Stochastic Modeling and the Theory of Queues
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Module - 1
Lecture - 4
What is a Stochastic Process?

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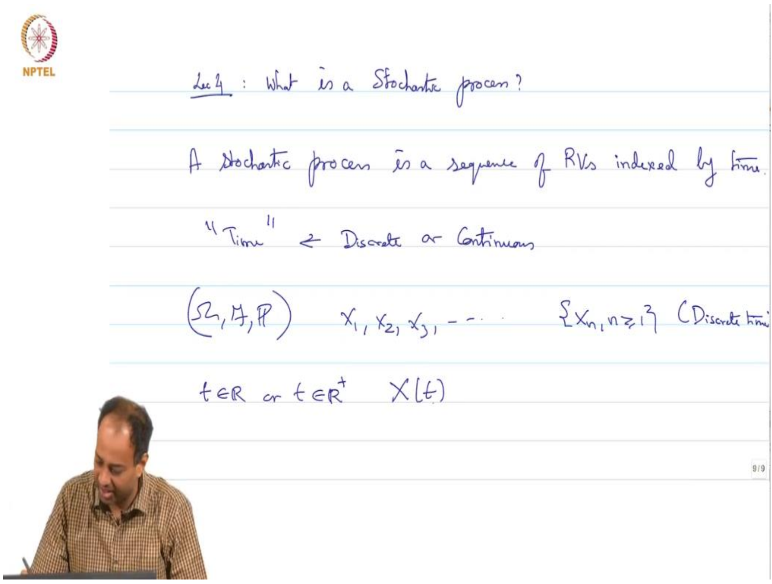

What is a Stochastic Process? See, a stochastic process is nothing but a sequence of random variables, where the sequence, the index of the sequence has the interpretation of time. So, you can say that a stochastic process is a sequence of random variables indexed by time. Now, this time, what I am talking of as time, could be discrete or continuous. So, there may be some; the stochastic process maybe; you may be observing it only every minute or every hour or whatever.

In that case, the time index is a discrete index. So, it will be indexed, for example by integers. Or it could be continuous, in which case it could be indexed by R or R +. You may be; if you are continuously observing something, as opposed to sampling it at integer times or whatever. So, this time index could be discrete or continuous. So, if the time index is discrete, it is exactly really the sequence that we have already seen.

So, you have some Omega, F, P. All of this, the Omega, F, P is already defined. From this Omega, F, P, a sequence X 1, X 2, X 3, ... realises. Every time a little omega is chosen, and the sequence X 1 of omega, X 2 omega, X 3 of omega realises, except that this now, this index now has the interpretation of time. That is all there is. So, you can just speak of; in the notation, we will just write; for a discrete time stochastic process, we will just write X n, n greater than or equal to 1 is a discrete time stochastic process, defined on this probability space.

It is nothing but a sequence of random variables. That, we have already seen. Clear? So, whenever little omega realises, you have a real sequence realising. For some other little omega, you get some other real sequence. So, a discrete; a stochastic process in discrete time is nothing but a sequence of random variables indexed by some natural numbers. And the continuous time stochastic process is also a sequence of random variables except this sequence is not indexed by n, but it is indexed by an uncountable index.

It is indexed by r or something. So, if continuous time; so, this is for discrete time. For continuous time, you have, it is a t belongs to R or t belongs to R +. X of t is a stochastic process. You can just view as; I mean, in continuous time, we write X bracket t. You could just as well write X subscript t; there is nothing wrong with that. The only difference between the previous case and the case where, I mean, the X of t; now this t is running over some uncountable index set R or R +, typically R or R +. In the previous case, it was running over a countable set, where it is the integers. So, what happens here is that;

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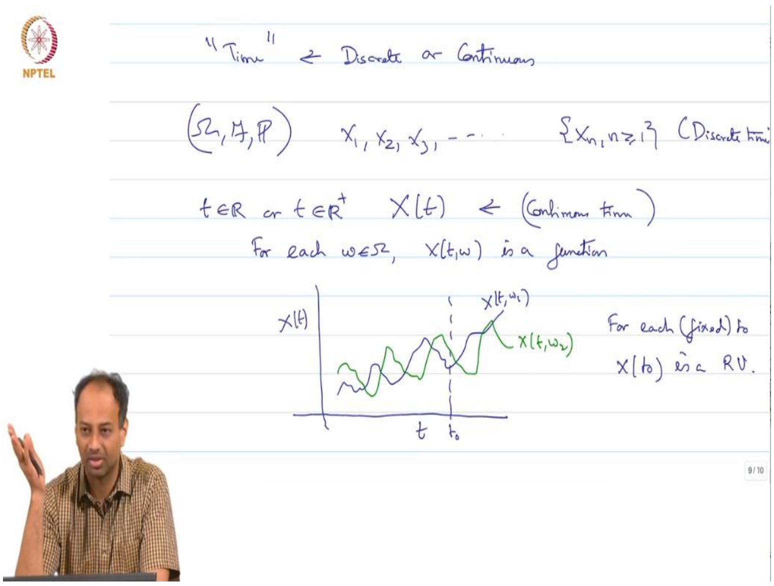

So, in this case; this is for the continuous time. For each realisation omega, X of t, omega is a, what is it? It is, you fix a little omega, X of t, omega becomes; is a function. So, for each little omega, you get a different function. So, just to draw out something. This is time. So, I am just drawing some continuous time stochastic process. So, let us say that some; so, this may be X of t, omega 1.

And for a different value of omega, you may get a; this could be X of t, omega 2. So, I want to emphasise that there is only; so, there is a little omega that realises, and the entire sequence X t as indexed by a continuous time t realises. So, basically an entire function realises. I am drawing this over R +, but it could just as well be R or something, or a boundary interval on R. All that is okay. So, each time a little omega realises, you get a different function.

Now, on the other hand, if you fix a particular value of t nought; for each fixed t nought, X of t nought is what? Now, fixing a t nought, you are looking at what is the X of t nought. X of t nought will depend on little omega. So, X of t nought is a random variable. See, just like in the discrete case, if you fix some i; X i is a random variable. Here, i is not a integer, but some t nought; that is all.

So, bottom line is, X of t is a stochastic process, means that, for each realisation little omega, you get a different function at a high level. And if you just fix a t, you get a random variable. It is after all just a sequence of random variables indexed by that t. As of course, you fix both t nought and omega, you get a number. You could just get a number. So, that is really what a stochastic process is. It is a sequence of random variables indexed by time.

The time could be discrete or continuous; that is all. Now, in this course, I said that we will deal with discrete stochastic processes. We will basically just deal with Poisson processes, renewal processes, and Markov chains. These are basically the 3 stochastic process we will study. Now, in Markov chains, we are going to study Markov chains which have state spaces which are finite or countable.

So, in those cases, the value taken by the stochastic process will be constrained in a discrete set. And in the case of Poisson process or renewal process, the time index is continuous, but the values taken by the stochastic process again will be constrained in a discrete set. It will be integer valued or some countable set value, taking values in a countable set. So, in this course, we are not going to encounter continuous time, continuous state, continuous valued random process like Brownian motions and all that, we are not going to study, only discrete stochastic process; either n or the values taken by the process will be in a discrete set.