Integrated Photonics Devices and Circuits
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Lecture – 10
Fundamentals of Lightwaves: EM Waves 1D Optical Waveguide: Ray Optic Model

Hello everyone so, today I am going to discuss about 1D optical waveguide. Just to get an idea that how this optical waveguide can be design and what is their working principle. Since we have learned that metal plates cannot be used for wave guiding or metal waveguides cannot be used for optical waveguide because of losses etcetera and dimensional reduction. And towards that end, we will take a peek model called ray optics model.

Not directly we will solve Maxwell's equation to understand that how waveguide operates, rather we will try to understand with a very basic principle how 1D optical waveguide works and you can visualize prime.

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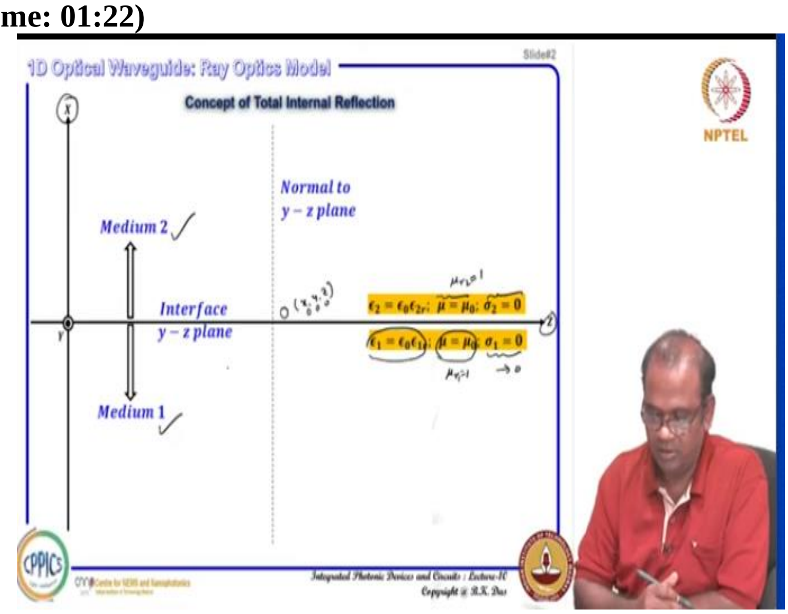

1D Optical Waveguide: Ray Optics Model

Concept of Total Internal Reflection

So, for that purpose, let us consider to understand that let us consider 2 medium, medium 1 and medium 2. They are separated by interface with a yz plane, this is your x axis, this is your z axis, this is y axis. So, that means x axis, y axis, z axis like that and then the medium 1 is characterized by its permittivity epsilon 1, epsilon 0, epsilon 1r. So, that is the relative permittivity for the medium 1 and it is considered a nonmagnetic material.

So, mu r = 1 and dielectric material where we can consider the conductivity plane is to 0 very low. So, similarly we consider another dielectric medium to where that is also sigma 2 equal to tends to 0 nonmagnetic. So, again mu r2 this is mu r1, mu r2 also equal to 1 and permittivity is epsilon 0 epsilon 2r. So, that is how we can define 2 media and they are meeting at yz plane.

Let us consider that one normal to the yz plane is scaled that O that can be you can consider coordinate x, y, z any x, y, z that can be 0 0 0 something like that. This is the medium 2 media, media 1, media 2 and I would like to understand what happens electromagnetic wave propagating from medium 1 towards medium 2 whether we can get a certain reflection. So, called total internal reflection similar to what we could get in a dielectric metal interface that is what the idea we would like to understand.

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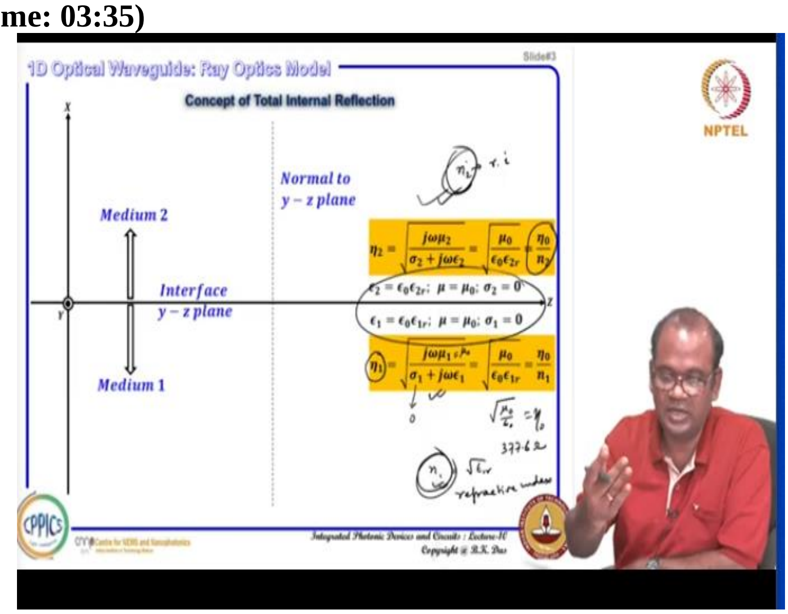

1D Optical Waveguide: Ray Optics Model

Slide#3

Concept of Total Internal Reflection

$$\eta_2 = \sqrt{\frac{j\omega\mu_2}{\sigma_2 + j\omega\epsilon_2}} = \sqrt{\frac{\mu_0}{\epsilon_0\epsilon_{2r}}} = \frac{\eta_0}{n_2}$$

$\epsilon_2 = \epsilon_0\epsilon_{2r};\ \mu = \mu_0;\ \sigma_2 = 0$

$\epsilon_1 = \epsilon_0\epsilon_{1r};\ \mu = \mu_0;\ \sigma_1 = 0$

$$\eta_1 = \sqrt{\frac{j\omega\mu_1}{\sigma_1 + j\omega\epsilon_1}} = \sqrt{\frac{\mu_0}{\epsilon_0\epsilon_{1r}}} = \frac{\eta_0}{n_1}$$

$\sqrt{\frac{\mu_0}{\epsilon_0}} = \eta_0$

377.6 Ω

n_1 $\sqrt{\epsilon_{1r}}$ refractive index

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Now, let little bit going ahead if I try to find out the so called impedance eta value eta 1 we know this is the definition of eta 1 for a given material characterized by epsilon 1 mu 1 and sigma 1 for a given frequency omega is like this. So, whenever I am putting these one equal to sigma 1 = 0, then mu 1 equal to say mu 0 because mu 0 mu r = 1 then we can get say this one 0 j j will cancel. So, it will be omega omega will cancel. So, you will be getting mu 0 times epsilon 1 = epsilon 0 epsilon 1r.

So, again you know that mu 0 / epsilon 0 is the eta naught so, that is the constant for the free space integer impedance for the free space we know the value is 377.6 or something like that free space. So, we can write this one equal to eta naught / n 1 where n 1 is nothing but square root of epsilon 1r. So, dielectric constant if you are just consider as long as the dielectric constant is real, the square root also will be real, we call it a refractive index of the material of the medium 1 refractive index.

So, similarly similar fashion this is also dielectric material medium, so we can also find out eta 2 is nothing but eta naught / n 2, n 2 is the refractive index so the medium 2 again refractive index. So that means if it is a dielectric media and we want to see how electromagnetic wave behaves in the interface we only need to know n 1 and n 2, n 1 and n 2 if it is known then we can we will be able to explain how electromagnetic wave behaves in the interface. So, instead of all these parameters we have to remember only n 1 and n 2 if it is a dielectric medium.

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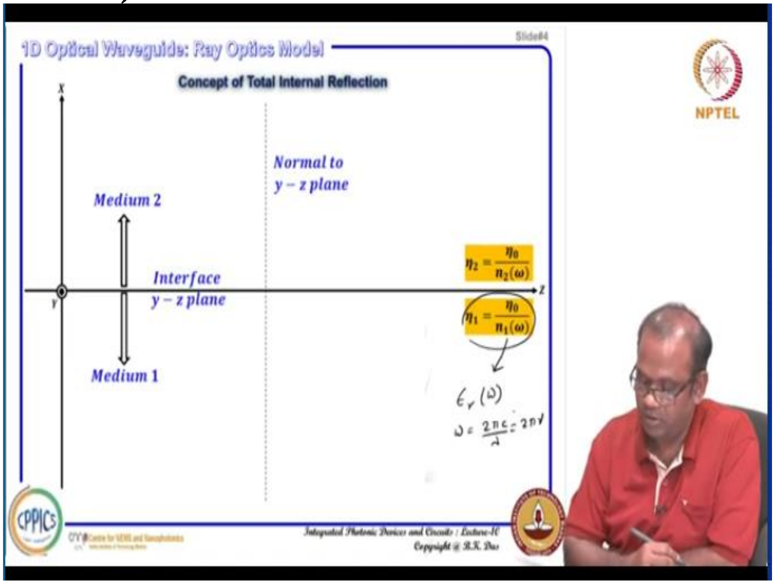

1D Optical Waveguide: Ray Optics Model

Concept of Total Internal Reflection

$\eta_2 = \frac{\eta_0}{n_2(\omega)}$

$\eta_1 = \frac{\eta_0}{n_1(\omega)}$

$\epsilon_r(\omega)$

$\omega = \frac{2\pi c}{\lambda} = 2\pi\nu$

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Now that is what we have written eta 1 obviously, that will be frequency dependent because refractive index is frequency dependent because we have soon earlier that epsilon r can be frequency dependent. So, omega = 2pi c / lambda or 2pi nu angular frequency nu is the linear frequency, lambda is the wavelength. Similarly eta 2 we have written like that.

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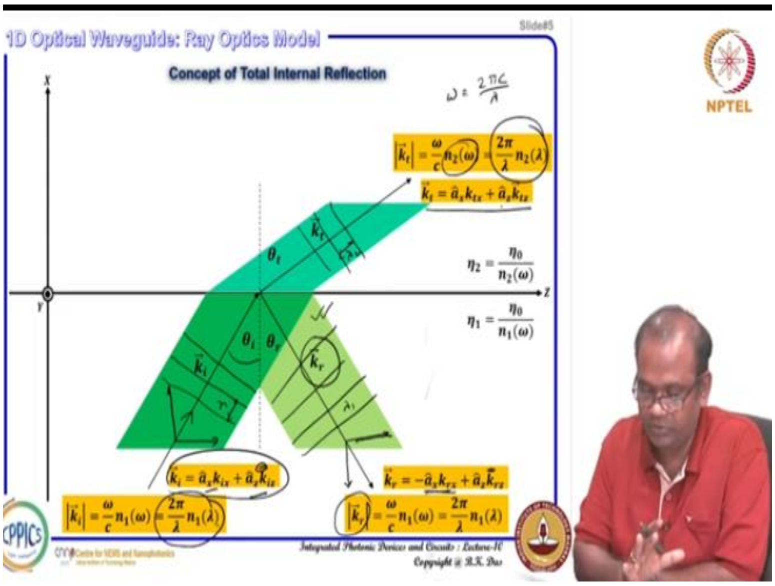

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1D Optical Waveguide: Ray Optics Model

Concept of Total Internal Reflection

$\omega = \frac{2\pi c}{\lambda}$

$|\vec{k}_t| = \frac{\omega}{c} n_2(\omega) = \frac{2\pi}{\lambda} n_2(\lambda)$

$\vec{k}_t = \hat{a}_x k_{tx} + \hat{a}_z k_{tz}$

$\eta_2 = \frac{\eta_0}{n_2(\omega)}$

$\eta_1 = \frac{\eta_0}{n_1(\omega)}$

$\vec{k}_i = \hat{a}_x k_{ix} + \hat{a}_z k_{iz}$

$|\vec{k}_i| = \frac{\omega}{c} n_1(\omega) = \frac{2\pi}{\lambda} n_1(\lambda)$

$\vec{k}_r = -\hat{a}_x k_{rx} + \hat{a}_z k_{rz}$

$|\vec{k}_r| = \frac{\omega}{c} n_1(\omega) = \frac{2\pi}{\lambda} n_1(\lambda)$

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Now let us see what actually happens, you just consider electromagnetic wave is plane wave propagating in k i direction making an angle with the normal theta i, k i is the propagation vector wave vector and you can consider this is just a plane wave parallel phase front and you can sketch so, that you can consider this as your lambda in that media. So, when plane wave comes and make an incident in the interface so, we can assume that fraction of the power or field will be reflected along k r direction.

That will be also you can consider that is also plane wave that will be plane wave plane phase front, you can think of it is extended infinitely but for this understanding purpose we have limited the width. Similarly, a fraction of the electromagnetic wave power that will be transmitted to the second media and that is also will be going to be also plane wave it is going like that and that will be the lambda 2 here this is lambda 1 here and that will be lambda 2 here it will be lambda 1 here.

Because the medium is same lambda 1 and here will be lambda 2 and one thing we should keep in mind that this k i can be written as this one we can consider the k i is lying in the xz plane the screen is the xz plane in the screen itself k i. So, that is why you have x component and z component this is not vector sign this will not be there. Similarly so you can decompose into z component and x component.

Similarly the k r we can decompose into z component and x component that is what reflected to wave vector k rx it is negative x direction and this one is the positive direction k object will be there. So, that means, the difference between incident wave and reflected wave is that difference of the k rx, x component is very worst that is it and we can define k i, k i is a wave vector, earlier we have discussed that will be omega / c n 1 omega or 2pi / lambda n 1 lambda that is a wave vector.

Depending on the medium, you have to n 1 and it is a mode so, we can ignore the direction. So, whatever value you are getting, you can say that k r also will be same just in the same medium. So, wave vector that means values wave number so called wave number will be same because it is in the same medium incident wave and reflected wave same medium. So, 2pi / lambda n 1 and in case of transmitted wave vector. So, if number will be getting omega / c instead of n 1 we are writing n 2.

So that can be omega equal to again same way we can write 2pi c / lambda c c cancelled so, this is the another way of the presentation and k t pictorially you can express like this. So, for the situation is set now we need to understand a little more what is that?

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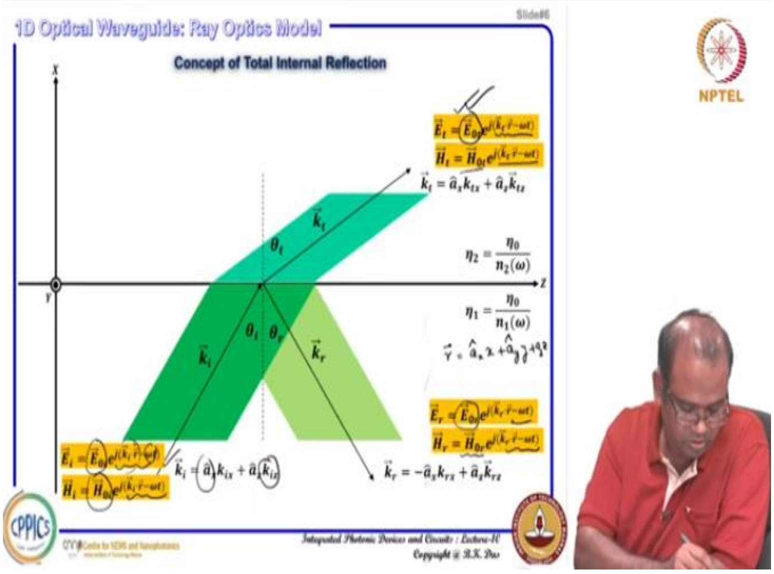

1D Optical Waveguide: Ray Optics Model

Slide#6

Concept of Total Internal Reflection

$\vec{E}_t = \vec{E}_{0t} e^{j(\vec{k}_t \cdot \vec{r} - \omega t)}$

$\vec{H}_t = \vec{H}_{0t} e^{j(\vec{k}_t \cdot \vec{r} - \omega t)}$

$\vec{k}_t = \hat{a}_x k_{tx} + \hat{a}_z k_{tz}$

$\eta_2 = \dfrac{\eta_0}{n_2(\omega)}$

$\eta_1 = \dfrac{\eta_0}{n_1(\omega)}$

$\vec{r} = \hat{a}_x x + \hat{a}_y y + \hat{a}_z z$

$\vec{E}_r = \vec{E}_{0r} e^{j(\vec{k}_r \cdot \vec{r} - \omega t)}$

$\vec{H}_r = \vec{H}_{0r} e^{j(\vec{k}_r \cdot \vec{r} - \omega t)}$

$\vec{E}_i = \vec{E}_{0i} e^{j(\vec{k}_i \cdot \vec{r} - \omega t)}$

$\vec{H}_i = \vec{H}_{0i} e^{j(\vec{k}_i \cdot \vec{r} - \omega t)}$

$\vec{k}_i = \hat{a}_x k_{ix} + \hat{a}_z k_{iz}$

$\vec{k}_r = -\hat{a}_x k_{rx} + \hat{a}_z k_{rz}$

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Let us think k i the wave, this is k i propagating in this direction, it has x component and z component. So, we can consider the incident wave has associated electric field wave. Electric field can be represented like this plane wave representation, it has amplitude, obviously, it will be a vector, we can define which direction it can be and associated magnetic field also will be there, but since they are plane wave, so phase factor will be same for electrical field and magnetic field.

So, k dot r - omega t this is the space dependent phase factor this is time dependent space factor likewise, if anything reflecting so, reflected amplitude I can define E 0r and we can like k r dot r omega t and k r omega t same space magnetic field and electric field you can write space will be same by the way this r is nothing but it can just consider any position vector a x x + a y y + a z z.

So, this is the case similarly, for transmitted wave also we can just say that transmitted wave amplitude will be this one that will be vector also space will be instead of k i, k r, k t we have written we have assumed that frequency does not change at all linear medium no frequency is going to be changed and associated magnetic field we can define like this.

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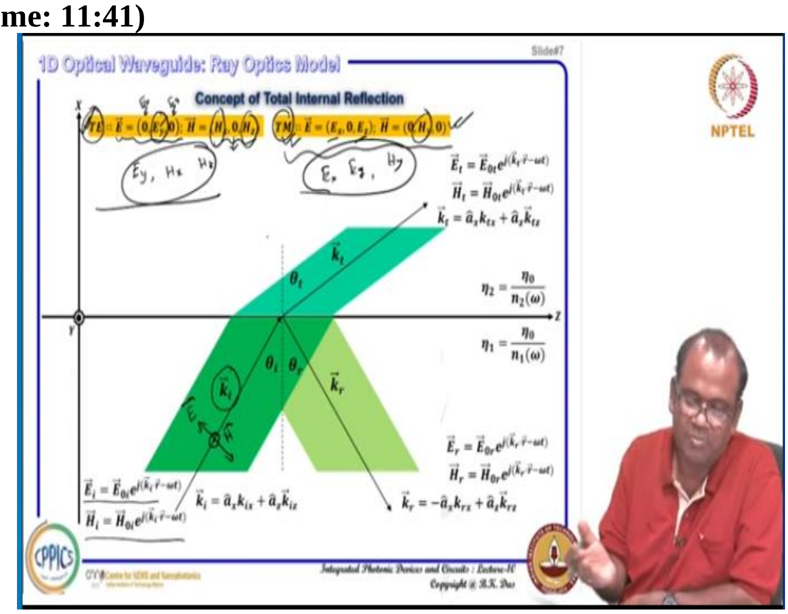

1D Optical Waveguide: Ray Optics Model

Concept of Total Internal Reflection

$TE :: \vec{E} = (0, E_y, 0);\ \vec{H} = (H_x, 0, H_z)$ $TM :: \vec{E} = (E_x, 0, E_z);\ \vec{H} = (0, H_y, 0)$

$\vec{E}_t = \vec{E}_{0t} e^{j(\vec{k}_t \cdot \vec{r} - \omega t)}$

$\vec{H}_t = \vec{H}_{0t} e^{j(\vec{k}_t \cdot \vec{r} - \omega t)}$

$\vec{k}_t = \hat{a}_x k_{tx} + \hat{a}_z \vec{k}_{tz}$

$\eta_2 = \dfrac{\eta_0}{n_2(\omega)}$

$\eta_1 = \dfrac{\eta_0}{n_1(\omega)}$

$\vec{E}_r = \vec{E}_{0r} e^{j(\vec{k}_r \cdot \vec{r} - \omega t)}$

$\vec{H}_r = \vec{H}_{0r} e^{j(\vec{k}_r \cdot \vec{r} - \omega t)}$

$\vec{k}_r = -\hat{a}_x k_{rx} + \hat{a}_z \vec{k}_{rz}$

$\vec{E}_i = \vec{E}_{0i} e^{j(\vec{k}_i \cdot \vec{r} - \omega t)}$

$\vec{H}_i = \vec{H}_{0i} e^{j(\vec{k}_i \cdot \vec{r} - \omega t)}$

$\vec{k}_i = \hat{a}_x k_{ix} + \hat{a}_z \vec{k}_{iz}$

So, next thing is that we have considered electric field, but as we mentioned that what direction of electric field we can consider for our discussion. We have 2 choices, one is called TE and another is TM. So, this is TM transverse electric field and transverse magnetic field. So, for transverse electric field we can consider E vector has only y component x component is not there. So, this is corresponding to E x component that is 0, z component that is also 0 I have represented and E y component only there.

So, E field I can consider that is actually oscillating along y direction that is your E field we can say that it is oscillating, it must be perpendicular to the k i that is correct. And magnetic field you can assume that you have to assume that magnetic field is also perpendicular to the k i vector and that should be perpendicular to the E vector also since E is oscillating along y direction I can say that magnetic field will have we should be in the xz plane.

So, that means I can consider magnetic field will have x component and z component no y component. So, that means, I can consider electron plane wave electromagnetic wave is having electrical filed E y component and magnetic field H x and H z that would be sufficient for electromagnetic wave propagation along k i direction, it can carry electromagnetic energy in that direction.

So that particular type of polarization will call that transverse electric field that means, transverse means it is actually perpendicular to the screen electric field is perpendicular to the screen y direction. Similarly, for TM polarization we can consider that magnetic field is perpendicular to the xz plane it is along y direction that means, in this case electric field is the tangential component to the interface and for TM magnetic field is the tangential component to the interface.

So, magnetic field is the perpendicular to the screen and if magnetic field is this direction, earlier case magnetic field electric magnetic field is like this here it was oscillating like this and the electrical oscillating in the y direction and now it is a reverse case you can consider this one is your so called this one will be your H field and this one will be your electric field. Electric field will be oscillating in the xz plane. So, x component z component is there in that case that will be called as a TM polarization.

So, in this case we will have E x component E z component and H y component so, that means all the 6 components can have in a plane wave, but we can decompose into TE polarization and TM polarization of this kind and we can treat them independently because they can carry energy independently they can be a independent solution. So, if some other polarization is there obviously, we can decompose into TE and TM.

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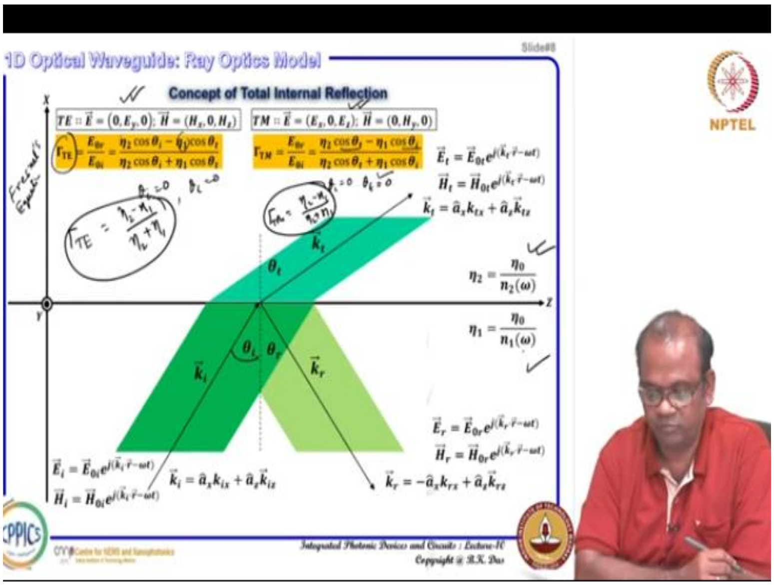

1D Optical Waveguide: Ray Optics Model

Slide#8

Concept of Total Internal Reflection

$TE :: \vec{E} = (0, E_y, 0);\ \vec{H} = (H_x, 0, H_z)$

$TM :: \vec{E} = (E_x, 0, E_z);\ \vec{H} = (0, H_y, 0)$

$$\Gamma_{TE} = \frac{E_{0r}}{E_{0i}} = \frac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}$$

$$\Gamma_{TM} = \frac{E_{0r}}{E_{0i}} = \frac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$$

Fresnel's Equ.

$\theta_i = 0 \quad \theta_t = 0$

$$\Gamma_{TE} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

$$\Gamma_{TM} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

$\vec{E}_t = \vec{E}_{0t} e^{j(\vec{k}_t \cdot \vec{r} - \omega t)}$

$\vec{H}_t = \vec{H}_{0t} e^{j(\vec{k}_t \cdot \vec{r} - \omega t)}$

$\vec{k}_t = \hat{a}_x k_{tx} + \hat{a}_z \vec{k}_{tz}$

$\eta_2 = \frac{\eta_0}{n_2(\omega)}$

$\eta_1 = \frac{\eta_0}{n_1(\omega)}$

$\vec{E}_r = \vec{E}_{0r} e^{j(\vec{k}_r \cdot \vec{r} - \omega t)}$

$\vec{H}_r = \vec{H}_{0r} e^{j(\vec{k}_r \cdot \vec{r} - \omega t)}$

$\vec{E}_i = \vec{E}_{0i} e^{j(\vec{k}_i \cdot \vec{r} - \omega t)}$

$\vec{H}_i = \vec{H}_{0i} e^{j(\vec{k}_i \cdot \vec{r} - \omega t)}$

$\vec{k}_i = \hat{a}_x k_{ix} + \hat{a}_z \vec{k}_{iz}$

$\vec{k}_r = -\hat{a}_x k_{rx} + \hat{a}_z \vec{k}_{rz}$

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Next we try to see, you know, for example TE polarization and TM polarization if I consider and making an incident I can find using our boundary condition how much fraction of electric field will be electric field or electromagnetic wave will be reflected. So, using the boundary condition that equation is actually that reflection coefficient gamma TE is can be derived and first time Fresnel derived was using boundary condition that is why we call it as your Fresnel's equation of reflection for gamma TE.

And that is actually for any 2 media we have discussed earlier also that if 2 media and they are characterized by eta 1and eta 2, then that is actually a reflection coefficient depends on our incident angle theta i, theta t and so on. And for TM polarization everything is same only theta i and theta t they will be changed they are interchanged so, instead of cos theta you were writing cos theta t and here you are writing cos theta i otherwise same.

So, another important thing you can notice from this equation that numerator denominator the same just plus minus difference both for TE polarization, TM polarization and one more important thing interesting point you can note that suppose theta i = 0 that means normal incident this theta i = 0 obviously your theta t will be equal to 0. So, in that case your gamma TE will be equal to eta 2 - eta 1 / eta 2 + eta 1.

Similarly, here also if we put theta i = 0 theta t also will be equal to 0 then gamma TM also will be eta 2 - eta 1 / eta 2 + eta 1. So, that means, for normal incident TE polarization and TM polarization the reflection coefficient is same, but if it is a little bit angled then reflection coefficient for TE and TM polarization is different that is one of the application people use for various purpose sometimes polarization filter purpose people used just to have to choose right combination of material dielectric 1 and dielectric 2 we are not going into that direction instead we will move forward.

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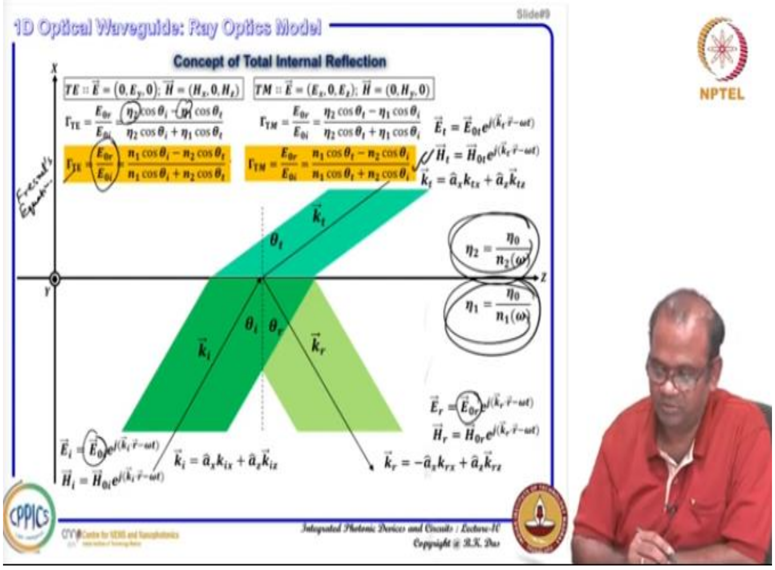

1D Optical Waveguide: Ray Optics Model

Slide#9

Concept of Total Internal Reflection

$TE :: \vec{E} = (0, E_y, 0);\ \vec{H} = (H_x, 0, H_z)$

$TM :: \vec{E} = (E_x, 0, E_z);\ \vec{H} = (0, H_y, 0)$

$$\Gamma_{TE} = \frac{E_{0r}}{E_{0i}} = \frac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}$$

$$\Gamma_{TE} = \frac{E_{0r}}{E_{0i}} = \frac{n_1 \cos\theta_i - n_2 \cos\theta_t}{n_1 \cos\theta_i + n_2 \cos\theta_t}$$

$$\Gamma_{TM} = \frac{E_{0r}}{E_{0i}} = \frac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$$

$$\Gamma_{TM} = \frac{E_{0r}}{E_{0i}} = \frac{n_1 \cos\theta_t - n_2 \cos\theta_i}{n_1 \cos\theta_t + n_2 \cos\theta_i}$$

$\vec{E}_t = \vec{E}_{0t} e^{j(\vec{k}_t \cdot \vec{r} - \omega t)}$

$\vec{H}_t = \vec{H}_{0t} e^{j(\vec{k}_t \cdot \vec{r} - \omega t)}$

$\vec{k}_t = \hat{a}_x k_{tx} + \hat{a}_z \vec{k}_{tz}$

$\eta_2 = \frac{\eta_0}{n_2(\omega)}$

$\eta_1 = \frac{\eta_0}{n_1(\omega)}$

$\vec{E}_r = \vec{E}_{0r} e^{j(\vec{k}_r \cdot \vec{r} - \omega t)}$

$\vec{H}_r = \vec{H}_{0r} e^{j(\vec{k}_r \cdot \vec{r} - \omega t)}$

$\vec{E}_i = \vec{E}_{0i} e^{j(\vec{k}_i \cdot \vec{r} - \omega t)}$

$\vec{H}_i = \vec{H}_{0i} e^{j(\vec{k}_i \cdot \vec{r} - \omega t)}$

$\vec{k}_i = \hat{a}_x k_{ix} + \hat{a}_z \vec{k}_{iz}$

$\vec{k}_r = -\hat{a}_x k_{rx} + \hat{a}_z \vec{k}_{rz}$

Fresnel's Equation

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Suppose it is a dielectric medium we have learned that instead of eta we can just the medium is characterized by refractive index only eat 1 and eta 2 can be represented by refractive index because conductivity is 0 nonmagnetic material so on. So, if I just substitute eta 2 with this one and eat 1 with this one then I get n 1 instead of eta 2 it will be n 1 cos theta i = n 2 cos theta t and n 1 cos theta i n 2 cos theta t.

This one you can remember the E 0r means reflected amplitude, this one is E 0r divided by E 0i that means, what is the reflected amplitude and you are normalizing with incident amplitude that is your reflection coefficient. So, now the Fresnel’s equation in terms of refractive index, so if you know the refractive index and if you know the incident angle then you can find out what is the reflection coefficient for TE polarization and what is the reflection coefficient for TM polarization?

There are different because theta i theta t little bit different when it is angle their reflection coefficient is a difference but dielectric nonmagnetic material to material medium interface the reflection coefficient we can easily find out so far so good.

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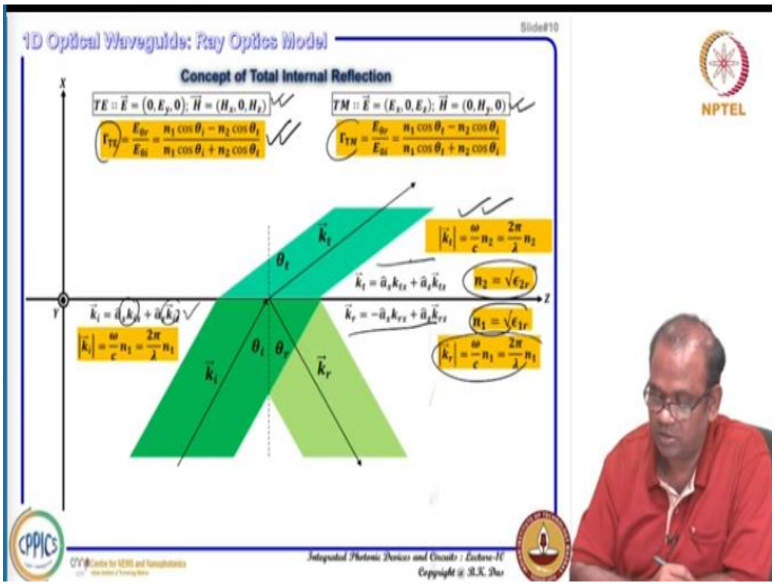

1D Optical Waveguide: Ray Optics Model

Concept of Total Internal Reflection

$TE :: \vec{E} = (0, E_y, 0); \vec{H} = (H_x, 0, H_z)$

$\Gamma_{TE} = \frac{E_{0r}}{E_{0i}} = \frac{n_1 \cos\theta_i - n_2 \cos\theta_t}{n_1 \cos\theta_i + n_2 \cos\theta_t}$

$TM :: \vec{E} = (E_x, 0, E_z); \vec{H} = (0, H_y, 0)$

$\Gamma_{TM} = \frac{E_{0r}}{E_{0i}} = \frac{n_1 \cos\theta_t - n_2 \cos\theta_i}{n_1 \cos\theta_t + n_2 \cos\theta_i}$

$|\vec{k}_t| = \frac{\omega}{c} n_2 = \frac{2\pi}{\lambda} n_2$

$\vec{k}_t = \hat{a}_x k_{tx} + \hat{a}_z \vec{k}_{tz}$

$n_2 = \sqrt{\epsilon_{2r}}$

$\vec{k}_i = \hat{a}_x k_{ix} + \hat{a}_z \vec{k}_{iz}$

$|\vec{k}_i| = \frac{\omega}{c} n_1 = \frac{2\pi}{\lambda} n_1$

$\vec{k}_r = -\hat{a}_x k_{rx} + \hat{a}_z \vec{k}_{rz}$

$n_1 = \sqrt{\epsilon_{1r}}$

$|\vec{k}_r| = \frac{\omega}{c} n_1 = \frac{2\pi}{\lambda} n_1$

So now, let us move on I have just written down same thing, let the reflection coefficient for TE polarization and you should keep in mind that TE polarization I am just retaining these equations, this component so, that the things you can remember easily so, you can correlate TM. So, this is gamma TE reflection coefficient gamma TM. So, theta i theta t dependent as well as n 1 and n 2 dependent.

Now, 2 more thing I have just retained here, k t earlier we have discussed we are retaining and we are we can define that n 2 is nothing but dielectric constant same thing whatever we have discussed, we are just keeping here everywhere just to follow up some of these parameters we need for the discussion that is why I detained here. So, all these k vector so I can have longitudinal component and the transverse component and longitudinal component transverse means along the x direction longitudinal means, z direction components are there for incident wave reflected wave for transmitted wave fine.

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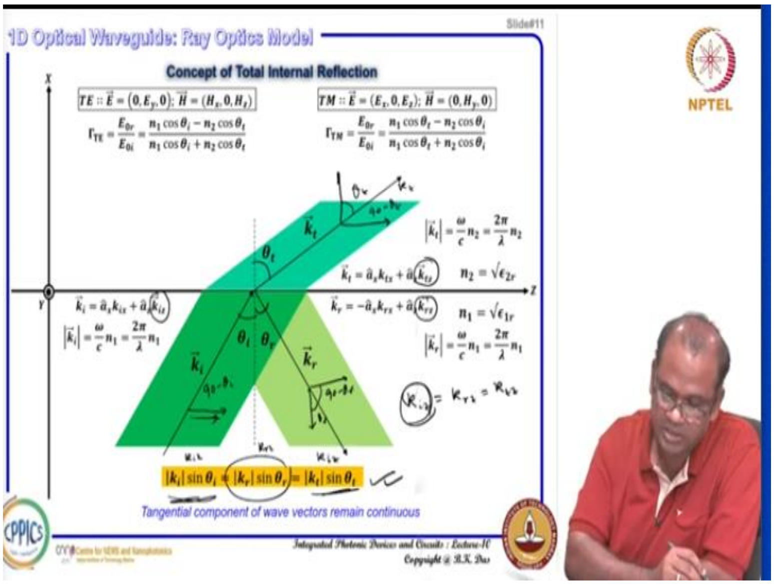

Slide#11

1D Optical Waveguide: Ray Optics Model

Concept of Total Internal Reflection

$TE :: \vec{E} = (0, E_y, 0);\ \vec{H} = (H_x, 0, H_z)$

$$\Gamma_{TE} = \frac{E_{0r}}{E_{0i}} = \frac{n_1\cos\theta_i - n_2\cos\theta_t}{n_1\cos\theta_i + n_2\cos\theta_t}$$

$TM :: \vec{E} = (E_x, 0, E_z);\ \vec{H} = (0, H_y, 0)$

$$\Gamma_{TM} = \frac{E_{0r}}{E_{0i}} = \frac{n_1\cos\theta_t - n_2\cos\theta_i}{n_1\cos\theta_t + n_2\cos\theta_i}$$

$|\vec{k}_t| = \frac{\omega}{c}n_2 = \frac{2\pi}{\lambda}n_2$

$\vec{k}_t = \hat{a}_x k_{tx} + \hat{a}_z k_{tz}$ $\quad n_2 = \sqrt{\epsilon_{2r}}$

$\vec{k}_i = \hat{a}_x k_{ix} + \hat{a}_z k_{iz}$

$|\vec{k}_i| = \frac{\omega}{c}n_1 = \frac{2\pi}{\lambda}n_1$

$\vec{k}_r = -\hat{a}_x k_{rx} + \hat{a}_z k_{rz}$ $\quad n_1 = \sqrt{\epsilon_{1r}}$

$|\vec{k}_r| = \frac{\omega}{c}n_1 = \frac{2\pi}{\lambda}n_1$

$k_{iz} = k_{rz} = k_{tz}$

$|\vec{k}_i|\sin\theta_i = |\vec{k}_r|\sin\theta_r = |\vec{k}_t|\sin\theta_t$

Tangential component of wave vectors remain continuous

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So, now one important thing you must understand that this k iz, k rz, k tz they are all tangential component of the wave vector and from the boundary condition, we know that they are basically same k iz. So, this would be continuous k rz and k tz they are equal. Now, what is k iz? k iz if you see if it is this direction, so, if this is theta this is 90 – theta i. So, k iz is component basically this component is k i times sin theta i.

So, that is actually k iz this one means k iz and this one is k rz because if you just z component means, if this is theta r this would be theta r so, this would be 90 - theta r. So, that will be k rz means k rz is k r sin theta r. Similarly, k t so, you have this is z direction this is your x direction. So, if this one these theta t this is theta t, so, this is 90 - theta t so, if it is k t so, k t sin theta t that will be your; this is k rz this is k tz.

So, longitudinal components that means transverse in the interface the transverse component of the wave vector they must be continuous that also you can prove using the boundary conditions, any standard textbook again electromagnetic theory textbook, you can get that one I just borrowed directly that thing fine.

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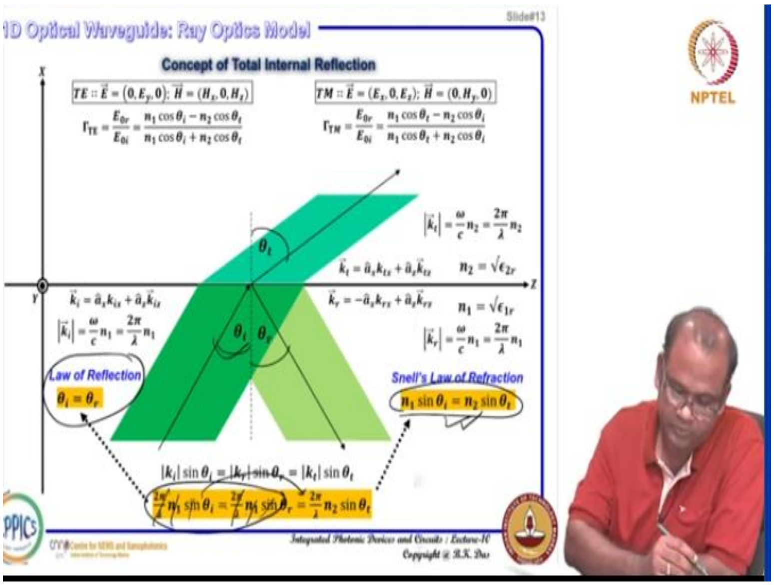

1D Optical Waveguide: Ray Optics Model

Concept of Total Internal Reflection

$TE :: \vec{E} = (0, E_y, 0); \vec{H} = (H_x, 0, H_z)$

$$\Gamma_{TE} = \frac{E_{0r}}{E_{0i}} = \frac{n_1 \cos\theta_i - n_2 \cos\theta_t}{n_1 \cos\theta_i + n_2 \cos\theta_t}$$

$TM :: \vec{E} = (E_x, 0, E_z); \vec{H} = (0, H_y, 0)$

$$\Gamma_{TM} = \frac{E_{0r}}{E_{0i}} = \frac{n_1 \cos\theta_t - n_2 \cos\theta_i}{n_1 \cos\theta_t + n_2 \cos\theta_i}$$

$\left|\vec{k}_t\right| = \frac{\omega}{c} n_2 = \frac{2\pi}{\lambda} n_2$

$\vec{k}_t = \hat{a}_x k_{tx} + \hat{a}_z \vec{k}_{tz}$ $\quad n_2 = \sqrt{\epsilon_{2r}}$

$\vec{k}_i = \hat{a}_x k_{ix} + \hat{a}_z \vec{k}_{iz}$

$\left|\vec{k}_i\right| = \frac{\omega}{c} n_1 = \frac{2\pi}{\lambda} n_1$

$\vec{k}_r = -\hat{a}_x k_{rx} + \hat{a}_z \vec{k}_{rz}$ $\quad n_1 = \sqrt{\epsilon_{1r}}$

$\left|\vec{k}_r\right| = \frac{\omega}{c} n_1 = \frac{2\pi}{\lambda} n_1$

θ_t θ_i θ_r

Law of Reflection

$\theta_i = \theta_r$

Snell's Law of Refraction

$n_1 \sin\theta_i = n_2 \sin\theta_t$

$$|k_i| \sin\theta_i = |k_r| \sin\theta_r = |k_t| \sin\theta_t$$

$$\cancel{\frac{2\pi}{\lambda}} \cancel{n_1} \cancel{\sin}\theta_i = \cancel{\frac{2\pi}{\lambda}} \cancel{n_1} \cancel{\sin}\theta_r = \frac{2\pi}{\lambda} n_2 \sin\theta_t$$

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Next thing is what is that? So, this one a little bit modified, so k i we know that k i is this one 2pi / lambda n 1, k r is 2pi / lambda n 1 k i k r they are same in the same media and k t instead of k t I am writing 2pi / lambda n 2. So, if I just use this thing this formula this condition then I get 2 things if I just compare these 2 2pi / lambda 2pi / lambda will cancel n 1 n 1cancel sin sin cancel so, theta equal to theta that is actually we know as the law of reflection.

And if we compare these 2 then you see n 1 sin theta i = n 2 sin theta t this is known as Snell’s law of refraction that is this thing law of reflection Snell’s law of refraction that means, that relates incident angle to refracted angle and this relate incident angle to reflected angle that is a law of reflection this one actually should be equal to this one and this theta t and theta i related with the law of reflection that actually we get from the Maxwell's equation also the Snell’s law. That is our school level we learned that now, here we are understanding how that can be extracted from the boundary condition.

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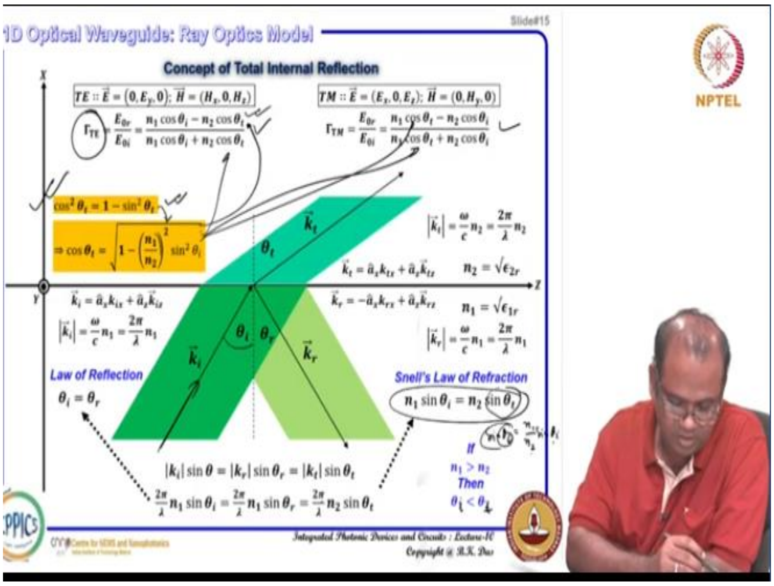

1D Optical Waveguide: Ray Optics Model

Slide#15

Concept of Total Internal Reflection

$TE :: \vec{E} = (0, E_y, 0);\ \vec{H} = (H_x, 0, H_z)$

$\Gamma_{TE} = \frac{E_{0r}}{E_{0i}} = \frac{n_1 \cos\theta_i - n_2 \cos\theta_t}{n_1 \cos\theta_i + n_2 \cos\theta_t}$

$TM :: \vec{E} = (E_x, 0, E_z);\ \vec{H} = (0, H_y, 0)$

$\Gamma_{TM} = \frac{E_{0r}}{E_{0i}} = \frac{n_1 \cos\theta_t - n_2 \cos\theta_i}{n_1 \cos\theta_t + n_2 \cos\theta_i}$

$\cos^2\theta_t = 1 - \sin^2\theta_t$

$\Rightarrow \cos\theta_t = \sqrt{1 - \left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_i}$

$|\vec{k}_t| = \frac{\omega}{c} n_2 = \frac{2\pi}{\lambda} n_2$

$\vec{k}_t = \hat{a}_x k_{tx} + \hat{a}_z \vec{k}_{tz}$ $\quad n_2 = \sqrt{\epsilon_{2r}}$

$\vec{k}_i = \hat{a}_x k_{ix} + \hat{a}_z \vec{k}_{iz}$

$|\vec{k}_i| = \frac{\omega}{c} n_1 = \frac{2\pi}{\lambda} n_1$

$\vec{k}_r = -\hat{a}_x k_{rx} + \hat{a}_z \vec{k}_{rz}$ $\quad n_1 = \sqrt{\epsilon_{1r}}$

$|\vec{k}_r| = \frac{\omega}{c} n_1 = \frac{2\pi}{\lambda} n_1$

Law of Reflection

$\theta_i = \theta_r$

Snell's Law of Refraction

$n_1 \sin\theta_i = n_2 \sin\theta_t$

$\sin\theta_t = \frac{n_1}{n_2}\sin\theta_i$

If

$n_1 > n_2$

Then

$\theta_i < \theta_t$

$|k_i| \sin\theta = |k_r| \sin\theta_r = |k_t| \sin\theta_t$

$\frac{2\pi}{\lambda} n_1 \sin\theta_i = \frac{2\pi}{\lambda} n_1 \sin\theta_r = \frac{2\pi}{\lambda} n_2 \sin\theta_t$

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So, next thing is that what is the new thing in here, this one suppose you consider this thing. For example, if you considered n 1 greater than n 2 following this equation, if n 1 greater than n 2, then theta i should be less than theta t. So, theta i should be less than theta t we put fully considered this n 1 is greater than n 2. So, in that case, theta i theta i this should be theta i must be greater than theta t that is should be theta t.

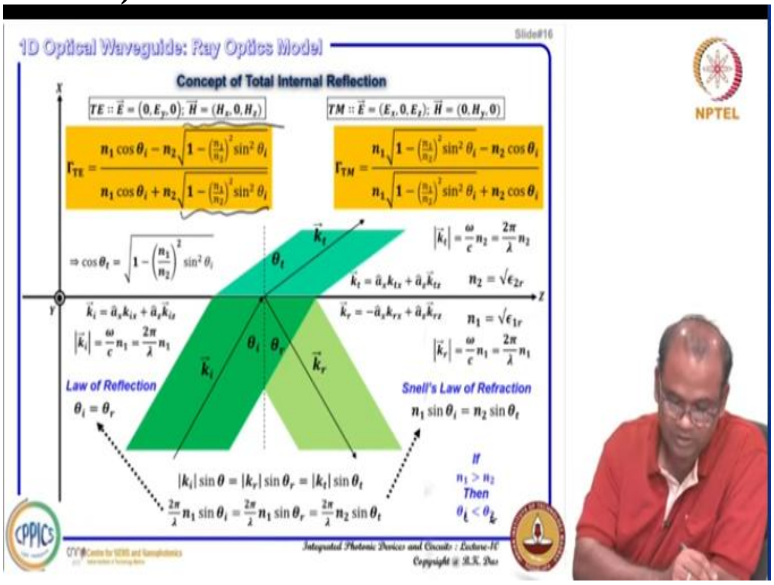
So that means, in this case particularly coincidentally the sketch is actually the specific condition n 1 greater than n 2 that is why it is you see theta i and theta t theta i is basically less than theta t. So, for this case it is suitable for this condition n 1 greater then n 2. So, after that so next thing is that here, what is this? Now if we use of course here, you will write type error here theta i and this will be theta t.

Now, if you see this equation let us concentrate this equation so, here you have theta i theta t both equation in both theta i and theta t. But normally you know any interface what is happening normally depends on how we are signing our electromagnetic wave it can be plane wave, So, theta i is known to us, but theta i to theta t that is actually related by Snell's law. So, I can actually find theta t by using this equation. So, the sin theta t can be expressed as sin theta t = n 1 / n 2 sin theta i.

So, if I know sin theta i theta i I know then I can find out theta t so, this cos theta t if I can replace by theta i using Snell's law I know that we can we know that this trigonometric relationship we can use cos square theta t = 1 - sin square theta t. Then cos theta t = square root of sin square theta t sin square theta t means n 1 / n 2 square sin square theta i so, n 1 / n 2 whole square sin square theta i.

So, now theta t I am representing in terms of theta i n 1 and n 2 so, theta t depends on not only theta i depends on the refractive index on media 1 and 2. So, this cos theta t value I can insert here, this value I can insert here, this value I can insert here, this value I can insert here. So, all this value we can insert then I can get our reflection coefficient in terms of only theta i and n 1 n 2 that is straightforward just we have used this Snell's law, because Snell's law it is proved from the boundary condition we have we can take it granted

(Refer Slide Time: 27:05)



1D Optical Waveguide: Ray Optics Model

Slide#16

Concept of Total Internal Reflection

$TE :: \vec{E} = (0, E_y, 0); \vec{H} = (H_x, 0, H_z)$

$TM :: \vec{E} = (E_x, 0, E_z); \vec{H} = (0, H_y, 0)$

$$\Gamma_{TE} = \frac{n_1 \cos\theta_i - n_2\sqrt{1 - \left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_i}}{n_1 \cos\theta_i + n_2\sqrt{1 - \left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_i}}$$

$$\Gamma_{TM} = \frac{n_1\sqrt{1 - \left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_i} - n_2 \cos\theta_i}{n_1\sqrt{1 - \left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_i} + n_2 \cos\theta_i}$$

$$\Rightarrow \cos\theta_t = \sqrt{1 - \left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_i}$$

$|\vec{k}_t| = \frac{\omega}{c} n_2 = \frac{2\pi}{\lambda} n_2$

$\vec{k}_t = \hat{a}_x k_{tx} + \hat{a}_z \vec{k}_{tz}$ $\quad n_2 = \sqrt{\epsilon_{2r}}$

$\vec{k}_i = \hat{a}_x k_{ix} + \hat{a}_z \vec{k}_{iz}$

$|\vec{k}_i| = \frac{\omega}{c} n_1 = \frac{2\pi}{\lambda} n_1$

$\vec{k}_r = -\hat{a}_x k_{rx} + \hat{a}_z \vec{k}_{rz}$ $\quad n_1 = \sqrt{\epsilon_{1r}}$

$|\vec{k}_r| = \frac{\omega}{c} n_1 = \frac{2\pi}{\lambda} n_1$

Law of Reflection

$\theta_i = \theta_r$

Snell's Law of Refraction

$n_1 \sin\theta_i = n_2 \sin\theta_t$

$|k_i| \sin\theta = |k_r| \sin\theta_r = |k_t| \sin\theta_t$

$\frac{2\pi}{\lambda} n_1 \sin\theta_i = \frac{2\pi}{\lambda} n_1 \sin\theta_r = \frac{2\pi}{\lambda} n_2 \sin\theta_t$

If
$n_1 > n_2$
Then
$\theta_i < \theta_t$

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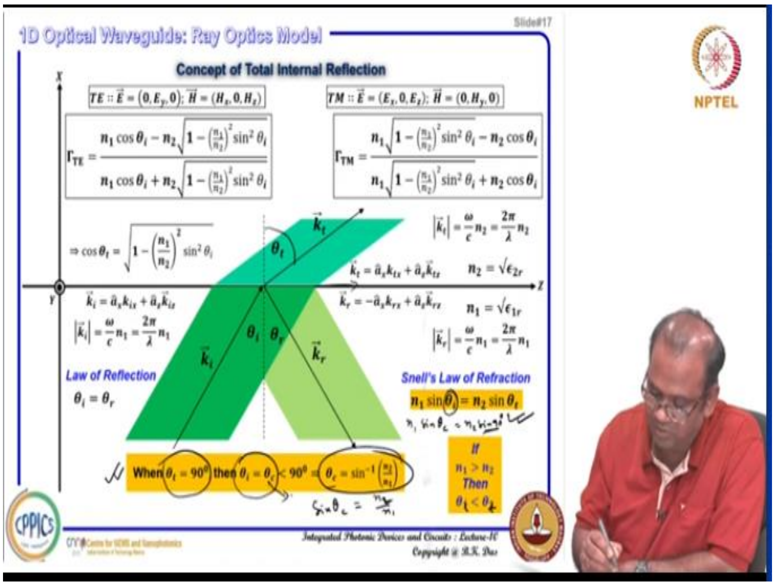
Next thing let us concentrate this one so, after inserting cos theta t this is the value of cos theta t whatever the things we have put here we have just written down there. So, then obviously, you just keep on doing this theta i theta t and this type error is continuing. So, copy paste.

(Refer Slide Time: 27:29)



1D Optical Waveguide: Ray Optics Model

Concept of Total Internal Reflection

$TE :: \vec{E} = (0, E_y, 0);\ \vec{H} = (H_x, 0, H_z)$

$TM :: \vec{E} = (E_x, 0, E_z);\ \vec{H} = (0, H_y, 0)$

$$\Gamma_{TE} = \frac{n_1 \cos\theta_i - n_2\sqrt{1-\left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_i}}{n_1 \cos\theta_i + n_2\sqrt{1-\left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_i}}$$

$$\Gamma_{TM} = \frac{n_1\sqrt{1-\left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_i} - n_2 \cos\theta_i}{n_1\sqrt{1-\left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_i} + n_2 \cos\theta_i}$$

$$\Rightarrow \cos\theta_t = \sqrt{1-\left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_i}$$

$|\vec{k}_t| = \frac{\omega}{c} n_2 = \frac{2\pi}{\lambda} n_2$

$\vec{k}_t = \hat{a}_x k_{tx} + \hat{a}_z \vec{k}_{tz}$ $\quad n_2 = \sqrt{\epsilon_{2r}}$

$\vec{k}_i = \hat{a}_x k_{ix} + \hat{a}_z \vec{k}_{iz}$

$|\vec{k}_i| = \frac{\omega}{c} n_1 = \frac{2\pi}{\lambda} n_1$

$\vec{k}_r = -\hat{a}_x k_{rx} + \hat{a}_z \vec{k}_{rz}$ $\quad n_1 = \sqrt{\epsilon_{1r}}$

$|\vec{k}_r| = \frac{\omega}{c} n_1 = \frac{2\pi}{\lambda} n_1$

Law of Reflection

$\theta_i = \theta_r$

Snell's Law of Refraction

$n_1 \sin\theta_i = n_2 \sin\theta_t$

$n_1 \sin\theta_c = n_2 \sin 90°$

If $n_1 > n_2$ Then $\theta_i < \theta_t$

When $\theta_t = 90°$ then $\theta_i = \theta_c < 90° \Rightarrow \theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$

$\sin\theta_c = \frac{n_2}{n_1}$

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NPTEL

So, now let us move what is that special condition we are considering this one when suppose you want to get a theta t = 90 degree and that follows this equation. So, theta i is always less than theta t so, you can find a theta i value where theta t can be 90 degree when theta i can be less than 90 degree but theta t can reach to 90 degree because theta i is always less than theta t because n 1 is greater than n 2.

So, if I find theta t = 90 degree that time whatever the value of theta t theta i I get from the equation Snell's law that I will call as a theta c that must be less than 90 degree. So, that is called critical angle. So, this will be I am putting n 1 sin theta c = n 2 sin 90 degree. So, sin 90 degree this one is 1 so, I can find sin theta c = n 1 n 2 / n 1 that means theta c = sin inverse n 2 / n 1. So, critical angle is the angle of incidence corresponding to refracted angle 90 degree according to the Snell's law.

So, according to the Snell's law, if you consider to have your refracted angle theta t is exactly 90 degrees that means, your wave must reach through the interface in this direction in a practical way, so, that particular situation to happen you should have theta c angle incident angle theta = theta c = sin inverse n 2 / n 1.

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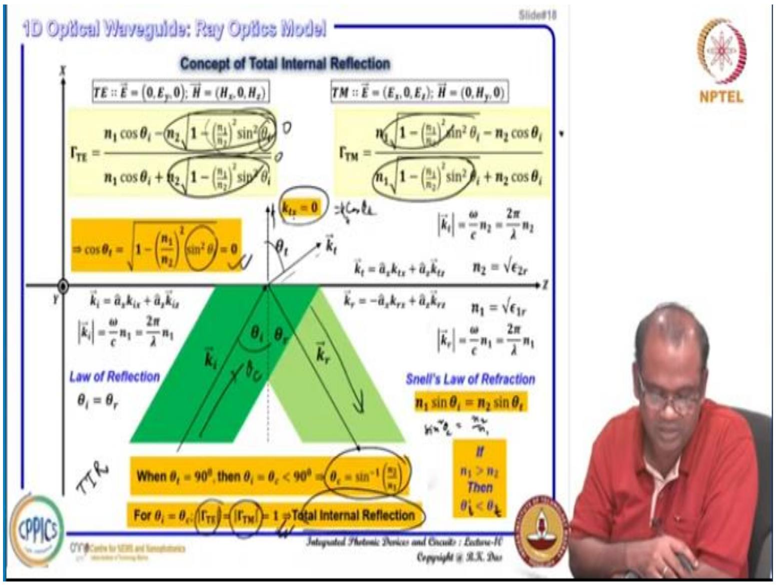

1D Optical Waveguide: Ray Optics Model

Slide#18

Concept of Total Internal Reflection

$TE :: \vec{E} = (0, E_y, 0); \vec{H} = (H_x, 0, H_z)$

$TM :: \vec{E} = (E_x, 0, E_z); \vec{H} = (0, H_y, 0)$

$$\Gamma_{TE} = \frac{n_1 \cos\theta_i - n_2\sqrt{1-\left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_i}}{n_1 \cos\theta_i + n_2\sqrt{1-\left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_i}}$$

$$\Gamma_{TM} = \frac{n_1\sqrt{1-\left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_i} - n_2 \cos\theta_i}{n_1\sqrt{1-\left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_i} + n_2 \cos\theta_i}$$

$k_{tx} = 0$

$$\Rightarrow \cos\theta_t = \sqrt{1-\left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_i} = 0$$

$|\vec{k}_t| = \frac{\omega}{c} n_2 = \frac{2\pi}{\lambda} n_2$

$\vec{k}_t = \hat{a}_x k_{tx} + \hat{a}_z \vec{k}_{tz}$

$n_2 = \sqrt{\epsilon_{2r}}$

$\vec{k}_i = \hat{a}_x k_{ix} + \hat{a}_z \vec{k}_{iz}$

$|\vec{k}_i| = \frac{\omega}{c} n_1 = \frac{2\pi}{\lambda} n_1$

$\vec{k}_r = -\hat{a}_x k_{rx} + \hat{a}_z \vec{k}_{rz}$

$n_1 = \sqrt{\epsilon_{1r}}$

$|\vec{k}_r| = \frac{\omega}{c} n_1 = \frac{2\pi}{\lambda} n_1$

Law of Reflection

$\theta_i = \theta_r$

Snell's Law of Refraction

$n_1 \sin\theta_i = n_2 \sin\theta_t$

If $n_1 > n_2$ Then $\theta_i < \theta_t$

When $\theta_t = 90^0$, then $\theta_i = \theta_c < 90^0 \Rightarrow \theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$

For $\theta_i = \theta_c$: $|\Gamma_{TE}| = |\Gamma_{TM}| = 1 \Rightarrow$ Total Internal Reflection

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So, for that situation to happen we have 2 things if theta i = theta c, if theta i equal to you are putting theta c that when sin square theta c = n 2 / n 1. So, when theta r = theta c sin square theta c = n 2 / n 1. So, that means this value will become 1, so 1 – 1 is 0 that second term is completely 0 for it is happening 0 this is also going to be 0 and this is also going to be 0 this is also going to be 0. So, in that case this cos theta t this value particularly this value is 0 I have written.

So, if that is the case then you will pass down numerator and pass down denominator is leftover. So, if you just compare this one n 1 cos theta / n 1 cos theta, so gamma TE = 1, so gamma TE = 1 will be getting 1. Similarly, for gamma TM it will be minus because this minus sign depends on because you are considering magnetic field in the here you are considering electrical and magnetic fields when they are changing up the reflection one of the field has to be reversed in the boundary.

They will be oscillating with omega t but at that boundary when you are considered using boundary condition at that particular instant upon reflection what happens so, that is a minus sign, that is why I put a mod of gamma TM that is also 1. So, I can find the critical angle when electromagnetic wave propagates up from the denser medium having the refractive index n 1 2 rare and medium I can find an angle theta c defined by sin inverse n 2 / n 1.

I can have reflection coefficient 1 that means everything will be reflected back whatever is going everything will be reflected back. So, if everything reflected back what will be transmitted then 0. So, one interesting thing is that if this is k t I am just considering k t, 90 degree we have considered but when theta c is completely theta c is actually satisfying. So, the theta t has to be 90 degree that means, your k tx will be 0 that means in this direction the x directional component must be equal to 0.

Obviously, it is k t then this k tx will be normally cos theta t, k t cos theta t but cos theta t as you can see cos theta c is equal to this one and if you are putting theta equal to theta c that will be equal to 0. So, k t = 0 that means, you cannot see any wave propagating along the x direction somehow it will not give a k tx component is not there. So, no energy will be flowing along the x direction. So, everything is reflecting back that is called total internal reflection.

So, everything is coming to the first medium itself nothing is in the second medium in principle as of now. So, I have written total internal reflection sometimes it is called TIR total internal reflection. Obviously, this type error just correct theta i theta t, sometimes you can consider theta 1 and theta 2 second media that is medium 1 medium 2, you can consider also that will work.

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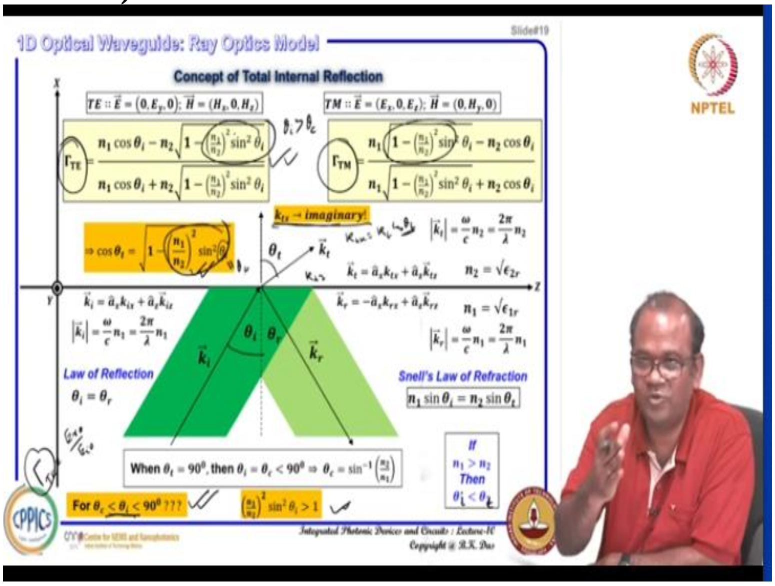

1D Optical Waveguide: Ray Optics Model

Slide#19

Concept of Total Internal Reflection

$TE :: \vec{E} = (0, E_y, 0); \vec{H} = (H_x, 0, H_z)$

$TM :: \vec{E} = (E_x, 0, E_z); \vec{H} = (0, H_y, 0)$

$$\Gamma_{TE} = \frac{n_1 \cos\theta_i - n_2\sqrt{1 - \left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_i}}{n_1 \cos\theta_i + n_2\sqrt{1 - \left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_i}}$$

$\theta_i > \theta_c$

$$\Gamma_{TM} = \frac{n_1\sqrt{1 - \left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_i} - n_2 \cos\theta_i}{n_1\sqrt{1 - \left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_i} + n_2 \cos\theta_i}$$

$\Rightarrow \cos\theta_t = \sqrt{1 - \left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_i}$

$k_{tx} \rightarrow$ imaginary!

$|\vec{k}_t| = \frac{\omega}{c} n_2 = \frac{2\pi}{\lambda} n_2$

$\vec{k}_t = \hat{a}_x k_{tx} + \hat{a}_z \vec{k}_{tz}$

$n_2 = \sqrt{\epsilon_{2r}}$

$\vec{k}_i = \hat{a}_x k_{ix} + \hat{a}_z \vec{k}_{iz}$

$|\vec{k}_i| = \frac{\omega}{c} n_1 = \frac{2\pi}{\lambda} n_1$

$\vec{k}_r = -\hat{a}_x k_{rx} + \hat{a}_z \vec{k}_{rz}$

$n_1 = \sqrt{\epsilon_{1r}}$

$|\vec{k}_r| = \frac{\omega}{c} n_1 = \frac{2\pi}{\lambda} n_1$

Law of Reflection

$\theta_i = \theta_r$

Snell's Law of Refraction

$n_1 \sin\theta_i = n_2 \sin\theta_t$

When $\theta_t = 90^0$, then $\theta_i = \theta_c < 90^0 \Rightarrow \theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$

If $n_1 > n_2$ Then $\theta_i < \theta_t$

For $\theta_c < \theta_i < 90^0$???

$\left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_i > 1$

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Now, let us concentrate a little bit differently, you just considered this one concentrate on this one. So, theta i I can increase and I can reach to theta c and theta c i know that theta t will be 90 degree. Now, suppose theta i somewhere greater than theta c so, if greater than theta c but theta i maximum limit will be less than 90 degree theta i somewhere greater than theta c less than 90 degree. So, in that case what will happen?

So, if you see this value if you are just considering n 1 square this value I am just concentrating on this one n 1 / n 2 square 2 theta c then it is n 2 / n 1 that will be 1 exactly theta i whenever putting theta c that time it is becoming 0 and below theta c that will be actually lower less than 1. So, cos theta t value will be there that is why you will see some transmitted wave is there but exactly our theta c cos theta t is has to be 0 that is why k tx will be normally earlier we have discussed that will be 0.

But when theta c greater than theta i greater than theta c these value will become more than 1 so, that means, this cos theta t will become imaginary, this one will become imaginary. This 1 - 1 that is if cos theta is becoming imaginary k tx will be imaginary because I know that k tx = k t cos theta t. So, If cos theta t is becoming imaginary when theta i greater than theta c then k tx is imaginary.

So, I can have k tz will be there also and k tx will also will be there, but k tx is imaginary that means, in the x direction you can see some wave field will be there, but since the propagation vector component that direction imaginary that means, that direction it will be attenuating. As if you know the propagation constant gamma = alpha + beta if something imaginary is there that means, it will be attenuating. So, field will be there but it will be attenuating in the x direction.

So, attenuating it will be very quickly it will be becoming 0, so that energy will not be flowing along the x direction also. So, if that is the case, I would let us concentrate on this one also. So, this one will be also higher than 1 when theta is greater than theta c, if theta i greater than theta c, so this one will be higher than 1. So, this will be also becoming imaginary and this one also will become imaginary. So, as a whole I see that this reflection coefficient gamma TE and gamma TM is suddenly becoming complex.

So, reflection coefficient is complex reflection coefficient is nothing but so I can say that E r0 / E i0 that means, reflected wave and this one is gamma TE when this is becoming complex that means upon reflection you see if this is complex upon reflection you can see that it is not only reflecting completely, but it may had some kind of phase I am not sure yet whether it is completely reflecting or not at least we see that reflection coefficient is complex, k tx is imaginary.

So, at least it is indication that along the x direction energy will not flow it will be attenuating it is like a skin depth in the metal it will be attenuating. So, what happens to gamma TE, how to calculate gamma TE?

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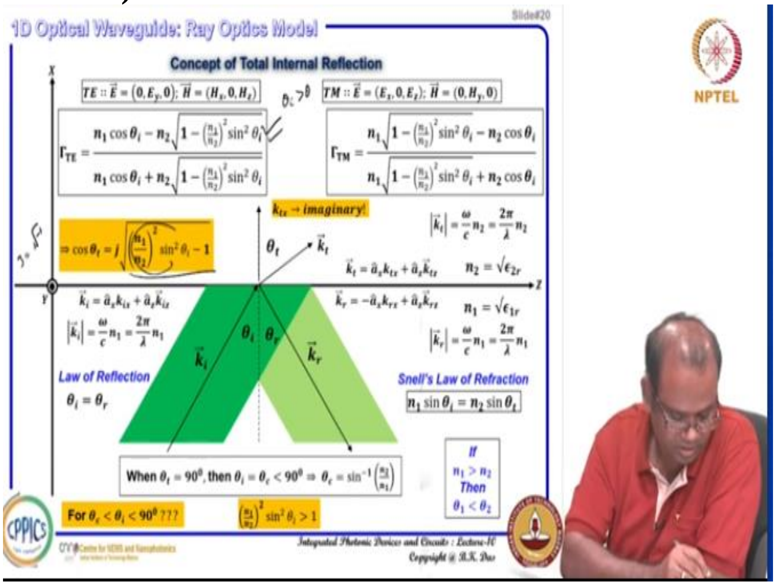

1D Optical Waveguide: Ray Optics Model

Slide#20

Concept of Total Internal Reflection

$TE :: \vec{E} = (0, E_y, 0); \vec{H} = (H_x, 0, H_z)$

$\theta_i > \theta$

$TM :: \vec{E} = (E_x, 0, E_z); \vec{H} = (0, H_y, 0)$

$$\Gamma_{TE} = \frac{n_1 \cos\theta_i - n_2\sqrt{1 - \left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_i}}{n_1 \cos\theta_i + n_2\sqrt{1 - \left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_i}}$$

$$\Gamma_{TM} = \frac{n_1\sqrt{1 - \left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_i} - n_2 \cos\theta_i}{n_1\sqrt{1 - \left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_i} + n_2 \cos\theta_i}$$

$k_{tx} \rightarrow$ imaginary!

$$\Rightarrow \cos\theta_t = j\sqrt{\left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_i - 1}$$

$|\vec{k}_t| = \frac{\omega}{c} n_2 = \frac{2\pi}{\lambda} n_2$

$\vec{k}_t = \hat{a}_x k_{tx} + \hat{a}_z \vec{k}_{tz}$ $\quad n_2 = \sqrt{\epsilon_{2r}}$

$\vec{k}_i = \hat{a}_x k_{ix} + \hat{a}_z \vec{k}_{iz}$

$|\vec{k}_i| = \frac{\omega}{c} n_1 = \frac{2\pi}{\lambda} n_1$

$\vec{k}_r = -\hat{a}_x k_{rx} + \hat{a}_z \vec{k}_{rz}$ $\quad n_1 = \sqrt{\epsilon_{1r}}$

$|\vec{k}_r| = \frac{\omega}{c} n_1 = \frac{2\pi}{\lambda} n_1$

Law of Reflection

$\theta_i = \theta_r$

Snell's Law of Refraction

$n_1 \sin\theta_i = n_2 \sin\theta_t$

When $\theta_t = 90^0$, then $\theta_i = \theta_c < 90^0 \Rightarrow \theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$

If $n_1 > n_2$ Then $\theta_1 < \theta_2$

For $\theta_c < \theta_i < 90^0$???

$\left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_i > 1$

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If I just do a little bit work on it, you see cos theta t I can write like this, because this one greater than 1 I have just taken j outside j = square root of minus 1. So, if this is j kt is imaginary I can just write this one for theta i greater than theta c that means, in complex form I can write, how to write?

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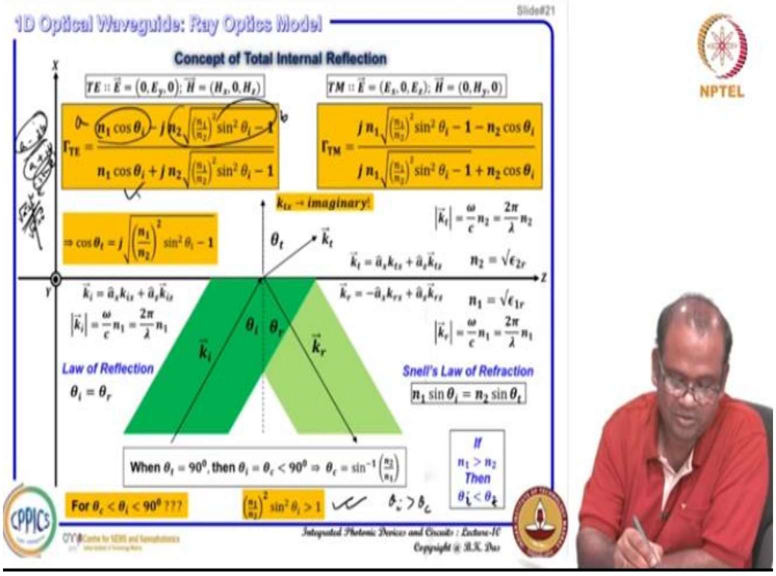

1D Optical Waveguide: Ray Optics Model

Slide#21

Concept of Total Internal Reflection

$TE :: \vec{E} = (0, E_y, 0); \vec{H} = (H_x, 0, H_z)$

$TM :: \vec{E} = (E_x, 0, E_z); \vec{H} = (0, H_y, 0)$

$$\Gamma_{TE} = \frac{n_1 \cos\theta_i - j\, n_2\sqrt{\left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_i - 1}}{n_1 \cos\theta_i + j\, n_2\sqrt{\left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_i - 1}}$$

$$\Gamma_{TM} = \frac{j\, n_1\sqrt{\left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_i - 1} - n_2 \cos\theta_i}{j\, n_1\sqrt{\left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_i - 1} + n_2 \cos\theta_i}$$

$k_{tx} \rightarrow$ imaginary!

$$\Rightarrow \cos\theta_t = j\sqrt{\left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_i - 1}$$

$|\vec{k}_t| = \frac{\omega}{c} n_2 = \frac{2\pi}{\lambda} n_2$

$\vec{k}_t = \hat{a}_x k_{tx} + \hat{a}_z \vec{k}_{tz}$ $\quad n_2 = \sqrt{\epsilon_{2r}}$

$\vec{k}_i = \hat{a}_x k_{ix} + \hat{a}_z \vec{k}_{iz}$

$|\vec{k}_i| = \frac{\omega}{c} n_1 = \frac{2\pi}{\lambda} n_1$

$\vec{k}_r = -\hat{a}_x k_{rx} + \hat{a}_z \vec{k}_{rz}$ $\quad n_1 = \sqrt{\epsilon_{1r}}$

$|\vec{k}_r| = \frac{\omega}{c} n_1 = \frac{2\pi}{\lambda} n_1$

Law of Reflection

$\theta_i = \theta_r$

Snell's Law of Refraction

$n_1 \sin\theta_i = n_2 \sin\theta_t$

When $\theta_t = 90^0$, then $\theta_i = \theta_c < 90^0 \Rightarrow \theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$

If $n_1 > n_2$ Then $\theta_i < \theta_t$

For $\theta_c < \theta_i < 90^0$???

$\left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_i > 1$ $\quad \theta_i > \theta_c$

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I can write this one this way, for this when theta i sin square theta i is greater than this one that means theta i obviously, when this is happening that is theta i must be greater than theta c this is theta i theta t. So, I have something like that you have something a and you can consider jb that means, this one you will be getting like a – j b / a + j b. So, numerator and denominator is just complex value complex conjugate.

And if complex conjugate value if you see in phase upon I can write a square + b square e to the power -j tan inverse b / a some angles will be there and denominator also same value with angle plus and minus angle numerator will be minus angle and same angle denominator depends upon it will plus angle. So, that means and absolute value also numerator and denominator they are same. So, that means I will be getting something interesting if I just a little bit modify this one then I will be getting nice thing this one.

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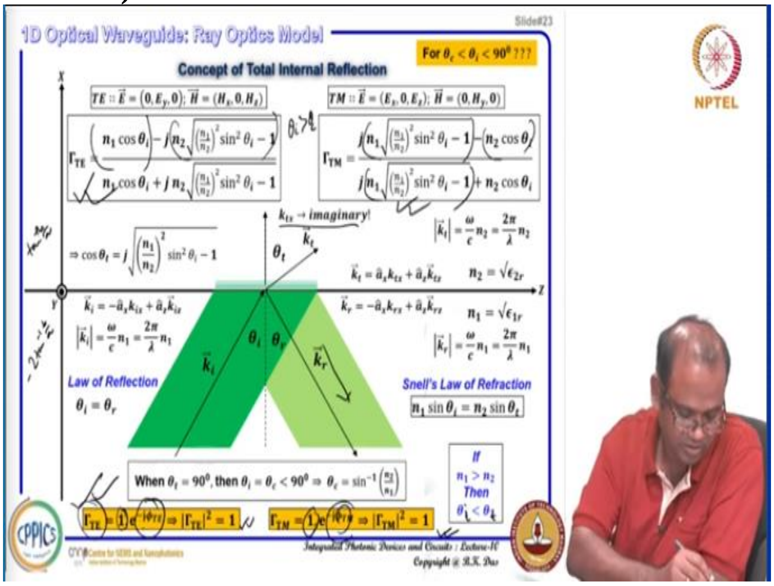

1D Optical Waveguide: Ray Optics Model

Slide#23

For $\theta_c < \theta_i < 90^0$???

Concept of Total Internal Reflection

$TE :: \vec{E} = (0, E_y, 0); \vec{H} = (H_x, 0, H_z)$

$TM :: \vec{E} = (E_x, 0, E_z); \vec{H} = (0, H_y, 0)$

$$\Gamma_{TE} = \frac{n_1 \cos\theta_i - j n_2 \sqrt{\left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_i - 1}}{n_1 \cos\theta_i + j n_2 \sqrt{\left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_i - 1}}$$

$\theta_i > \theta_c$

$$\Gamma_{TM} = \frac{j n_1 \sqrt{\left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_i - 1} - n_2 \cos\theta_i}{j n_1 \sqrt{\left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_i - 1} + n_2 \cos\theta_i}$$

$k_{tx} \rightarrow$ imaginary!

$$\Rightarrow \cos\theta_t = j\sqrt{\left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_i - 1}$$

$|\vec{k}_t| = \frac{\omega}{c} n_2 = \frac{2\pi}{\lambda} n_2$

$\vec{k}_t = \hat{a}_x k_{tx} + \hat{a}_z \vec{k}_{tz}$

$n_2 = \sqrt{\epsilon_{2r}}$

$\vec{k}_i = -\hat{a}_x k_{ix} + \hat{a}_z \vec{k}_{iz}$

$|\vec{k}_i| = \frac{\omega}{c} n_1 = \frac{2\pi}{\lambda} n_1$

$\vec{k}_r = -\hat{a}_x k_{rx} + \hat{a}_z \vec{k}_{rz}$

$n_1 = \sqrt{\epsilon_{1r}}$

$|\vec{k}_r| = \frac{\omega}{c} n_1 = \frac{2\pi}{\lambda} n_1$

Law of Reflection

$\theta_i = \theta_r$

Snell's Law of Refraction

$\boxed{n_1 \sin\theta_i = n_2 \sin\theta_t}$

When $\theta_t = 90^0$, then $\theta_i = \theta_c < 90^0 \Rightarrow \theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$

If $n_1 > n_2$ Then $\theta_c < \theta_t$

$\Gamma_{TE} = 1 e^{-j\phi_{TE}} \Rightarrow |\Gamma_{TE}|^2 = 1$

$\Gamma_{TM} = 1 e^{j\phi_{TM}} \Rightarrow |\Gamma_{TM}|^2 = 1$

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Let us concentrate this one this highlighted one so, gamma TE will be just one value and then little bit of phase because numerator denominator the same complex conjugate just simply complex conjugate, their absolute value will be same only there will be some phase angle. So, that phase angle I am considering phi t and if you just take this is the reflection coefficient for the amplitude reflection coefficient, but if it is reflectivity, if you just square it, then basically you have to take complex conjugate and multiply then it will be 1.

So, similarly, this is also complex and this is a complex conjugate so, this is your imaginary part and this is your real part, this is your imaginary part, this is your real part and plus minus sign is there. So, they are also you will be getting reflectivity, reflection coefficient 1 and certain phase will be there upon reflection some phases will be there. So, whenever theta i greater than theta c then you have total internal reflection but you have also additionally some phase part is there.

So, how to find the phase that you can straight forward just tan inverse b / a that will be the phase angle in the numerator and tan inverse b / a also phase angle in the denominator. So, that is why total phase change will be -2 tan inverse b / a, b I am considering this one imaginary part and a will be this one. So, that is straight forward phase upon you can just find out. So, I can see that when theta is greater than theta c there also I am getting everything reflected.

And you have k tx also imaginary and reflected wave this one is having certain kind of phase, what is that phase? If I just use this equation or this equation I can find out what is phi TE? And what is phi TM?

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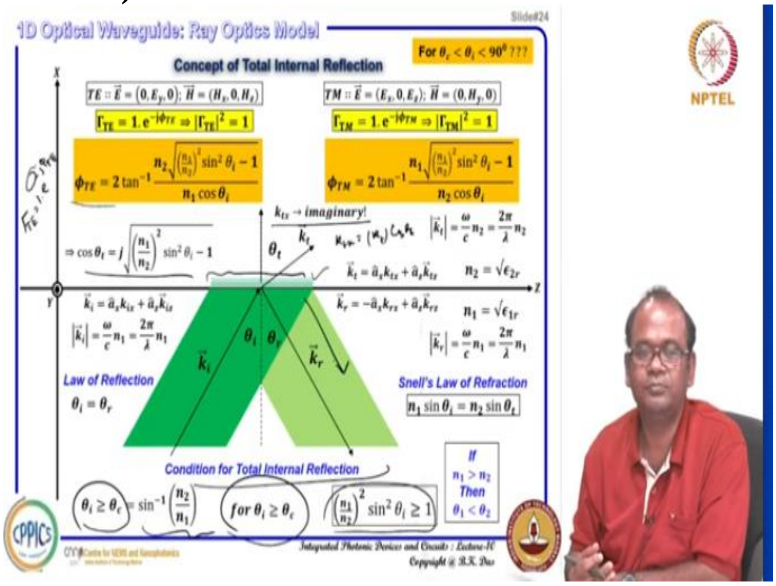

Slide#24

1D Optical Waveguide: Ray Optics Model

For $\theta_c < \theta_i < 90^0$???

Concept of Total Internal Reflection

$TE :: \vec{E} = (0, E_y, 0); \vec{H} = (H_x, 0, H_z)$

$\Gamma_{TE} = 1.e^{-j\phi_{TE}} \Rightarrow |\Gamma_{TE}|^2 = 1$

$$\phi_{TE} = 2\tan^{-1}\frac{n_2\sqrt{\left(\frac{n_1}{n_2}\right)^2\sin^2\theta_i - 1}}{n_1\cos\theta_i}$$

$TM :: \vec{E} = (E_x, 0, E_z); \vec{H} = (0, H_y, 0)$

$\Gamma_{TM} = 1.e^{-j\phi_{TM}} \Rightarrow |\Gamma_{TM}|^2 = 1$

$$\phi_{TM} = 2\tan^{-1}\frac{n_1\sqrt{\left(\frac{n_1}{n_2}\right)^2\sin^2\theta_i - 1}}{n_2\cos\theta_i}$$

$k_{tx} \rightarrow$ imaginary!

$$\Rightarrow \cos\theta_t = j\sqrt{\left(\frac{n_1}{n_2}\right)^2\sin^2\theta_i - 1}$$

$|\vec{k}_t| = \frac{\omega}{c}n_2 = \frac{2\pi}{\lambda}n_2$

$\vec{k}_t = \hat{a}_x k_{tx} + \hat{a}_z \vec{k}_{tz}$

$n_2 = \sqrt{\epsilon_{2r}}$

$\vec{k}_i = \hat{a}_x k_{ix} + \hat{a}_z \vec{k}_{iz}$

$|\vec{k}_i| = \frac{\omega}{c}n_1 = \frac{2\pi}{\lambda}n_1$

$\vec{k}_r = -\hat{a}_x k_{rx} + \hat{a}_z \vec{k}_{rz}$

$n_1 = \sqrt{\epsilon_{1r}}$

$|\vec{k}_r| = \frac{\omega}{c}n_1 = \frac{2\pi}{\lambda}n_1$

Law of Reflection

$\theta_i = \theta_r$

Snell's Law of Refraction

$n_1 \sin\theta_i = n_2 \sin\theta_t$

If $n_1 > n_2$ Then $\theta_1 < \theta_2$

Condition for Total Internal Reflection

$\theta_i \geq \theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$ for $\theta_i \geq \theta_c$ $\left(\frac{n_1}{n_2}\right)^2\sin^2\theta_i \geq 1$

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Simply you can just write down phi TE = 2 tan inverse this one if you are writing minus sign here gamma TE = 1 into e to the power -j phi TE. So, minus sign we have included here so, here phi TE will be just plus 2 tan inverse b / a. So, in phi TM also you will be getting tan inverse b / a. So, the difference is that phase you see everything is same 2 tan inverse 2 tan inverse here n 2 / n 1 and here n 1 / n2 otherwise same.

So, for TE polarization and TM polarization you will see that there will be certain kind of phase difference upon reflection total internal reflection. So, this is how total internal reflection is possible condition for total internal reflection is theta i has to be greater than theta c greater than equal to theta c, theta c can be expressed sin inverse n 2 over n 1. So, for theta i is greater than theta c we know that this is actually n 1 / n 2 square sin square theta i that is greater than or equal to 1.

Because exactly theta c this is one, if theta greater than theta c then that will be greater than 1 and when that is greater than 1 cos theta t is becoming imaginary, cos theta t is becoming imaginary means k tx is imaginary, because k tx again each k t cos theta t so, that is imaginary. So, that means imaginary will not be flowing in this direction it will be decaying along extraction so, everything will be reflective. So, when it is decaying, that is why we are just decaying field you have shown here that is called actually Evanescent field.

So, now we know that if you create a certain medium interface medium 1 and medium 2 and if you had just if you know there are certain refractive indexes dielectric values dielectric constant, then you can identify angle range which is greater than theta c and less than 90 degree you can attain total internal reflection when total internal reflection I am talking it is penetrating in the second media like metal in the second media.

But in metal normally there is a sigma it will cause some kind of loss because of resistivity electron will move back and forth against a damping. So, some kind of heat loss will be there scattering loss, absorption loss will be there, but since it is a dielectric medium sigma = 0 in the second media, so, but field is it decaying so, energy will not be lost. So, everything will be reflected back ideally.

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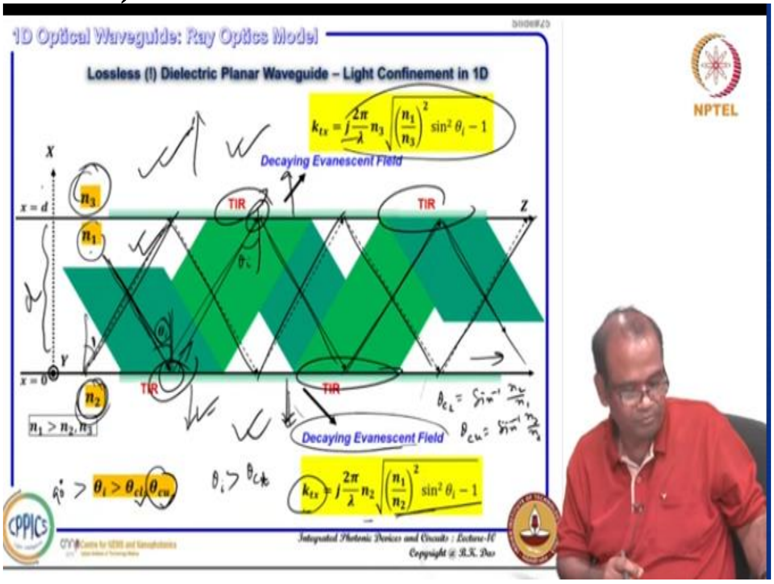

1D Optical Waveguide: Ray Optics Model

Lossless (!) Dielectric Planar Waveguide – Light Confinement in 1D

$$k_{tx} = j\frac{2\pi}{\lambda}n_3\sqrt{\left(\frac{n_1}{n_3}\right)^2\sin^2\theta_i - 1}$$

Decaying Evanescent Field

Decaying Evanescent Field

$n_1 > n_2, n_3$

$90 > \theta_i > \theta_{cl}, \theta_{cu}$

$\theta_i > \theta_{ck}$

$\theta_{cl} = \sin^{-1}\frac{n_2}{n_1}$

$\theta_{cu} = \sin^{-1}\frac{n_3}{n_1}$

$$k_{tx} = j\frac{2\pi}{\lambda}n_2\sqrt{\left(\frac{n_1}{n_2}\right)^2\sin^2\theta_i - 1}$$

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So, if that is the case, then you can think of lossless optical waveguide you can use medium 3, medium 1 and medium 2. Medium 3 is dielectric also you can actually characterize it by refractive index n 3 and medium 1 which is sandwiched between medium 2 and 3 and 2 and then 3 here and this medium 2 is will reach about just certain d this is like whatever you have consider like a earlier like a metal plate we are considering whether metal waveguide optical waveguide can be realized or not we discussed in the previous lecture.

So, similarly you can consider same thickness, but surrounded by sandwiched by between 2 in piloted extended dielectric medium characterized by n 2 and n 3 refractive indices. So, in that case you can think of a certain wave is coming with an angle theta i and if this theta i is greater than theta cl that means critical angle at the lower interface. So, theta cl lower interface in this interface that should be sin inverse n 2 / n 1 and theta cu upper sin inverse n 3 / n 1.

So, if your theta i somehow you set a theta i value incident angle that is greater than both theta cl and theta cu then you can get total internal reflection here as well as in the upper interface also. So, anything theta i greater than this one and less than 90 degree for example, then you can say that electromagnetic plane wave comes go back and forth and everywhere it will see total internal reflection similarly, complimentary other direction if it comes with the same angle for example theta i here, so it enter here it will go total internal reflection here also.

So, within a certain angle from this side and this side if it is entering they cannot actual propagate and can go like you can carry energy along the z direction that is a propagation direction of the waveguide. So, since everywhere you are having total internal reflection, total internal reflection, total internal reflection. So, you can have some kind of decaying Evanescent field because k tx in this medium it is n 2 it is j 2 pi over lambda n 2 this one.

Because k tx is equal to we can simply write cos theta t whatever cos theta t imaginary cos theta t you are getting that is what I have derived earlier also. Similarly, k tx in this medium it will be this one it is imaginary completely imaginary. So, since imaginary along the x direction it will be decaying after some time it will be 0 in this direction this would be 0. So, I can ensure that electric field electromagnetic wave it will be decaying in medium 2 and medium 3.

But, what is their inside in the core region which is actually x 0 to x d this typically in the order of lambda that thickness would be a 1550 nanometer we have shown on all the time equal to 45 degree typically if it is a glass it should be about 750 nanometers in the previous lecture we discussed

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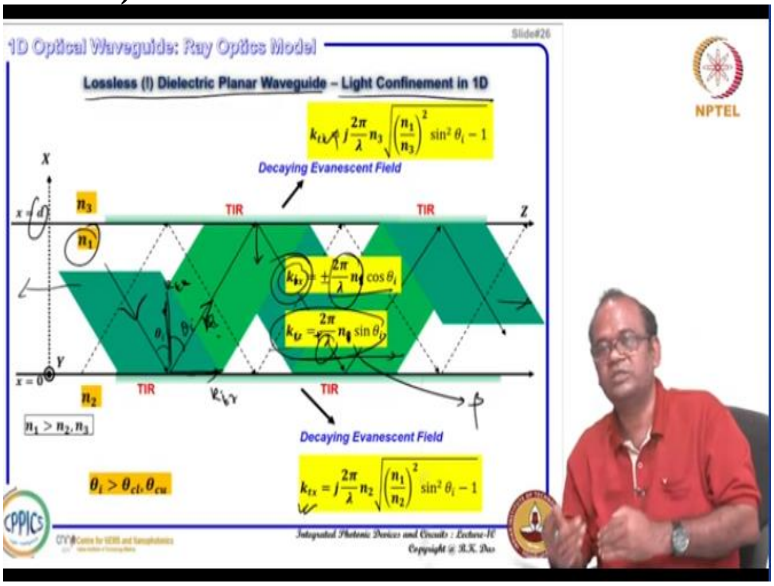

1D Optical Waveguide: Ray Optics Model

Lossless (!) Dielectric Planar Waveguide – Light Confinement in 1D

$$k_{tx} = j\frac{2\pi}{\lambda} n_3 \sqrt{\left(\frac{n_1}{n_3}\right)^2 \sin^2\theta_i - 1}$$

Decaying Evanescent Field

$$k_{tx} = \pm\frac{2\pi}{\lambda} n_1 \cos\theta_i$$

$$k_{tz} = \frac{2\pi}{\lambda} n_1 \sin\theta_i$$

Decaying Evanescent Field

$$k_{tx} = j\frac{2\pi}{\lambda} n_2 \sqrt{\left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_i - 1}$$

$n_1 > n_2, n_3$

$\theta_i > \theta_{cl}, \theta_{cu}$

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So, what happens in the middle? In the middle you have k tx you have one k tx will be there and k tz and here also k tx, k tz. So, that means k tx will be one is positive x direction another is negative direction. So, this is your k 1 this should be n 1 this should not be n 2 because in this medium refractive index is n 1. So, k tx will be plus minus 2pi / lambda n 1 and k tz will be 2pi / lambda n 1 sin theta i.

If this is theta i so we can have k tx equal to this one this is also theta i so, this is k tx and this is k tz, so k tz equal to this one actually k so, not k t here k i should get here what about longitudinal things, so, this is actually k ix and k i and this is actually k iz incident angle k i incident it is coming so, you can say that this one. So, in that case, so simply clearly you can see that you have counter propagating k tx component, the wave component.

You can consider which is actually going along the x direction for positive direction and negative direction. So, 2 types of wave vector so, that means, in the x direction you can have a standing wave because of the k tx component. And k tx will be 2pi / lambda n 1 that is actually obviously real that is not imaginary. So, that wave will be there but it is counter propagating it will create some kind of standing room that is actually called mod depending on the type value certainty theta you will be allowed I will be discussing that.

And in the longitudinal direction you can have this k tz that means whatever k tz you are considering that k tz or k iz or inside whatever value that is actually 2 pi / lambda n 1. I can simply write if I considering just k ti do not need to write I just confused in the previous slide that transmitted wave whatever happening here. So, here actually transmitted that is why k tr writing, but inside it is typically k i. So, in that case you get this one that will be only positive sign.

So that means in the forward direction it will be propagating obviously, you can launch from this side and total internal reflection can happen so, negative sign also it will be propagating. So, in that case k tz will be this one and this one sometimes we call it as a beta that means, this k transmits a component inside the core that actually contributes to field confinement along the x direction and this component of the vector that actually helps to carry energy along the wave by direction.

So that is how you can actually really design a lossless dielectric plane or waveguide so, light confinement in 1D with this I close today, I will follow up this one to understand the how this modes distribution, different types of modes, how many modes it can support for a given value of d, and for a given value of lambda. Those things we will be discussing in the next lecture. Thank you very much.