Introduction to Time – Varying Electrical Networks
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Lecture No. 76
Introduction to noise in LPTV Networks

Good evening, and welcome to advance electrical networks. This is lecture 47. So, so far in this course, we have seen how to analyze LPTV systems. We found how to analyze systems where the output of the LPTV system is sampled and the sampling rate happens to be the same as the rate at which the system is varying. We also saw the corollary last time where we showed that if you drive an IPTV system with a modulated sequence, it you can find you can think of the output as being the result of driving the same modulated sequence through a linear time invariant filter.

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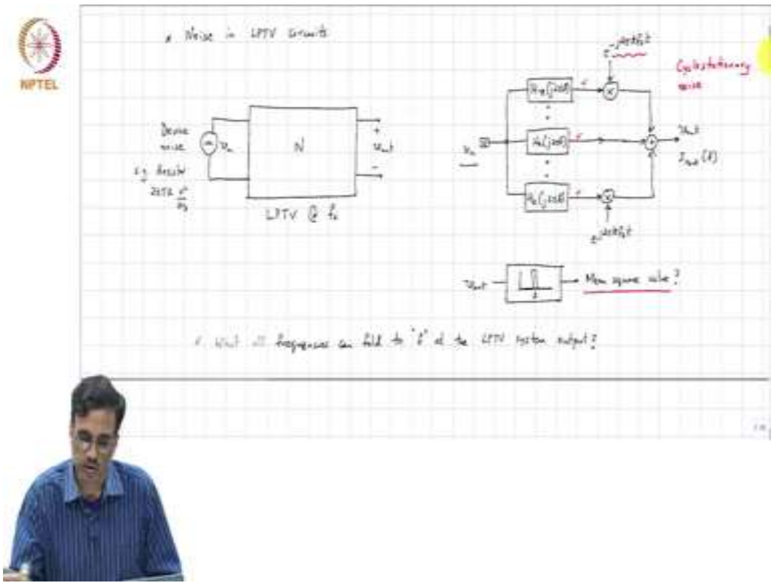

Today, we will discuss noise in every LPTV circuits. So, and the premises is the same so we have an LPTV system N which is LPTV at fs. And most of the time, we are dealing with noise sources, which are usually which are device noise sources. And these noise sources can be modeled by some noise spectral density. If this is a resistor, for example, the two sided spectral density is 2 ktr volt square per hertz.

If its transistor noise, it is 2 kt times GM time some correction factor, two-thirds typically or maybe more, depending on the device construction. So, the bottom line is that you can think of device noise is being, as being some spectral density that is known from the physics of either the device or the component. And in this case I am just assuming that this is an input noise voltage, and the output of the system is some v out.

And just like how we did with the time invariant case, we also can do this with the time varying case. And the idea is the following. So, if you draw the block diagram, you have Vin, and then it is going through h0 of j 2 pi f, h sub k of j 2 pi f and this gets multiplied by e to the j 2 pi k fs times t, this is h sub minus k of j 2 pi f, e to the minus j 2 pi k fs times t. We add all these things. This is v out.

The question is what is the noise power at a frequency f here? So, we are interested in S v out of. So, in other words, what we are interested in is the following. We basically take v out, pass it through a narrowband filter centered at some frequency f and we measure the mean square value. And the only, the difference between, the two differences between what happens in a time invariant circuit and a time variant circuit.

In a time invariant circuit, well, if you have an output noise component at a frequency f that can only come from an input frequency and a noise frequency which is at f. Now, we have to worry about not merely an input noise at or the power of the input noise at f.

Student: Plus.

Professor: Very good. So, you also have to worry about noise at multiples of f plus minus k times fs. And then the other thing is now, because you have to worry about multiple, noise at multiple input frequencies, we obviously have to worry about the transfer functions from those frequencies to the output. So, that is one aspect.

The other thing is that when we are talking about a time invariant network, the noise at the output of an LTI system driven. See, if you look at Vn, it turns out that device noise or resistor noise for instance, if you look at the power in a narrow frequency band, whether you measure it now or whether measure at a time t later, it turns out that the property the statistical properties of the noise do not change. So, such a noise is called stationary.

Now, if you take a stationary random process and process it through a linear time invariant system well, the gain of the time invariant system does not change with time. So, the output noise is also stationary, that basically means its properties do not change with time. So, if we have any stationary, if you look at this block diagram, then clearly all the noise at all these points is all stationary.

However, when you take this noise and multiply it by a sine wave, then the noise no longer becomes stationary, it only, I mean, because at some point in time, let us say the cosine is 0, then how much ever noise you have at the at the input, the output is 0. So, in other words, the mean and the variance and other statistical properties of the noise will depend on time, because you are multiplying it by a you are taking a sequence.

I mean a noise process which is stationary is not varying with time and you are passing it through an amplifier or a gain, which is changing with time. So, it is only natural that the output also has statistical properties that change with time. So, and it turns out. And because our gain is varying periodically, the statistical properties of the noise of the output also change periodically, and such processes random processes are called cyclostationary noise.

But it so turns out that most of the time when you are making measurements you are looking for power in a very, very narrow band. So, you take this process which is varying rapidly with time and you pass it through a narrowband filter. And what is the job of a narrowband filter? A narrowband filter is say simply averages out the variations and you will get something which is a constant. So, what do you measure the mean square value that you measure at the output is an average of what happens throughout the complete cycle.

And as you can imagine, the process of noise computation is in a LPTV network is a lot more involved than the LTI case simply because even if you had only one source, you have to worry about?

Student: All the source.

Professor: We have to worry about the strength of the noise at all f plus k fs as well as the transfer functions from those frequencies to the output at f. So, the question is what all frequencies can alias. In other words, so, what all frequencies can fall to a certain frequency f at the output of the LPTV system? So, to do that, what would we do, what would you do?

Student: We can use the receiver.

Professor: We can use, if you know it off the top of your head, that is good. Otherwise, we can use the concept of inter-reciprocity.

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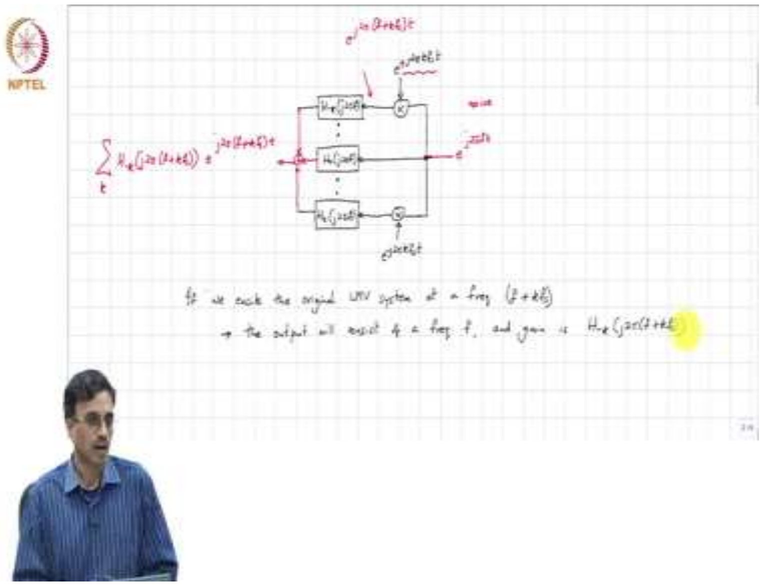

And so, if we did that, well, what will we need to do? We will, we will turn the signal flow graph the other way around the summing node becomes a pickoff point. So, and you have to, the again has to become reverse time, and you excite it with e to the j 2 pi ft, and this becomes a summation. And what do you expect to see here? So, it is the j 2 pi f t, here becomes e to the j 2 pi f plus k fs times t. And what you get here is h sub minus k of.

Student: J 2 pi f plus k fs times e.

Professor: j 2 pi f plus k fs times.

Student: J 2 pi.

Professor: E to the j 2 pi.

Student: F plus k fs.

Professor: F plus k f s times t. And now we have a sum over all case. So, basically what this is telling us is that, if you if you excite, if we excite the original LPTV system at a frequency f plus k times fs what will happen? The output will consist of a frequency f and gain of this path is h sub minus k of j 2 pi f plus k fs. Intuitively this seems you know this is very convincing. If you could f plus k fs it go, the output has to be at f. So, you have to put in an input frequency which is f plus k fs and go through k down conversions to get to the output. So, this is now the gain from the, from an input at f plus k fs to.

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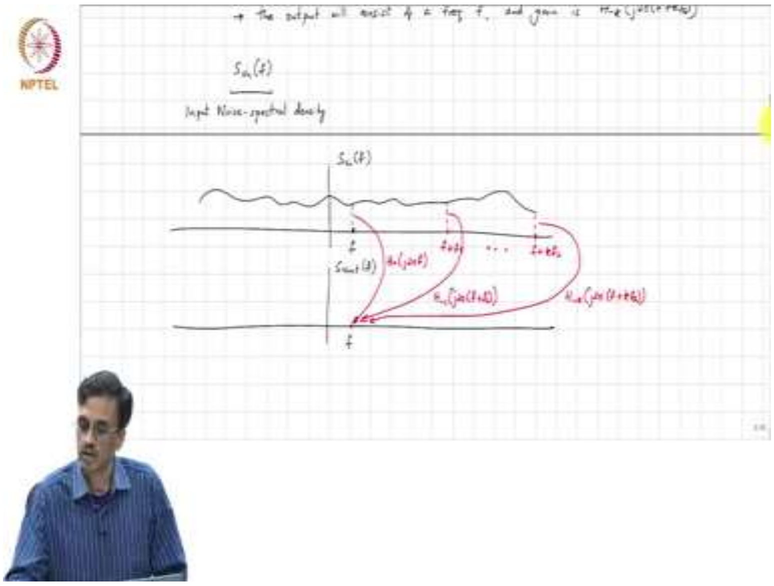

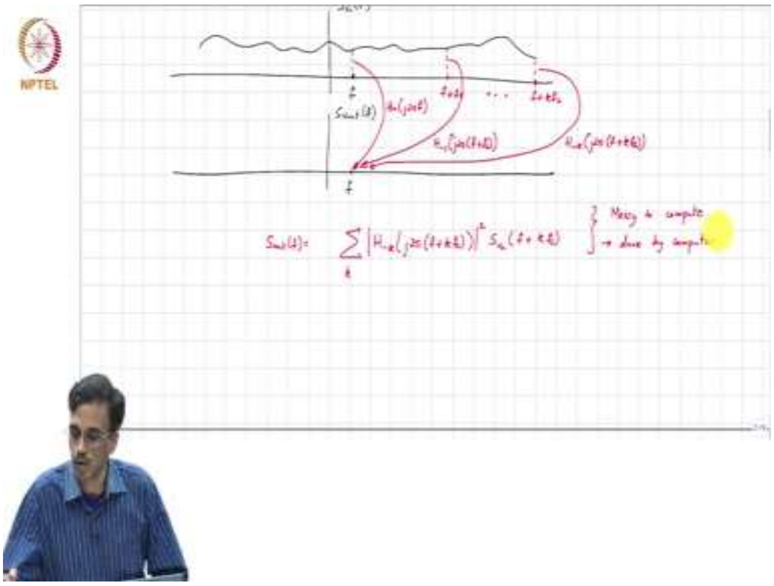

So, let us say the noise spectral density S Vn of is known. So this is the input noise spectral density. What comment can we make about the, the output? So, if we want to find the total noise power at f at the output, what do we think we should do? Well, let us take a look at this. So, let us say, so I assume this is symmetric. So, this is the output frequency. So, this is let us assume, so this is S Vn of f and S V0 of v out of f. So this is f so, we have. So, what is the transfer function from there to there?

Student: Start of j 2 pi f.

Professor: H0 of j 2 pi f. But it is not just this tone that will fold you will have f plus fs, blah, blah, blah, in general, f plus k fs. And this will fold over with again, h sub minus 1 of j 2 pi.

Student: F plus fs.

Professor: F plus fs. And this will fold over with a gain of h sub minus k of j 2 pi f plus k fs. So, what comment can you make about the output power spectrum? If you put a narrowband filter at f and measure the average power that is coming out, you what you would expect to see is S v out of f which is sum overall k h mod h sub k, h sub minus k of j 2 pi f plus k fs mod square times S Vin and f plus k fs.

Alright? And before even for one calculation, I mean for one input noise source, you can see that this is a very messy affair. Fortunately, there are build there are simulators that will do the calculation for you. And a big part of the calculation is finding all these h sub minus ks of all this stuff, and that is easily done using the adjoint network.

Now, so, what do you call messy to compute. And therefore, usually, you resort I mean, hand calculations you forget about it and do it with a computer using a adjoint techniques, to find all these transformations.