular pattern of electric field, particular pattern of magnetic field and they will be traveling with phase velocity so called phase velocity will be ω / β_{naught} . That will be actually I can consider $V_p = \omega / \beta_0$ that fundamental mode what would the phase velocity β_{naught} would be $2\pi / \lambda$ times n effective 0 fundamental mode effective index.

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So, similarly I can also consider for the $m = 1$ if mode $m = 1$ then you can calculate what is the κ_c because κ_c again you get from n_{eff} you have the value of a n_{eff} use that n_{eff} n_{eff} will be this one for $m = 1$ that value also given here for H equaled 1 micron. So, once you know n_{eff} , you can get κ_c , you can get κ_d you can get κ_s , once you know κ_s , κ_c , κ_d then you can find E_y for the first order mode that variation will be you can find this one and this one will be in the core region and this will be in the lower cladding region.

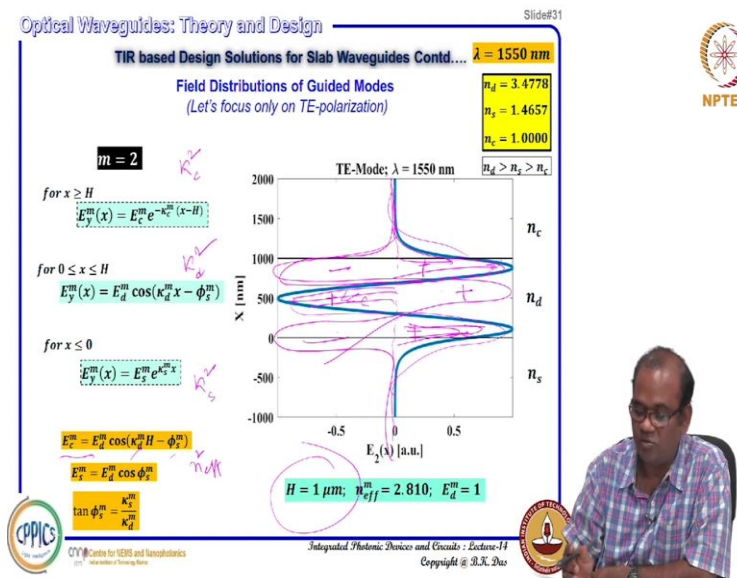
So, after solving if you try to plot the electric field as a function of x you will see this one this is your main position then electric field this is actually electric field if you see electric field strength in the first half electric field will be you see the field direction is this direction at any instant of time if you see field direction you will see at any instant of time this is something like this. So, and then it electric field will be the site that will be negative electric field that is negative sign and this will be positive.

So, this is positive this will be negative and slowly, slowly as time goes on $e^{j\omega t}$ will be there. So, this amplitude if I considering $t = 0$ at t rolls down so that will be actually reducing, reducing, reducing and that will be also reducing at any point if you are reducing and then it will start moving in other direction. So, one half to another half it will be actually so you will be getting this type of pattern again.

And so that will be oscillating back and forth with the frequency of optical frequency the frequency ω with that frequency that will be oscillating but all the time you will see that upper half the electric field will be positive and lower half will be electric field negative at any point this is if it is considered one propagation direction at any point you are considering like that.

So that is how mode distribution again this mode if you see how it is propagating along z direction then we have to write $\omega t - \beta_1 z$ you have to calculate once $m = 1$ β_1 will be equal to $2\pi / \lambda$ times $n_{\text{effective}}$. So, once we know that then we know how the first order mode also that will be propagating.

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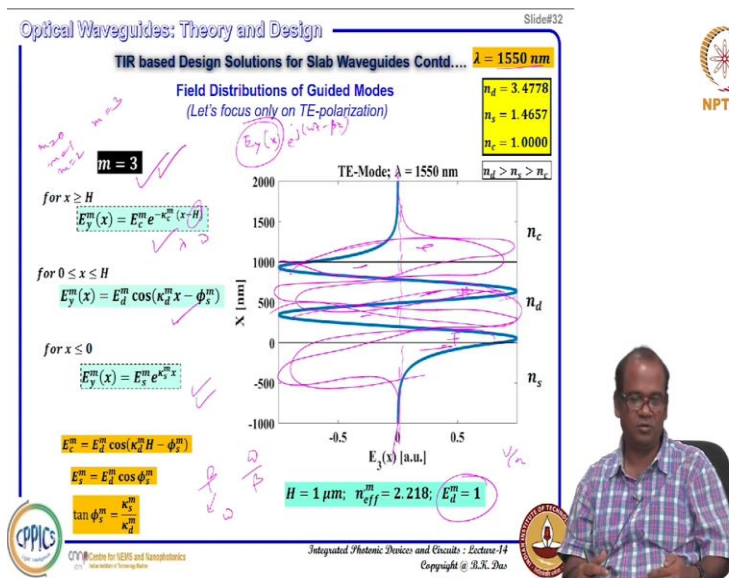
Then for $m = 3$ we have 4 modes for $H = 1$ micrometer there will be 4 modes that 4 modes solutions we could find and that mode 2 field distributions will be this one again you are getting like this that $m = 2$ that means you can again calculate κ_c , κ_d , κ_s once you put $n_{\text{effective}}$ value in κ expression and once you get this one corresponding values for the E_c you can find out all the κ_d ϕ_s you just put down.

And then E_c that for $m = 1$ you can find even you can just plot. And in that case also you see electric field you get one more situation. So, you get this is positive electric field positive

direction at any instant of time if you see this is positive and in the middle it is showing negative direction electric field. So that way it will be oscillation standing wave will be created like this and next instant you can see that after ϕ phase shift after T y 2 time you will see electric field will be something going like this and going like this and going like this.

So, it will be like oscillating standing wave, it will be oscillating like this it will be oscillating like this but at any instant of time if you see if this is plus the side this is plus the middle side will be minus. A middle side becoming plus then this side will be minus this minus so that that is again it will be oscillating with the optical frequencies that is $m = 2$.

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Then the last mode if you see $m = 3$ same way if you just see you get one more situations this will be say for example plus, plus you can consider this is actually 0 point electric field 0 point. So, at any position at $z = 0$ or something like that. So, because you know that E_y you are considering $E_y x e^{j(\omega t - \beta z)}$ you can consider at $z = 0$ and $t = 0$ then only you are plotting this one $E_y x$.

So, $E_y x$ is this these are the expressions these are the expression if you see that at any instant of time at any position along the propagation direction you see this is plus, this is plus, this minus, minus and as time rolls this will be oscillating like a standing wave along the x direction. So, you

can see something like this it will be oscillating energy will be something like this. So, this is 3 nodes anti nodes we will be seeing that maxima, maxima, maxima.

If you the intensity pattern you will be some time it will be averaging over time. So, you will be getting once spot like this here and another spot will be here and then another spot will be here then another you will be getting 4 spots so 4 spot means mode number 4th mode, 4th mode means you are starting from $m = 0$ so that is the first mode $m = 1$ 2nd mode, $m = 2$ 3rd mode, $m = 3$ 4th mode so 4th mode means vertical direction along x direction.

If you the intensity you get 4 spot similarly if you check here you would be seeing 1 spot 2nd spot 3rd spot standing wave pattern and no anti mode using like that like a standing wave pattern along the vertical direction, so 3 spot and you see the 4th spot . So, with this I just stop here for today's lecture now we know how to find out field distribution and once you know beta and you know what the phase velocity is also.

And you can normalize one if it is E_d m is suppose this mode carrying certain field amplitude volt per meter or some there amplitude electric field volt per meter so you can consider 1 volt per meter if it 5 volt per meter than you have to multiply 5 times this one this amplitude will be go 5 times if next time volt per meter then it will be 10 times, now I am considering 1 this is normalized one and all other parameters known to me so I know now.

Then I can find out this is of course calculated for $H = 1$ micro meter and at 1550 nanometer all of them they are lambda dependent or omega dependent right and H dependent H is always there every where H is there so H value depending on the H value depending on the lambda value depending on the refractive indexes extra you can find how many solution how many modes will be supported and what would we are phase velocity almost all the information known. So, now we will be discussing next that.

All the information known what about the model discussion that is effective index if it is calculated for lambda that 1550 nanometer the same effective index will be they are other wave lengths are not that I will be discussing the dispersion basically I will be trying to find out

whether this beta value is a function of omega are not I have calculated for lambda are particular omega so if you are changing omega value for same wave guide same 1 micro meter thickness.

Then how it will be propagating what would be relationship because that is actually very, very important to understand how device will be working in a complex circuit. With this I just stop today for this class.