# Digital System Design

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## Algebraic Simplifications

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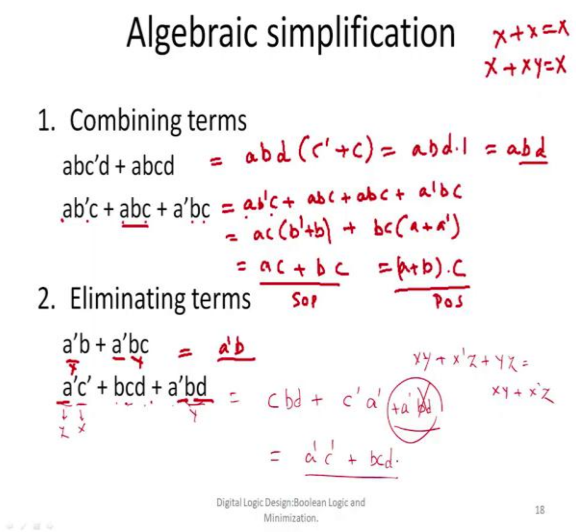


The other thing what we will discuss is we will try to see how we can do algebraic simplification. So, basically you have seen in this lecture that these expressions would be if I need to implement them, then I would require AND and OR gates and more easy number of terms that means the larger. So, let us say there is a sum of product term, so more easy number of terms that means, a larger would be the gate size of my OR gate.

And larger is the number of literals that means number of, more number of terms also means more number of let us say AND gates here and more literals means size of each gate is also more. So, we have to minimize both, we have to minimize literals as well as we want to minimize number of terms. So, how can we do that? Again, we have to use our algorithms, sorry, theorems which you have studied yesterday. So, in our previous lecture, so let us take some of the examples.

So, there are 4 ways of simplifying. We have to see what all we can combine. So, anything we can combine that would help us in reducing the term. So, let us say this is the example abc dash d abcd. So, if I see this, I can see that abd abd, so this particular set of literals is found in both the terms, because abd is there in both the terms, so I can also write it as a b and d, I can take it common and then because of law of distribution, I can write it as c dash plus c.

Now, we know that c dash plus c is equal to 1, so I can have a 1 over here, abd and 1. Multiplying 1 with any product term does not have any effect, so I can also call it abd. So, this way I am not only able to reduce the number of terms but also a number of literals. So, here number of terms were 2, now here number of term is 1. And number of literal in each term is also reduced. So, here the number of literals are 3.

So, let us say the, some other example here. So, here if I see this, then I can do, there are 2 possibilities here, one is that I can make if I see what is common here, so ac is common with these two terms and bc is common with these two terms. So, but I also know that x plus x is equal to x, so I can write it the other way, a b dash c plus abc plus abc. So, here I have replicated this term because it does not make any impact in terms of writing our Boolean algebra, so a dash bc.

So, here we see that a and c both are common in this terms, so I can write a and c b dash plus b. And, and I can again take a and c as common factor here and then a call b plus b dash. So, again b plus b dash would be 1, there is something wrong here, ac here I am going to take b and c actually, something is also wrong, so I need to erase this. So, here I can see that b and c can be taken common and a plus a dash would be the factor which will come out. So, b dash plus b will become 1, so I can write this as ac and here a plus a dash would be can be eliminated, so this will become bc.

So, if you see, I can, this is the sum of products or if I write it like this a plus b and then multiplying it with c, this will become sum, product of sum. So, this is product of sum expression and this is sum of products and this is product of sum. So, any of these expressions we can, we have derived from this place. So, this is how we can combine terms and then essentially reduce some other terms. So, combining two terms to make single term.

Now, there could be some other method where we can directly eliminate some other terms. So, for example, this a dash b plus a dash bc. So, what should we do? Here we see a dash b is there, a dash b is there in this term as well as this term, so if we remember our theorem which is X plus XY is equal to X, so we can use that here and we can call it this are elimination theorem, so this elimination theorem we can directly use and we can say this is equal to a dash b, because a dash b I can consider as X and this I can consider c we can consider as Y.

Let us take another example of eliminating terms. So, in this example we see that we have a a dash c dash plus bcd plus a dash bd. So, we see that bd is here as well as here and a dash is here as well as here and c dash, c dash is here and c is here. So, this help us in recalling our consensus theorem, and this consensus theorem can be written as let us say for our recalling, let us write it like this. XY plus X dash Z plus YZ equal to XY plus X dash Z. So, here this YZ term is actually redundant.

Now, if we see that, can we apply here this consensus theorem? So, we can see that we have to take c as X and bd we can take as Y and c we can, a we can take as a dash, a dash we can take as Z. So, we can rewrite this whole thing as our X is c dash and Y is this, so we can write c dash, sorry, this is Y this we, this according to, so according to this, we have this X and Y. So, Y is our bd, so we can write a dash and this, what is our Y? Y is our bd, so X is our c.

Let us write it again. So, our X is c. So, we will write X as c and Y is our b and d plus we can write X dash, X dash is c dash and Z is our a dash plus Y and Z, Y is our bd and Z is our a dash. So that means this particular term is redundant and we can write this, this whole term is redundant and we can write it as a dash c dash plus bcd. So, this is how if we use consensus theorem, then we can also remove some of the terms which are not required.

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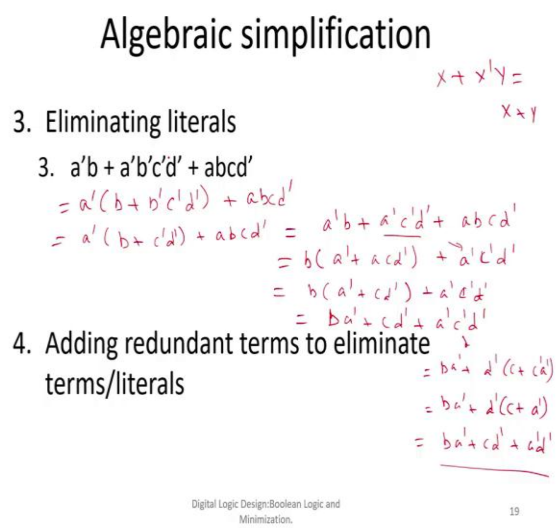


So, other than eliminating terms and we can also sometime eliminate literals. So, let us take this example a dash b a dash b dash c dash d plus a b c d dash. So, what so we see and how do we start. So, one thing we see here that a dash I can take common and I can see, I can at least see that there is a b and b dash so maybe we can try out something. let us try out, a dash we have taken common, b plus b dash c dash d dash and this we have left as such, a b c d dash.

Now, if you remember b plus b dash, so this could be used elimination theorem X plus X dash Y equal to X plus Y. So, we can say here a dash, this can be written as b plus c dash d dash plus a b c d dash. Now, I can again write it like this, so which is equal to a dash b plus a dash c dash d dash plus a b c d dash. Now, if I see it here, b is something which is common between this and this and we have a possibility of a dash here and a, so maybe we can try it, try out an optimization here.

I can take b common a dash plus this is a c d dash, this I can write here a dash c dash d dash equal to now this again could be optimized using this elimination theorem that means b a dash plus cd dash plus a dash c dash d dash. I can completely again write it like b a dash plus cd dash plus a dash c dash d dash. So, this could be one of the, so we do not see any other opportunity here. So, for example, d dash is here as well as here, but a dash c dash plus c we cannot optimize much out of that.

Yes, so there is further optimization possible, so I can further write b a dash plus d dash c plus c dash a dash. So, I can say this as b a dash plus d dash c plus a dash which is equal to b a dash plus c d dash plus a dash d dash. So, now I do not see any further possibility because if d dash is taken common, c and a dash are independent terms and here if a dash is taken common, b plus d dash, they are again independent terms, so we cannot optimize it further, but you see the total number of terms are still three, 1, 2, 3, here also number of terms are three, but because we are able to eliminate literals, so this expression is much more simplified form than this expression.

So, this is how we can use these algebraic simplifications. And you can see in the examples we have taken, in so other examples we have done elimination sometime we add a sum redundant terms to eliminate. So, using all these 4 methods by combination of terms, by elimination of terms, by elimination of literals and sometime we add redundant terms so that we can eliminate some other terms. So, these 4 methods can be used in combination and in certain order so that in any order actually, in any order so that we can have a algebraic simplification.

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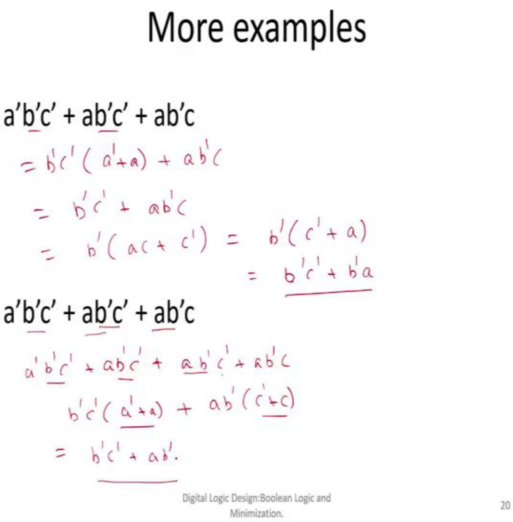


So, let us take one more example. So, let us say we want to have this a a dash b dash c dash and a b dash c, a b dash c, I want to simplify this. So, what do we see? I see b dash and c dash is a common term here so I can call b dash c dash and then a dash plus a plus a b dash c and now I can write b dash c dash plus a b dash c. So, if I want to further optimize, then, so on the other hand, so there is a further possibility also that I can take b dash as common and then say a c plus c dash which can say the using the elimination theorem I can say b dash c dash plus a which means b dash c dash plus b dash a.

So, let us take the same example again. So, can we have other method or some other way of optimizing or this is the only method which we did. Here, if we see that this a dash c dash is common between this, sorry, b dash c dash is common between these two terms and a b dash is common between these two terms so I can write it this particular term I can write it again, a dash b dash c dash plus a b dash c dash plus again I will write a b dash c dash plus a b dash c.

So, if I see this b dash and c dash these two are common here so I can say b dash c dash a dash plus a is the factor. Now, here I say a and c dash, sorry a b dash can be taken common and c dash plus c are the factors. So, I can write further, I can eliminate this, I can eliminate this, b dash c dash plus a b dash. So, using both the methods we are able to achieve this as a minimum expression.

Now sometime it may be possible that we are able to reach a minimum method one particular way of simplification, but you have to remember that each whenever you are going through one particular method of simplification or you have taken one approach, so it cannot be undone, so basically using that maybe you have removed one particular literal, one particular terms. Now, if the term which you have reduced could be help to reduce some other terms in the expression, so that opportunity sometime could be lost.

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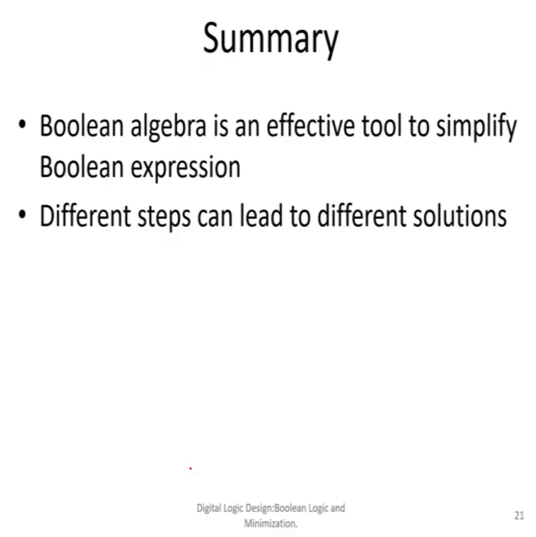


So, although this algebraic simplification, all algebraic simplifications are very effective method to simply any expression. But the major challenge is that different steps can lead to different solution sometime. And we do not know the most important thing is we do know whether the expression which we are trying to we have finally achieved whether this is the most simplified form or there is something which is even more simplified which is possible.

So, that is why we need to study more systematic method which will use possibly the same Boolean algebra laws, Boolean algebra theorems, but it is more systematic so that we can guarantee that this is the solution which can be achieved and this is the minimum solution. So there is some sort of guarantee. So, in the end we can conclude that a Boolean algebra, whatever theorem we have learned in Boolean algebra, so those theorems are very effective mean of simplifying Boolean expressions.

Although you have seen that the process may not be very effective sometime because number of steps may lead to different results, although it is important to learn that Boolean algebra because even if we come up with a systematic approach, the underlined principle used would be from the Boolean algebra. So, if we understand those principles of Boolean algebra, they can effectively use to come up with the systematic approach of simplification or optimization of Boolean expressions, Boolean functions. So, we will see these optimized method or systematic method in our next lecture. Thank you very much.