Digital System Design
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Lecture No. 03
Binary Number System - 1

Hello students, in our previous lecture we understood how analogue signal is converted into digital signal and we also seen the importance of 0 and 1 means of binary representation and how robust and other resilient system would be a binary system. Now, we would like to answer an important question in this particular lecture. The question is how to represent binary, how to represent information in binary form?

So, to help to understand this particular question itself, so we have two particular choice two particular sub questions. One question is, let us say a 2 bit number, a 2 bit number can have four states 00 01 11 and 10. So, these 4 states to which number each of these states should map. So, we can say that our two bit number can represent four numbers 01234 0123. Now, whether 00 should represent 0 or 01 should represent 0 and similarly 0123 how these 4 numbers would be mapped to 00 01 11 10.

So, one thing is clear, it is going to be a one to one mapping So, that means, no two numbers can map to the same decimal number or similarly no two decimal number can be mapped to the same binary number. So, we have to have some sort of one to one mapping but which mapping is most natural and this problem will become more complex if we will have more number of bits, let us say we have 5 number of bits or 10 20 30, the 30 bits are there, the combinations are 2s power 30.

So, which combination will correspond to what digital number or what decimal number is the question we would like to answer in this lecture. To answer this question, we can learn from history. Historically, each culture or each every region in the world they have developed independently, because they have developed independently so, they develop their own language systems that is why every part of world has their own languages.

Now, as people invented languages, they also find the need to count numbers, to count number of people, to count number of objects etcetera. So, that is why they invented their own number systems.

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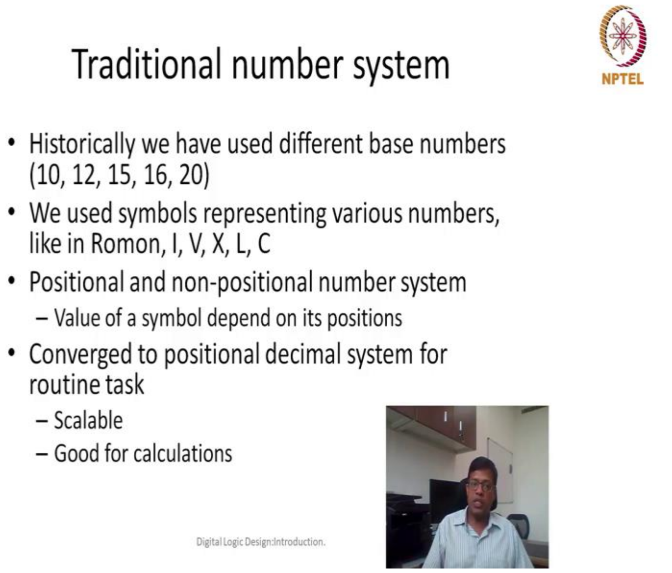

And if we observe many of these number systems, most of them, they were based on some sort of a base system or like in Indian system, we have typically based things on to base of 10, but at some other places like if we want to calculate weight, so in weight we had a different system where we had different notion like 5 kg and then 40 kg etcetera.

So, similarly at different places of world in part of Europe base system of 12 was quite common and Spain and France like places they had a base system of 16 as is quite common. So, similarly at other places base of 15, base of 20. So, all of these were common base systems and now, how do you counted large numbers? So, some, many people, they at many places, they used multiplication or position based number system, so where each number where each position in the whole number represent the, like value would be different.

While there were other systems where different symbols represent different values. For example, Roman numbers in Roman numbers, I means 1 and V means 5, X means 10, and L means 50. C means 100. So, they keep on inventing different symbols for different numbers. So, if you want to count a very large number, then there would be a different symbol for each word. So, each symbol would carry its own value.

So, based on this system now, we can also classify only these two parts one is position based, another is non-position based. So, in position based number system like there will be fixed set of symbols, for example in decimal, so there are fixed set of symbols there are 10 symbols 0123456789. Now, based on where that particular symbol is there in the number it will determine its value.

So, if it is at the unit's place, the value is same as the symbol, but if it is at decimals, if it is at tens place, the values multiplied by 10, if it is at 100s place, the values multiplied by 100. While in known positional system like a Roman like your value is determined by what symbol you are using, it does not matter where the symbol is. So, let us say XXX is 30. So, it does not matter where X would lie, so but there is also some relation between if X is before L or after L, the value would be different value. But it's X value would still be 10 in whether it is after L or before L, but overall value of the number would be different based on the position.

Now, after inventing so many type of number systems across the word. Finally, we have converged onto position based decimal system. The reason is that using these 10 digits 0123456789, we can represent any large number, you need not to define a new symbol for a large number. Whatever is the largest number of the word can be defined using only these 10 symbols.

Additionally, what have been observed that, for example Roman numbers or known position based on numbers, even any number which was of different base than 10 calculation was difficult. Like addition, subtraction, multiplication finding square root, so all of these things were slightly difficult if we were using some other base other than 10. So that is why finally we have converge onto a position based decimal system, where the base was 10 and each position carries its own value.

So, based on this learning from the history, we will simply use this position based, position based system. So, instead of decimal it would be binary, so where there are only two symbols 0 and 1, so it would be position based binary system that would be that we will use for our purpose.

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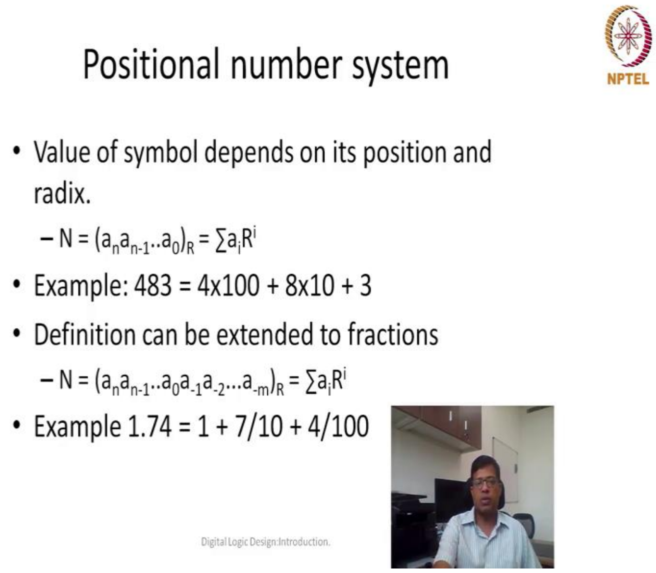

Now, let us try to see theoretically, like if value depends on the symbol, as we can define it by this particular equation. So in this equation, n is equal to let us say there, there are n numbers, there is N plus 1 symbols are there, symbols are fixed so let us say if it is, we can also generically defined as our radix system, so R is the total number of symbols which are present. So, let us say R are the number of symbols and the number has N minus 1 such digits.

So, this number can be represented as, so addition of ai position, value position at ith position into r is to power i. So, this looks complicated, but let us try to understand it with the decimal system. So, let us say we have, 483 is a number. So, now at units place we have 3, at units place we have 3 and at 10s place we have 8, so 8 would be multiplied by 10 is to power 1 and 4 would be multiplied by 10 is to power 2.

So, radix was 10 or base was 10. So that is why each number like as we go from left to right or right to left as we go from right to left, so LSP would be multiplied will always be unit's place and from LSB we go towards MSB, the power of this base will keep on incrementing by 1. So, here power of base means R is one here power of base means R is 2. So, 10 is power to power 2 is 100 so, keep on going like this.

Since we have understood or we have seen this particular system since our elementary classes we are so apt, we have so adapted to the system that it comes naturally for us to read numbers in decimal places and that is why throughout the course, we will talk mostly about the whenever we see numbers we will see in terms of decimal numbers only. If it is explicitly specified, it would be specified the number is represented in form of binary or some other radix, but this formula is generic. So, it is applicable for any radix, any particular number.

So, this position base number system is not only applicable only for integers, but it can also be used for fractions as we know like. So, if I want to extend this formula for fractions, then it will become, let us say a minus 1. So, like position number minus 1, position number minus 2 up to question number minus m. So, here in this number there are N plus 1 digits before the decimal point and M digit after decimal point, but the formula still remains same.

So, the most important thing is that at the decimal your position is the pivot point, where you are starting counting of your radix powers. So, for example here if I would like to see any number which has a sort of decimal, let us say 1.74. So, one means units place and this is before decimal. So, my LSB will start counting only at the decimal place. So, if it is before decimal, then I will start counting it like LSB and then I will go towards the most significant byte LSB means Least Significant Byte and MSB is Most Significant Bit.

So, by my powers of R will start incrementing after like before this decimal place and after this decimal place, it will start decrementing. Here 1.74 we understand that it is 1 plus a 7 by 10. So, means 10 is to power minus 1 and plus 4 by 100 means 4 into 10 is power minus 2. So in this way, we can represent any kind of a number being it absolute number integer or it is a fraction. So, let us try to extend this position based number system to binary system.

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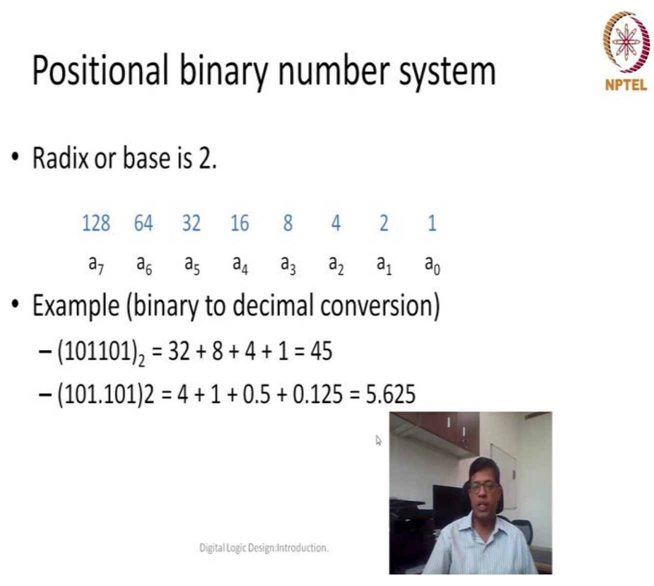

So, let us see it using this example. So in, when radix of base is 2 then it is a binary system, total number of symbols which are available to us is only 2 that mean 0 or 1 and as we move along to the positions my 0th place or units place will have a power of 1, it would be, it would have multiplication factor of 1 and then first digit, first bit, so this is 0th bit, this is first bit, first bit could be multiplied with 2 and second bit would be multiplied with 4, third bit will be multiplied with 8. So, that means 2 is to power 3, 2 is to power 5, 2 is to power 6, 2 is to power 7.

So this way, as we move from a least significant bit to more significant bit, power of two is kept on incrementing. Let us see it with an example that let us say we have some number which is a 101, 101 101 and I want to convert this binary number to a decimal number, so what it would be that I need to see from the least significant bit that is 1, so 1 would be multiplied by 1 and the next bit is 0, I am ignoring that and then at a2 place I have 1. So, a2 would be multiplied by 4 and then a3 is again 1, it is multiplied by 8 and a4 is 0, a5 is multiplied by 32.

So here, you see multiplication is not a multiplication, but it is actually a replacement. So, if bit is 1, its value is replaced, if it is 0, it's ignored. So, that way this particular number will become 32 plus 8 plus 4 plus 1. So, if I add all of them to 45. So this way, a binary number can be represented by using the same formula by calculating the power of or calculating the value at each place.

Now, if we would like to convert a number, which is a fractional number. So, let us take a fractional number example. So, it can be extended in the same way. Here, things would be like at before the decimal place, like all the integer part would be calculated in the same way we had calculated this binary part. So here, we will start with this 1 1 and after that, its value is 1 here.

Now, a1 is 0. So, we are ignoring this, we are not writing it in this calculation part and a2 is 1 a2 value is found to be 4. So, 4 plus 1 we can write this is the integer part. Now, what is the fractional part, fractional part is also 101 here. So, a minus 1 after decimal we have started minus 1. So, minus 1 means 2 is to power minus 1, 2 is to power minus 1 is 1 by 2, 1 by 2 is point 5. So, this 1 would mean a 0.5 in decimal.

Now, the second place a minus 2 is 0, we can ignore that and a minus 3 is 1. So, a minus 3 is 1 by 2 is to power 3, 1 divided by 2 is to power 3 that means 1 by 8, 1 by 8 is point 1 to 5. So, if I add all of them, it will become 5.625. So, this is how a fractional number can also be represented or converted to decimal, if I keep following the formula, which we mentioned in the previous slide. So, this formula can be used in generically. So, now after this, so we can summarise that this binary system can be represented and we also know how to convert if there is a binary number given then how we would convert it into a decimal number.

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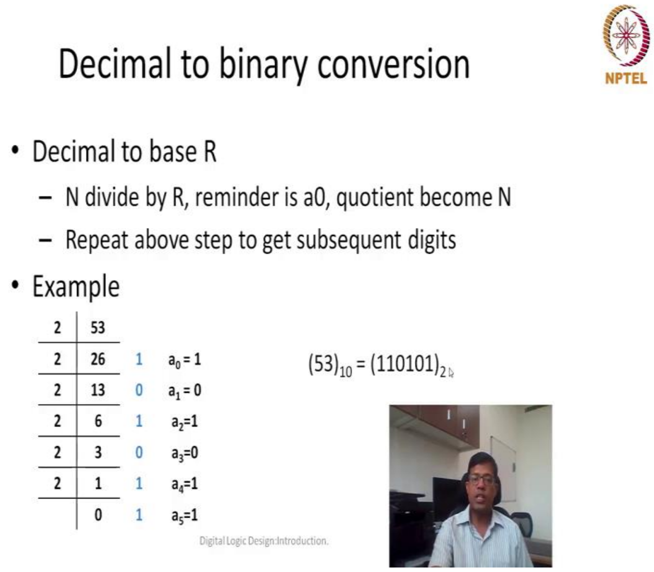

So, let us see the next thing that if decimal number is given, how to convert into a binary number or in general to any number with base R. So, most of you know this calculations, this has been done in your in your higher secondary classes or in your schools. So, let us revise it for the sake of completeness and for benefit of everybody.

So, let us say number n, which was represented in a decimal number and we want to convert into a base R. So, what we do is we divide this number and by R, whatever is the reminder that bit is a0, and the quotient become a new n and then we keep on calculating again divide n, a new n by R and then the goal, then the remainder is the next bit. So, the consequent bit, subsequent we can keep on generating during this. Let us see this with an example so that we can understand it better.

Let us say we have this number 53 which is represented in binary, in decimal. 53 is represented in decimal now we would like to convert it to binary, because it is binary base is 2. So, we have to divide every time this number by 2. Now initially we divided this 53 by 2, it will come out to be 26 and the remainder is 1 and because remainder is 1, so this will form the least significant bit 1, a0 will be equal to 1 and now new number is 26. This 26 has to be divided again by 2. By after division, the new number is 13 and quotient is 0, reminder is 0, so because remainder is 0, this will become the subsequent bit for that new number, for our binary number.

Now after 13 we again divide it by 2, quotient is 6 and remainder is 1, because remainder is 1, this is become subsequent binary bit and again we divide 6 by 2, it will be 3 here remainder is 0. So, my subsequent bit binary bit is 0 and again I divide 2 by this my number 3 by 2, remainder is 1 and remainder is 1 and quotient is also 1. So, because remainder is one my subsequent binary bit is 1 and again I divide this 1 by 2, here quotient is 0 and remainder is 1 because remainder is 1, I have represented next bit as subsequent binary bit as 1.

So, I the thing which has been known noted is that we are doing this process until we reach to the quotient equal to 0, till the number become 0, we are keep on dividing this number and the second thing to be noted is that we will receive first least significant bit and in the end, we will get more significant bit and every time we can keep on incrementing these bits.

So, in other words, I can rewrite this as this, 53 so this is also one representation we have to remember whenever 53 subscript 10 is written that means the number is represented in a decimal form and whenever subscript 2 is given, so that means number is represented in binary form.

So, in whenever we will talk in in questions or in exams or in slides, by default, we would be representing a number in a decimal form. So, that means there is no radix specified that means the number is in a decimal form. If radix is specified, then that would be the representation of that number or otherwise, we will write explicitly that the following number is a binary number or following number is a decimal number.

So, here are 53 in decimal has been written as 1, so this is the MSB 11010, 110101 and this is the decimal representation. So, this is how a number or an integer would be converted into a decimal number. Now, in a binary number, so let us say if there is this particular number is not a not an integer, but a fraction.

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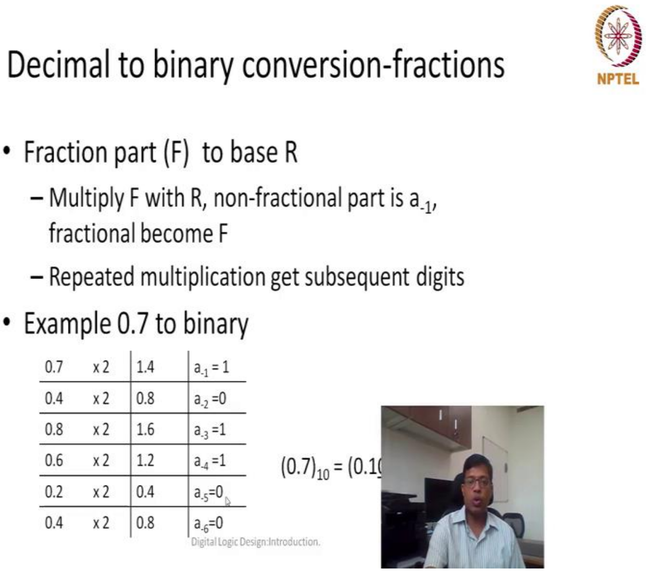

So, how to do it with a fraction? In fraction that the process would be difficult process would be a little bit inverse. So first of all, let us say we have received one particular number or let us say 3.5. So, we will separate will make two parts of this number 1 as a integer part, another as a fractional part, integer part we will do during this process, where we will keep on dividing and the fractional part will do using the process which we are going to discuss now.

So in for fraction, what we will do is we will keep on multiplying the number, multiply with the base R and whatever known fractional part we generate that will become a minus 1 and subsequent digits we keep on getting by multiplying by repeated multiplication and whatever is the non-fractional part will be the next subsequent digit. Again, let us see it with an example.

So, to make things clear and here because we would like to convert a decimal number into a binary number, so our ways would be too. So, let us say we want to convert a 0.7 which is represented in a decimal number to a binary number. So, first thing we do is as given in this formula, first 0.7 would be multiplied by 2, because it is multiplied by 2, we will get 1.4. Now, what is the integer part 1, non-fractional part is 1 and fractional part is 0.4. So, 1 we will take it as a minus 1 is 1.

Now the fractional part 0.4 we take and then again multiply, this is 0.8 and 0.8, because here known fractional part or integer part is 0, so we can say a minus 2 is 0 and we take the fractional part which is point 0.8, 0.8 would be again multiplied by 2, it became 1.6 and because it is 1.6 fractional part is 5 6 and integer part, non-fractional part is 1. So, a minus 3 is 1.

Again we take the fractional part point 0.6, 0.6 would be multiplied by 2 that become 1.2, here fractional part is 0.2 and non-fractional part is 1so, we can say a minus 4 is 1 and this point to again would be multiplied by 2 it will become 0.4 here non-fractional part is 0 and fractional part is 0.4. So, 0.4 we can take to the next row and 0.2 again we multiply with it become 0.8. So, this this particular part will now represent the binary number. Now, we can say that 0.7 in decimal is equal to 0.1 01100 in binary.

Now there is, for this particular number there is one observation that this number is a known a rational number. So, that means, this these bits in binary is non-rational because, 0.7 is not followed by 0, but this this is not completed. So, that means, here in the multiplication we are not received 0. So, that means, there would be some part which is keep on repeating. So, that repeating part we can observe here, you see this pattern point 0.4, 0.8, 0.6, 0.2 and because we received 0.4 again, so that means, this pattern is going to be continued.

So, these 4 digits, these 4 binary digits 0110 will keep on repeating. So, we can also say this number as 0.1 0110 where 0110 will keep on repeating itself to infinite. So, then it would be close to, then the value would be very close to 0.7. So, this is how we can convert a decimal number to a binary number be it a fractional or non-fractional.

Now, one more thing which I, we will finish in this, which we will try to understand in this lecture that is there any other base like in binary we have base of 2, is there any other base which is important for us in this particular course.