Introduction to Time – Varying Electrical Networks
Professor. Shanthi Pavan
Department of Electrical Engineering
Indian Institute of Technology, Madras
Lecture No. 78
Total integrated noise in networks with R, L, C and periodically operated switches

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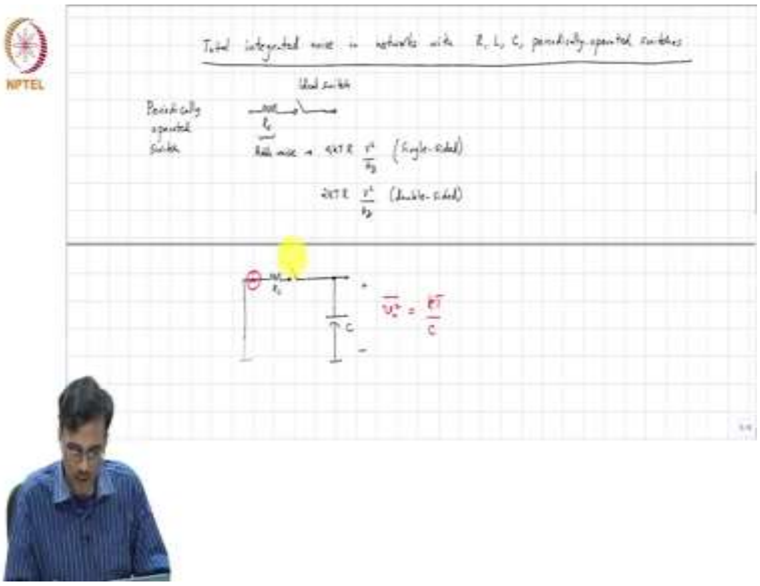

The next thing that I would like to draw your attention to is noise in a special class of LPTV networks. So, when we were doing, so this is total integrated noise in networks with R, L, C and periodically operated switches. Earlier, we have seen the total integrated noise in networks with only R, L, and C, now we also add periodically operated switches. So, remember that every periodically operated switch in practice basically can be thought of is having some series resistance and, ideal switch. And this rs adds noise, and its noise spectral density is 2 ktr volts square this is 4 ktr volt square per hertz. This is single-sided or 2 ktr volt square per hertz double-sided.

So, for example, here is an example circuit. So, let us say we have an ideal I mean a switch. This is the resistance of the switch, and this is a capacitor. The question is, what is the mean square? I mean clearly when you close the switch the there is an, I mean the resistor has got some noise. So, when the switch is closed, well this noise gets, goes through the network and is there on the capacitor. Some noise voltage exists across the capacitor. Then you suddently open the switch, and whatever noise there is on the capacitor remains there.

So, the therefore, there must be some means square you should be able to evaluate the mean square value of that noise. The average of the noise will be 0, so the mean square noise must be some value. And we would like to find out what that means square value is. We have already seen this earlier in context of I mean, sample and hold. And this basically what was the result? Very good. So, Vn square, the mean square noise across the capacitor is nothing but KT over C. Now, the question is if you have a more complicated network, so let us say this is some phi 1.

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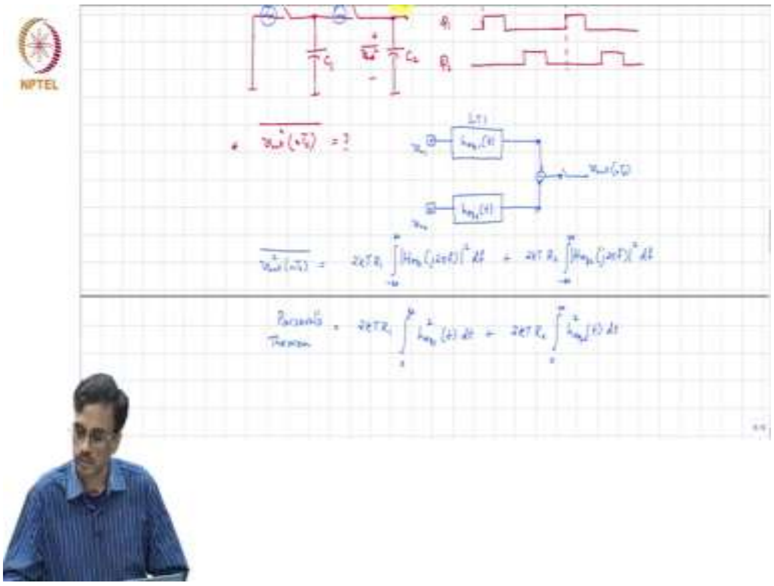

Now, the question is let us say, you have a switch. You have a capacitor, C1, you have another switch. You have another capacitor C2. Let us say, this is clocked by Phi 1 this is clocked by Phi 2. Phi 1 is something like this and Phi 2 is something like this. And they are periodically switched. So, this is say this is TS. And the question we would like to ask is what is for instance, what is the mean squared noise across C2?

So, there are several ways of doing this. And remember that we are only interested in the total mean square noise across the capacitor. Now, the question is, if we are only interested in the total mean square noise it does not seem to make sense to determine all the transfer functions, which are basically functions of frequency, and basically integrate the all of them to get the total mean square noise. And therefore work hard to get a lot of information, and then throwing away most of it, because you are only interested in the mean square noise.

This is exactly similar to the body noise theorem for time invariant networks. Now, it turns out that, for networks with R, L, C and periodically operated switches, we can use the same approach to simplify the noise. So, that is what we are going to be doing next. So, let us try and exploit what we know so far. Namely, that, let us try and find, I will continue with this example. Let us try and find let us say this v out. Let us try and find the mean square noise of this network, at, say, some n times ts. In other words, I am interested in the.

Student: Mean square noise.

Professor: Mean square noise at the output of the network.

Student: Time.

Professor: But this is the, the key point is that I have converted the mean square noise problem into something where I am only interested in the mean square of the sampled output of the LPTV network. So, in other words, therefore, you can basically use all our results on reciprocity, as well as the equivalent LTI filter. So, what are the noise sources?

Student: Vn 1.

Professor: Vn 1 and Vn 2. So, therefore, we have Vn 1, it goes through h equivalent 1 of t which is an LTI filter with an impulse response which is h equivalent 1 of t. Vn 2 likewise goes through h equivalent 2 of t. And then, you add these two and you sample the output and that will give you vout of nTs. You sample it at n times ts, you will get v out of nTs. And therefore, what is the mean square noise of nTs?

Student: Integrate it.

Professor: You simply integrate. It is basically the noise spectral density. Let us assume that Vn 1 this is a switch. So this is nothing but to 2 kt r1 integral minus infinity to infinity h equivalent 1 of j 2 pi f whole square df, plus 2 kt r2 integral minus infinity to infinity h equivalent 2 of j 2 pi f the whole square df.

Now by Perceval’s theorem. So, it does not matter whether you integrate in the time domain or the frequency domain, so this becomes the same as integral 0 to infinity h equivalent and 1 square of t dt plus 2 kt r2 integral 0 to infinity of h equivalent square 2 dt.

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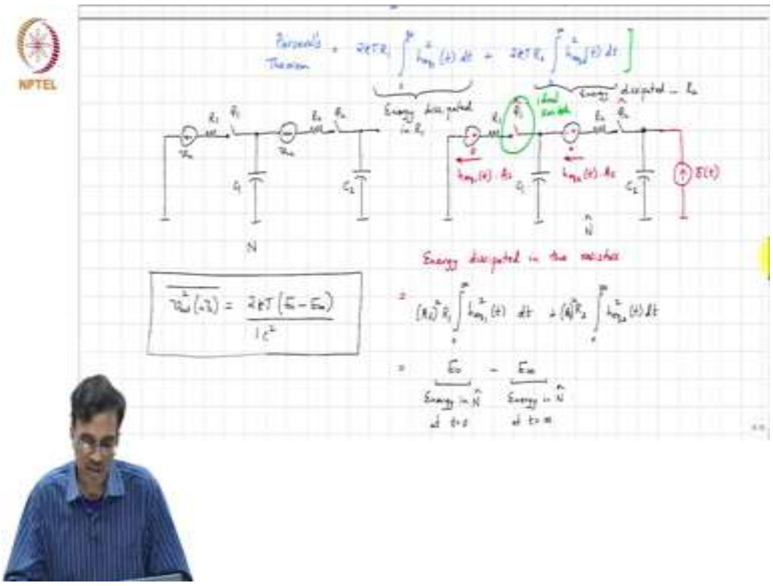

Now, how do you find the h equivalent 1 and h equivalent 2? We know how to do that already. So, remember this is our network. So, this is r1 this is Vn 1, which corresponds to the resistance of the first switch. This this is capacitance C1, Vn 2, r2, this is phi 2. And well to find h equivalent 1 and h equivalent 2 what will we do?

Student: We will apply the.

Professor: Our input is a voltage, the output, sampled output voltage we are sampling it at t equal to a timing offset t naught, which is equal to 0. So, what do we do we need to draw the adjoint network. So, this phi 1 hat and phi 2 hat, we remove these noise sources. These are all now 0 volt sources. What do we do?

Student: Apply impulse.

Professor: Apply a current impulse at t equal to 0 so, this is delta of t. And therefore, the current that flows through these voltage sources will have a shape that is similar to h equivalent of. But this , I remember that this is a, whatever flows in these sources is a current but to get h equivalent of t you have to divide that current by 1 coulomb correct we use 1 ampere second so that you will get the h equivalent 1 of t.

So, similarly, this is h equivalent 2 of t times. Remember, this is a current so this must be the current that you see will be this times ampere second. Does it make sense? And the reason is that this is a current, and it will have the same shape as the impulse response. Now, what comment can you make about the, if you so the energy dissipated in the resistors is what?

Student: Voltage across the ct initial is 1 by ct.

Professor: No, no, that is okay. But in terms of the currents flowing through them, what what comment can we make about the currents, about the energy being dissipated in the resistors?

Student: H equivalent square.

Professor: It is simply h equivalent 1 square t dt times r1 plus r2. And this integral has to go from 0 to infinity this is nothing but integral 0 to infinity h equivalent 2 square of t dt. And this must be equal to.

Student: 1 by c2.

Professor: Well, the impulse that you apply across the output port, will inject some energy E0 into the network at t equal to 0. So, this is nothing but energy in. So, if this is the network n, this is the network n hat, n hat at t equal to 0. So, then, as the network keeps running well there something happens to the energy, at t equals infinity there must be, there must be.

Student: Some energy.

Professor: There must be some energy in the network at. Now, all this remember applies to the adjoint network at t equal to infinity. So, this the difference between the energy at t equals 0 and t equal to infinity must be exactly the.

Student: Only dissipated in the resistors.

Professor: Must only be dissipated in the resistors, it cannot be dissipated anywhere else because the capacitor is lossless. If you had an inductor that would be last-less, this is an ideal switch. So, the switches are lossless. So, the only place where all this energy can be dissipated is the resistor.

So, therefore, we have seen therefore, that e infinity minus E0 minus E infinity must be this. But what are we trying to find? What we are trying to find is this quantity here, which is simply 2 kt. I mean, you can think of this as being proportional to.

Student: Energy dissipated.

Professor: Energy the energy dissipated in.

Student: Resistor r1.

Professor: In r1 and this is energy dissipated in r2. So, what do you say, I mean what do you what therefore can we say about the mean square noise?

Student: You can apply.

Professor: Mean square nice v out is nothing but 2 kt times E0 minus e infinity by 1 coulomb square. Why is that? So, this is actually.

Student: We are using the current.

Professor: Coulomb square and this must also be multiplied by actually I would call this ampere second. This is also ampere second.

So, the mean square noise of the output sequence sampled at n times ts is simply nothing but E0 minus e infinity by 1 column square. This is exactly the same form that we got for the time invariant network. So, whether it is time invariant or periodically time varying we get the same result.

Now, this is I mean remember, this v out of the mean square value of the output sequence sampled at integer multiples of ts, and this must be stationary, because it is not varying from it is the output you can think of it as the output of a linear time invariant filter. And therefore, this is basically not changing from sample to sample.

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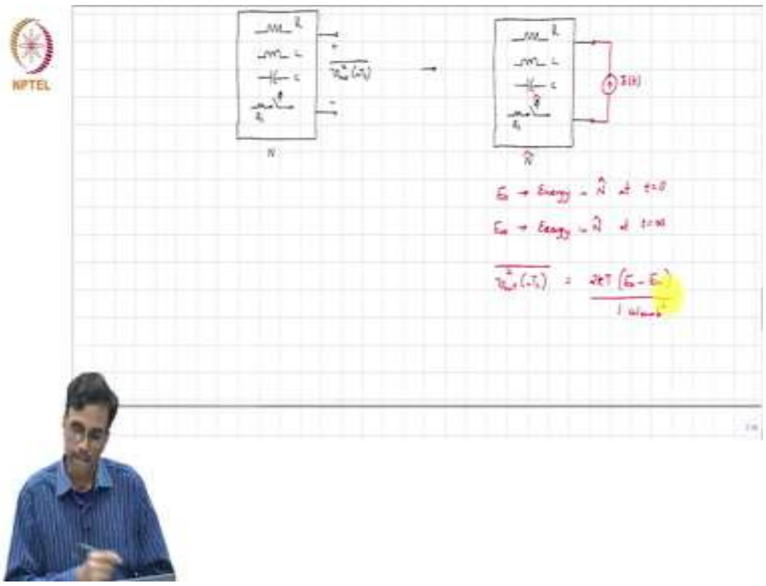

Now, what comment can we make if we wanted? So, to summarize therefore, if you have a network with R, L, C and periodically operated switches, and you want to find the mean square noise of the sequence, so what do you do? You first form the adjoint network. So, to do that, what do you do?

Well, you form the adjoint. So, your time reverse the timing of the switches, that's the only thing that you need to do, then, you inject an impulse current here E0 is the energy in n hat at t equal to 0, e infinity is the energy in n hat at t equal to infinity and mean square value of nTs is simply 2 kt times E0 minus e infinity by 1 column square.

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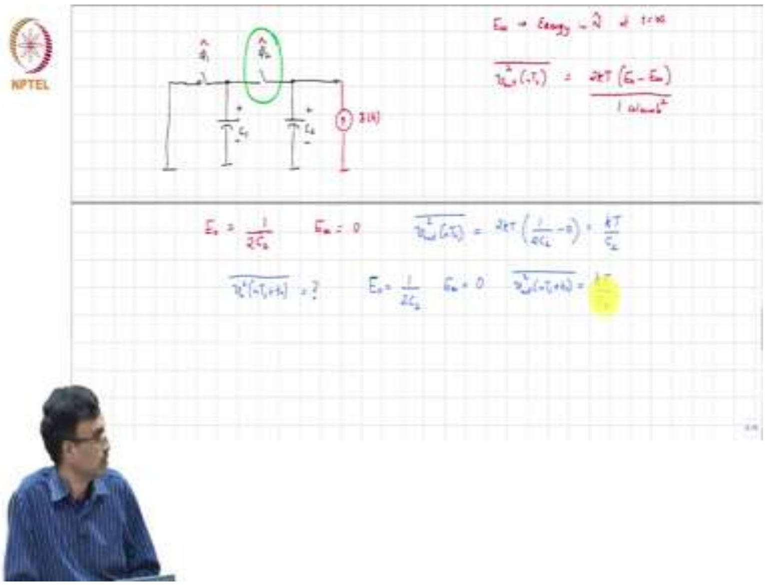

Now, let us do some examples. So, I am showing switches here without the resistance, but every switch is associated with the resistance. This is C2, this is phi 1, this is phi 2 and we are interested in the mean square noise here. So, what do we form the adjoint, you apply a current at t equal to 0. So, what is the E0?

Student: E0 is 1 by 2C.

Professor: 1 by. So, if you inject 1 coulomb charge 1 by C2 is the voltage developed. So, this basically becomes 1 by 2 C2 Q square by 2C. And E infinity what is the E infinity? Well, there is a voltage developed across C2 when phi 2 hat is high the charge is shared between C2 and C1 then C1 is connected to ground.

Student: So, it will discharge.

Professor: So, C1 will discharge. So, as you can see eventually charge is getting.

Student: Discharge from C2.

Professor: From C2 and also C1. So, eventually both the capacitors will have.

Student: 0.

Professor: 0 charge and therefore 0 energy, so infinity must therefore be 0. So, the mean square value nTs is nothing but 2 kt times 1 over 2 C2 minus 0 which is nothing but kt over C2. Now, what comment can we make? If for instance, if we wanted V naught square of mean square value of nTs plus t naught what would we do?

Student: We have to apply the impulse at the t naught we have to apply.

Professor: No.

Student: T. T plus t, t minus t naught.

Professor: What is t?

Student: Delta of t plus t.

Professor: We have to apply delta of t plus t naught. So, you have to apply the impulse at what time?

Student: At minus t naught.

Professor: At minus t naught. So, in this if you apply the impulse at minus at minus t naught what is going to change?

Student: Initial, like we have to find the initial charge.

Professor: Yeah, the initial charge. So, what happens? The only thing that changes is that I mean, this switch, which is basically is either an open switch or closed switch with resistance rs. In either case, what comment can we make about the voltage developed across C2?

Student: That is 1 by C2.

Professor: It is always going to be?

Student: 1 by C2.

Professor: 1 by C2, 1 by 2 C2. I mean, sorry the voltage developed across C2 is going to be 1 by C2. So, the initial energy is going to be E0 is still going to be 1 over 2 C2. E infinity is going to be 0. So, the mean square noise is going to be kt over C2. So, regardless of when you sample the output, the mean square noise is always going to be kt over C2.

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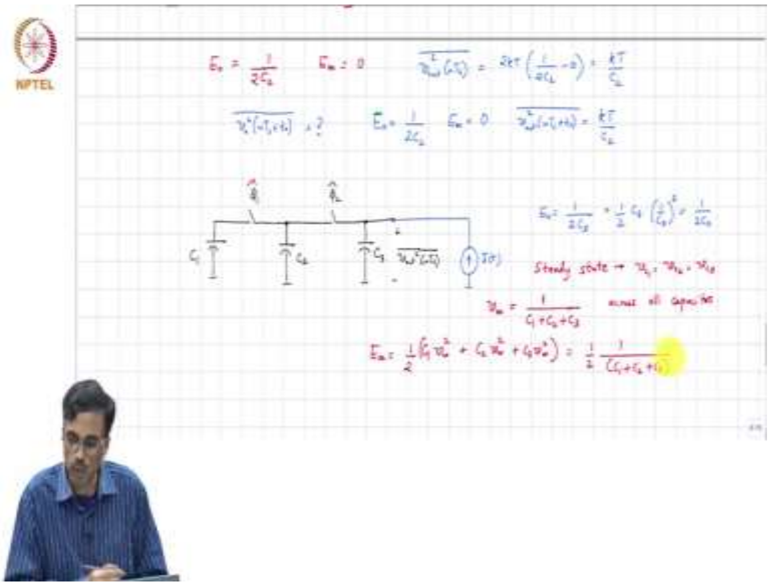

So, let me take one last example, where e infinity is not 0. Let us call this C1, this is C2 this is C3. So, what is the mean square noise here let us say of nTs. What will we do? We will form the adjoint then, so that basically is phi 1 hat, phi 2 hat then apply an impulse here delta of t. What is E0?

Student: 1 by 2 C2.

Professor: So, when you inject an impulse the voltage across C3 is 1 or C3 so, E0 is nothing but 1 over 2 C3, which is nothing but half C3 times 1 over C3 the whole square. Now, once the network started, I mean you let the whole thing operate what happens at t equal to infinity?

Student: Voltage at all the capacitors should be equally charged instead.

Professor: So, in steady state, the voltage across all the capacitors is identical. So, we see one equals VC 1 equals VC 2 equals VC 3. And what is that voltage?

Student: That initial voltage?

Professor: Initial. See there is nowhere for the charge to go. So, whatever charge you put in initially, which is 1 column will be distributed across.

Student: Across 3 capacitors.

Professor: Across 3 capacitors, and all the three capacitors have the same voltage, so the voltage will be 1 divided by C1 plus C2 plus C3 across all the capacitors. So, V infinity across all capacitors. So, what comment can we make about E infinity therefore?

Student: 1 over 2.

Professor: Half into C1 times V infinity the whole square plus C2 V infinity the whole square plus C3, V infinity the whole square, which is nothing but one half into C1 plus C2 plus C3 times V infinity the whole square so this is nothing, but 1 over 2 times C1 plus C2 plus C3.

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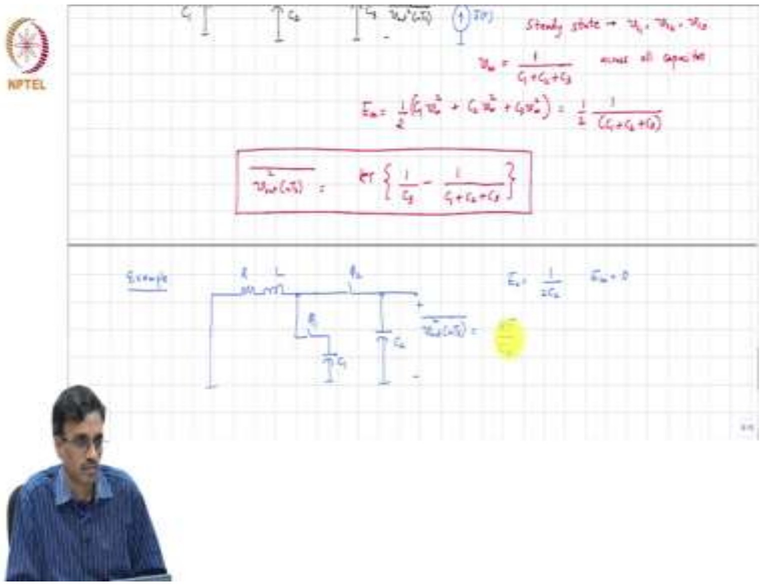

So, the mean square output noise is nothing, but it is kt times 1 over C3 minus 1 over C1 plus C2 plus C3. So, therefore, thanks to this observation and energy conservation. Just like how we saw in the case of linear time invariant networks, the process of evaluating the total mean square noise can be done actually by simply looking at the network without calculating any transfer function at all.

And, as you keep I mean, here is one final example. So, let us say so, this is phi 1, this is phi 2, this is R this is L, this is C1 and this is C2. And we are interested in the mean square noise here. What would this be by inspection?

Student: Kt by C2.

Professor: It will simply be kt over. So, E0 is nothing but 1 over 2 C2, e infinity is 0 and therefore, means square noise is simply kt over C2. And does not depend on L or C1 or R or anything else. So, with that I will stop.