Introduction to Time-Varying Electrical Networks
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Lecture 9
Inter- Reciprocity in networks with Operational Amplifier

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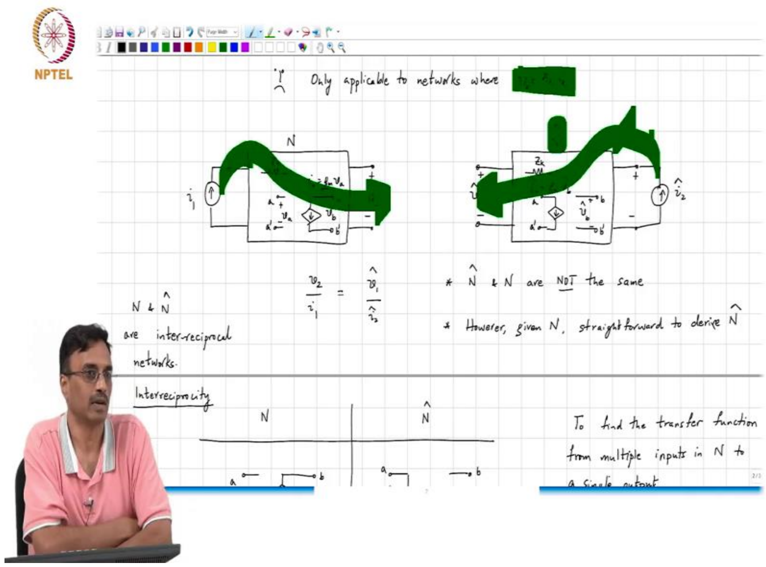

Professor: Yes.

Student: (())(00:25).

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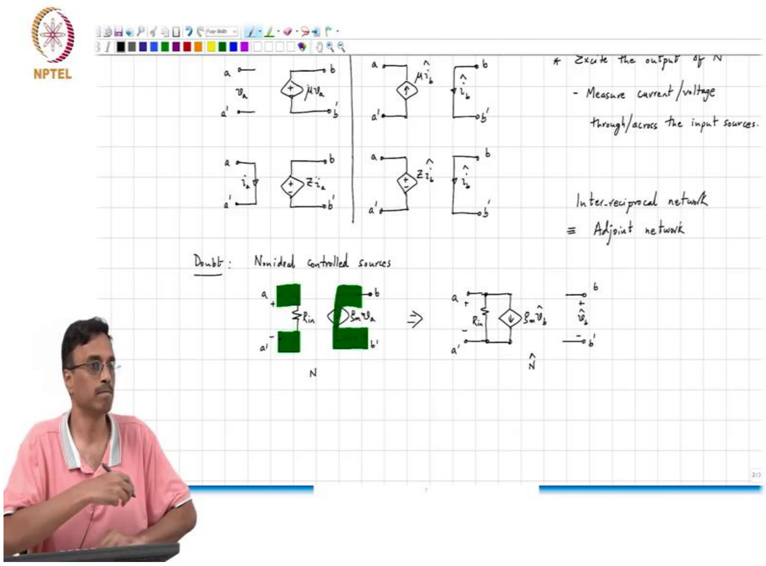

Well, so a question that was asked is. So, my control source is I know I mean let us say the controlled source is not ideal. So, let us say I have voltage controlled current source like this. So, this is an N, so let us call this R in, this is a, this is a prime, b b prime, gm v a.

So, any comments on what I should do in inter reciprocal network? Very good. Remember this is still an ideal controlled source. So, only you need to flip the controlled source and leave all the other non-idealities. I mean a non ideal controlled source is simply an ideal controlled source with some resistors you know some here and there.

So, all that you need to do is to flip. So, this will be R in, this will be gm times v b hat and this will be v b. So the inter reciprocal network, therefore is a way of exploiting reciprocity that works in all practical situations and how many of you use SPICE before or some form of SPICE. Big portion.

So, how many of you are familiar with noise analysis before? Do not be shy. I mean its ok if you have done it before. So, that when you, if you not done you will get to learn in this course. But those of you familiar with noise analysis you know that you can click or you can find the contributions from individual noise sources to the output.

That and tells the finding the transfer functions from individual noise sources to the output and that is all done it is not, it's not any more expensive to do that. It is just simply found using the inter reciprocal network and when we go little further, I will show you that or we will see together that when we write down matrices for, I mean circuits are all as you know on a computer (())(4:00) using matrix technique. It just turned out to be trivial transformation of that matrix of the circuit matrices and therefore, this is, this is routinely used in SPICE every time you run a SPICE simulation, noise analysis is used.

Some of you may also have been be aware of a less known feature of SPICE namely transfer function analysis. How many of you are aware of the AC analysis? Everybody is aware. What does the AC analysis do? You have a how many inputs and how many outputs? You have 1 input and you have you can click on any node and find the frequency response from that one input to output.

The transfer function analysis is a, is the same thing done upside down. You have you choose the output port and you can click on any independent source inside the network and it will show you the frequency response from that independent source to the single output and that is done again using the inter reciprocal network.

And there is also another name for the, inter reciprocal network, inter reciprocal network and that is that it is also called adjoint network and there is a good reason for this as again we will see few lectures down the line. At this point these are just two terms that you need to be familiar. Very good. So, let me you know you have done enough derivation, let us kind of see this as in action and small example, I mean and slight extension of this as you know and you will be immediately able to see a point.

For example, I mean, a when you do an analog circuit course, what is the other element that you keep seeing all the time? You have the four control sources you know, you know say resistors, capacitors, inductors there is something we spent the whole course deriving the ideal op amp.

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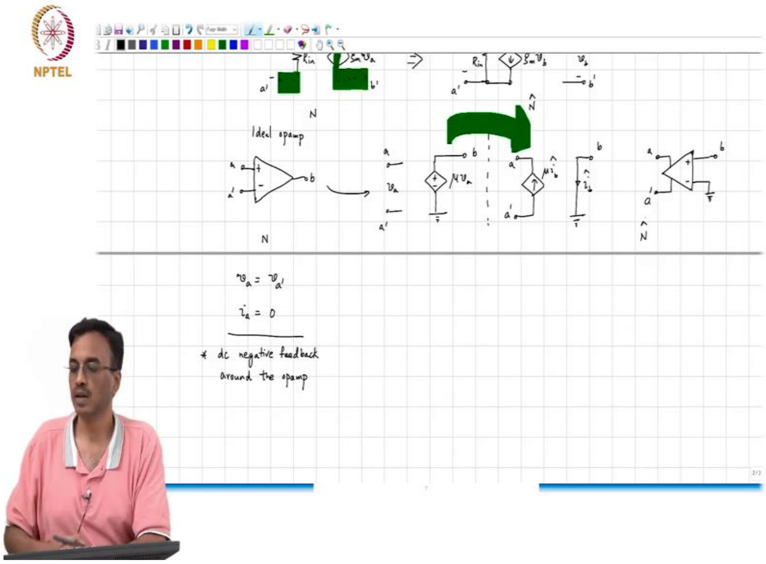

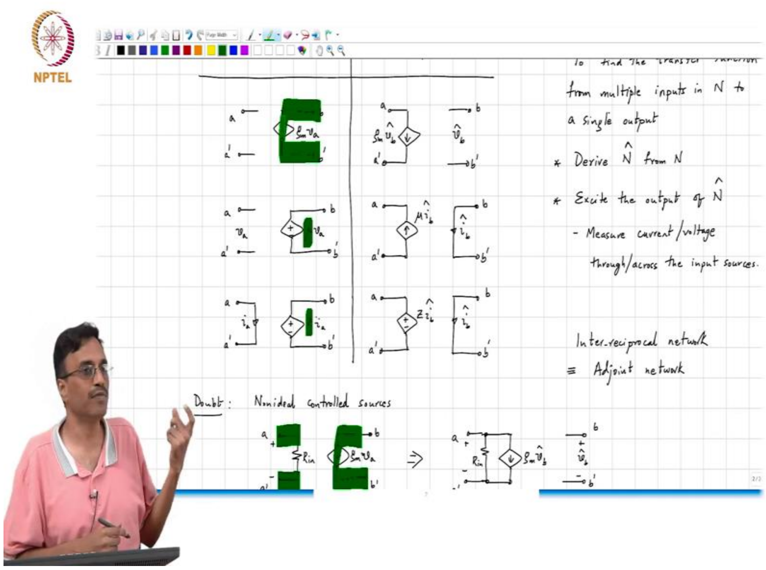

So, in N let us say you have an ideal op amp and if you have an ideal op amp what comment can we make about the properties of an ideal op amp? These are all this of course but when you embed the ideal op amp inside a circuit what comment can we make about the terminal voltages and current?

Student: (())(7:23)

Professor: So, basically yes anybody else? If you have an ideal op amp embedded inside a circuit what you do to analyze when you analyze the circuit?

Student: (())(7:36)

Professor: So, what constraints does the ideal op amp put? v a must be equal to v a prime and i a equal to 0 and what comment can we make about v b? Well, v b is dictated by the circuit and under what conditions are these valid? The most important thing to realize is that this is negative feedback or other dc negative feedback around the op amp. So, in N if you had an opamp ideal opamp what comment can you make about what you should do in the adjoint network? I mean what kind of I mean, I mean what kind of controlled source can you think of an op amp as?

Student: (())(9:18)

Professor: Actually, it is not necessary to think of it as a voltage control voltage source with infinite gain. You can think of it as a voltage control current source with an infinite trans conductance or a current control voltage source with infinite trans impedance or a current control current source with infinite gain. As the controlling parameter tends to infinity. I mean what is the intuition behind any of those assumptions?

Remember, if you have a controlled source which is embedded inside a circuit and a circuit is working all voltages and currents must be, must be finite. So, if this controlling parameter happens to be infinite and the voltage would be b and b prime is finite and mu is infinite what comment can you make about v a? Should be 0. Likewise, if the controlling parameter z is infinite and the current passing through the branch, I mean the voltage between b and b prime is finite, what comment can you make about a and a prime the voltage between a and a prime?

The voltage between a and a prime in the current control voltage source is anyway 0. What comment can you make about the current? Is also 0 and likewise, with the voltage control current source, if this current is finite and gm is infinite, it is easy to see that. Well, b the difference between a and a prime is, the voltage difference between a and a prime is 0. So, all that I would like to point out is that it is necessary for us to think of an ideal op amp as simply voltage control voltage source with, with infinite gain. You think of, you can think of it as any one of those four control sources with that controlling parameter tending to infinity.

Now, with that reasoning what comment can you make about the, the ideal op amp? If you want to replace the ideal op amp in the original network consisted of an ideal op amp? What could we replace it with in the adjoint?

Student: (())(11:48)

Professor: Ideal op amp with what distinction? Remember, an ideal op amp is slightly different from an ideal control source. In the sense that the controlling ports, terminals are both available. So, this is v a and let us say if I assume this to be a voltage controlled voltage source this is mu times v a. But, what comment can we make about the ideal op amp? Well from the picture you see that it is only b which is accessible. What is happening to a? I mean what is happening to b prime?

Student: (())(12:34)

Professor: b prime is internal to the opamp and is therefore not accessible. It is a ground but is a internal to the op amp but not accessible. So, this is b. Now, if you want to form the adjoint if you had something like this what would it, I mean let us do it step by step. What comment can you make if you make this. The adjoint network how would this look like? It's a current control current source with controlling port must have one terminal grounded. So, this is therefore i b hat. Let me just change the signs to be and what must be the control port?

There is a voltage controlled voltage source to the original network, so what did we say for the current controlled current source what must be the sense of the current? Going up or coming down? Going up. So, this is a, a prime and this is mu times i b hat and therefore, now we say well this is an ideal op amp, so what must mu tend to? Mu must tend to infinity. So, mu must tend to infinity. So, even though the original circuit had the original op amp had both the input terminals accessible but one output terminal not accessible. So in the inter reciprocal network what else will, what you see?

Student: (())(14:53)

Professor: Well, both the output terminals are accessible but only one input terminal is accessible. So this a and this is a prime and this as well. So, let us take an example, so this is N hat. Let us take a simple example to illustrate the point.

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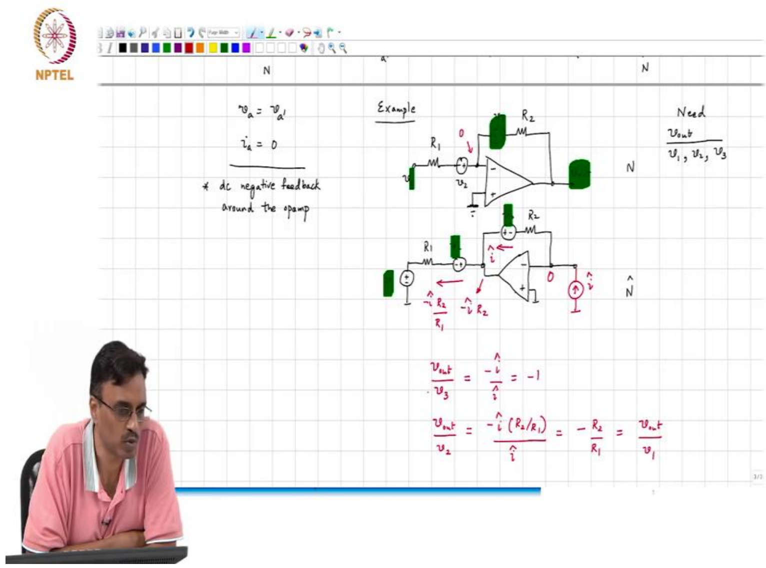

You may take a, in fact a trivial example that all of you are very familiar with let us call this. So, what is the problem? What would you like to do? Find transfer functions from way out to, I mean from v1, v2, v3 to v out. I mean this is a pretty straight forward example you do not really need the adjoint concept to go and do this.

But just as I said to illustrate the point. In reality you have much more complicated networks and therefore the adjoint is used a de facto. So, help me this is N. So, the first step is to draw N hat, so what are we want to replace the op amp by. In this network it is just incidentally turns out that one port of the, I mean the input port of the op amp is also one terminal grounded.

The output terminal is, the output port anyways has only one terminal accessible. Now, what you do? So, we draw the adjoint, so we have v1 which is 0. So, this is a voltage source. This is R1, this is v2, this is v3, this is R2. The only control source is the op amp. What are we supposed to do now? Yes, Alfred.

Very good. As Alfred, suggested we are going to do this and this is the output. So, first part of the job is done. We have found the adjoint network or the inter reciprocal network. What is the next thing to do? We need to find the voltage transfer functions. So, we need v out by v1, v2, v3. So, what you suggest now that we do? We exploit reciprocity. What you do? The voltage transfer functions in the original network now become? Voltage transfer function in the original network become?

Student: (())(19:44)

Professor: Current transfer function in the in the reciprocal or inter reciprocal network. So, what you suggest that I do? Excited with a current source i or let me call it as i hat. So, now it is a standard circuit analysis where does that i flow? What is this potential by the way? It is a virtual ground of the op amp that is 0. So, where does i hat flow? So, this is i hat and what is this voltage? Come on people.

Student: (())(20:41)

Professor: Minus, minus i hat R2, correct and therefore, what comment can you make about this current? Yes, what is that current, the voltage I mean can somebody tell me what that current is? Remember, all these voltage sources are I mean these are just names they are all 0 voltage, voltage sources.

Because we need to find the current through them. So, what is the current through R1? Minus i hat R2 and therefore, what comment can you make about v3 by, so let us what comment can you make about v out by v3 in the original network? It is simply the same as. Please think carefully and be careful about the signs. SIGNS. What is direction of the current through the voltage source v3?

So, v out by v3 in the original network is what in the inter reciprocal network? What do we need to do? What is it?

Student: (())(22:40)

Professor: I am not looking for the answer, what is the input and what is the output we are looking at?

Student: (())(22:50)

Professor: The current into the positive terminal of v3 is our output quantity. So, that is the current flowing into the positive terminal of v3 is? Minus i hat. So, that is basically minus i hat divided by i hat which therefore is minus 1 and likewise, what comment can you make about v out by v2 in the original network? What is the output way looking at?

We need to find the current flowing through the positive terminal of v2 in the inter reciprocal network And that happens to be minus i hat times R2 by R1 that divided by. This must be minus R2 by R1 and is evident that the current flowing through v2 and the current flowing through v1 are the same.

And therefore, this must also be equal to v out by v. So, in one shot I mean we found all the three transfer functions. But if we did not know this adjoint concept what would we probably have done is assume these two do not exist and what is the transfer function from v1 to v out?

Minus R2 by R1 and then if you open your eyes carefully and see then you would see that transfer function v1 and v2 are in series and therefore there is no point in doing analysis all over again. It is the transfer function from v1 to v out is the same as the transfer function from v2 to v out and you will say that is also minus R2 by R1.

From v3, what is the transfer function from v3 to v out in the original network? Well, you assume v1 and v2 to be 0. This potential the virtual ground potential will be well, you would say this is 0 and this would be, this would be?

Student: (())(25:57)

Professor: v out would be minus v and that is you know fortunately also what is predicted by the adjoint. So, to summarize therefore the concept of inter reciprocity is basically a profoundly important one and is used all the time in the routinely used in the circuit simulators. How many of you are aware of this before? I mean I can understand that undergrad is not seeing this but our, so as you can imagine even experience students here have not seen this.

And it’s also quite strangely also not known to most circuit designer that, that you can use reciprocity with a small twist and it is not some exotic thing you know that is only there in the textbooks. It is actually there routinely when you do SPICE on your circuits this is what is being used.

In fact, you will see going forward that is actually I was mentioning its its almost a trivial manipulation of the matrices that that allows us to do, to do this and I will see going forward later in this course when we start talking about time varying circuit. You will find that same concept is also used to derive the whole bunch of interesting things.