Introduction to Time – Varying Electrical Networks
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Lecture No. 74
Finding the equivalent LTI filter of a sampled LPTV system with offset sampling

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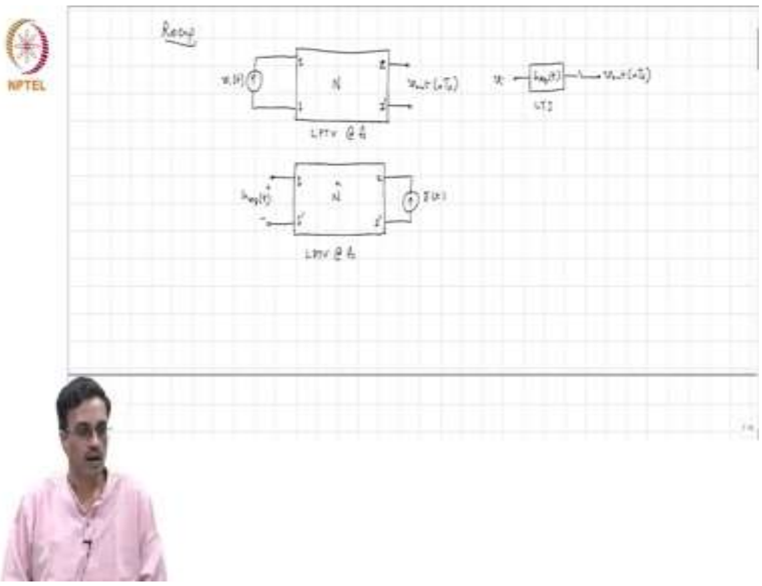

A quick recap what we were doing the last class. So, we did a few examples where we applied the principle that we have been discussing so far. Namely, that if you are interested only in the output samples taken at n times Ts when your drive and LPTV network with an input.

So, we saw that this is equivalent to taking that input vi of t and exciting an equivalent LTI system with vi of t and sampling the output of that at n times Ts. And how to find h equivalent of t? We basically said, we form the adjoint network n hat LPTV hat fs and excite the output port with delta t and the voltage that is developed is h equivalent of t.

So, today, and we saw the example of how one can use this to advantage in a switched Rc network as well as the first order continuous time delta sigma converter. If you have not seen a continuous time delta sigma converter before, then you just think of that system as simply an LPTV system and we saw h equivalent of t.

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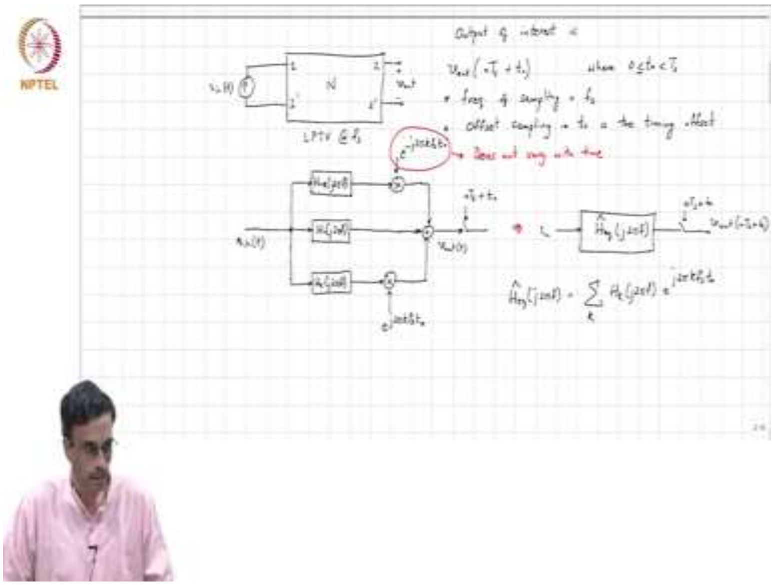

Today we will look at a different situation where for instance, again, concerning sampled LPTV networks. And I would say it is simply an extension of what we have seen already. So, we have a network, again, we have an input current and an output voltage. And we are, let us say we are not interested in v out of nTs, but we are interested in. So, the output of interest is v out of nTs plus some t naught where t naught is greater than equal to 0, less than or equal to, less than Ts.

So, in other words the frequency of sampling is the same as that at which the network is varying. Except that earlier we were interested in sampling.

Student: Sampling at adjoints.

Professor: Sample at multiple. I mean, at 0 Ts and so on. Now, a reasonable question to ask is what happens when you sample with an offset? So, this is called offset sampling and t naught is the timing offset. And as usual, we have seen I mean, so remember that the LTI system or LPTV system is h sub k of j 2 pi f, e to the j 2 pi k fs times t, h sub minus k, j 2 pi f, e to the minus j 2 pi k fs into t.

So, this is let us call this some in of t, this is v out, but we are only interested in, v out of nTs plus t naught. So, in other words, we are sampling this output, this is v out of t and this is being sampled nTs plus t naught. So, as usual as we have done before we will simply move the sampler inside. And so, the moment you sample this you basically will now get j 2 pi k, k fs times t naught.

And likewise, here also you will get this times t naught. And the reason is that when you evaluate this factor at nTs plus t naught, that nTs becomes a integral multiple of 2 pi and this is just what is remaining is just t naught. Correct. So, now, does this depend on time or is this independent of time?

Student: It is a complex number.

Professor: It is a complex number, but this is independent of time. So, does not vary with time, and is therefore it is simply a fixed number, and therefore, you can think of this therefore, as taking in of t passing it through a h equivalent of h equivalent hat of j 2 pi f. I use the hat because it is different from what we obtained when t naught was 0, and the output of this is a sampled at nTs plus t naught. So, you get v out of nTs plus t naught. And what is h equivalent hat of j 2 pi f is simply the sum over all k of h sub k of j 2 pi f, e to the power j 2 pi.

Student: k fs.

Professor: k fs times t naught. Sanity check, if t naught is 0 you should simply get some of the harmonic transfer functions.

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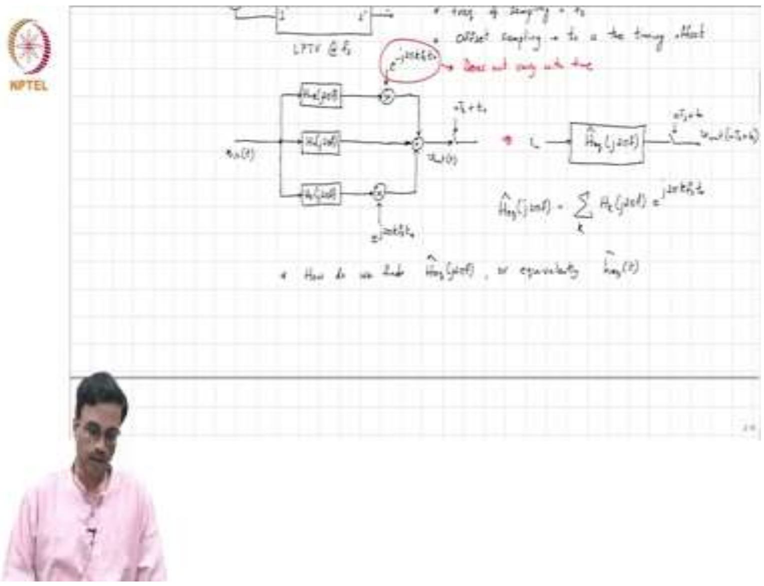

Now, the question is so, as usual the question is, how do we find h equivalent hat of j 2 pi f or equivalently the time domain he equivalent hat of t.

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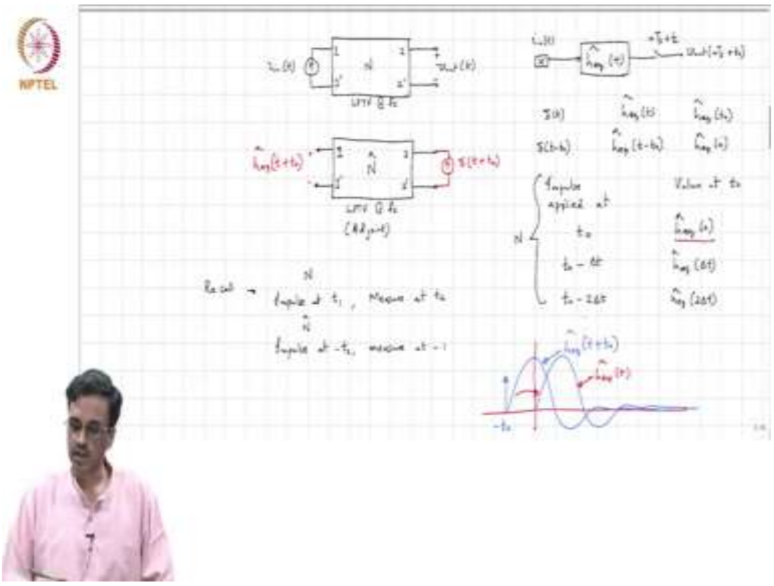

So, our situation is like this. So, we have an LPTV system. Let us say vi of t and this is the network and we have vo of t or let me just use since we are talking about currents and voltages let us do that. So, this is 1, 1 prime 2, 2 prime v out of t. And this is a sampled at.

So, we are interested in finding the samples. And we know that this is exactly equal into finding I mean equivalent to an LTI filter whose impulse response we want to find. So, if you excite if in of t is and so, the impulse response of the equivalent. So, let us say Iin of t is the input, you have some LTI system h equivalent hat of t and the output is sampled at nTs plus t t naught. So, this is v out of nts plus t naught.

So, if I want to find, so if I apply an input delta t the output waveform here will be h equivalent hat of t. So, the sample here will be, the first sample here will be h equivalent hat of t naught. But if I want h equivalent of h equivalent to hat of 0 what should I put in? At what time must excite the system?

Student: That is t plus t naught.

Professor: You cannot have t plus t not in the.

Student: Before we have done.

Professor: No, no, no. At what time should I apply the impulse so that I get h. When I sample the output at t naught I get h equivalent of 0.

Student: Delta of t minus t naught.

Professor: That is so if I apply a delta of t minus t naught. In other words, I must apply the impulse at?

Student: After t naught.

Professor: At t naught. Correct. So, if I apply the impulse at t naught then what will I get here? I will get h equivalent hat of.

Student: T minus t naught.

Professor: T minus t naught. So, if I sample it at T naught I will get h equivalent hat of 0. Do you understand this? So, alright. So, in other words so impulse applied and sample value at t naught let me just do that. So, if apply the impulse at t naught the sampled value of the output at t naught will be.

Student: H equivalent of 0. And h equivalent at 0.

Professor: 0. Now, if I want H equal. If I want the sample to be, if I wanted to sample to be h equivalent hat of delta t with delta t is some small number a small time at what time should I apply the input? Do you understand the question?

Student: I understood sir. Like, we have to apply that t naught plus t naught minus delta t.

Professor: Very good. We have to apply it at naught minus?

Student: Delta t.

Professor: Delta t. Correct. If you apply, if you want to see h equivalent hat 2 delta t you must apply minus t naught minus 2 delta t and so on. So, therefore, so again, we see that we are applying we are changing the inputs, but our output measurement is always the same. So, that basically means that you can exploit the adjoint again, as expected. We did the same thing with the case when t naught 0 now we will do this, it is the same. So, this is basically n and this is n had and this is the adjoint.

This is also LPTV at fs and this is the adjoin network. And this is 2 and this is 2 prime. And what should I do? Now in the once you, I mean, we already know how to find the adjoint, you do to time reversal of all the control signals and so on. And so, remember, if you apply an input at the t1, so, regional or rather let me call this for a network n if you apply the impulse at t1, measure at t2, it is the same as you will get the same result if in the adjoint impulse at minus t2 measure at.

And you must have applied the impulse at the output port of the adjoint. So, what should we do now? So, in this experiment, this is corresponding to n we are going on changing the instants at which the input is applied, but the measurement remains the same. So, in the adjoint the input has always to be applied at what time?

Student: T naught. Minus t naught.

Professor: Minus t naught. So, we will apply therefore a current of delta of t plus t naught. And if you measure the output of the adjoint at t naught what will you get at I mean, to get when will you see h hat equivalent of 0 in the adjoint?

Student: Minus t naught.

Professor: At minus t naught. So, what is the waveform that you will get there at the output?

Student: H hat equivalent of.

Professor: H hat equivalent of?

Student: T plus t naught.

Professor: T plus t naught. So, in other words in the adjoint what do you do is if I want to draw a picture what do you like to do is, you apply an input at minus t naught and what will happen? Well the output waveform will do something like this. So, what is the, to get h equivalent the hat of this is h equivalent hat of t plus t naught that basically means that h equivalent hat of t has been moved to the left by t naught, so h equivalent hat of t therefore, is you take this so, this is going to be you shift it to the right here is the h equivalent hat of t.

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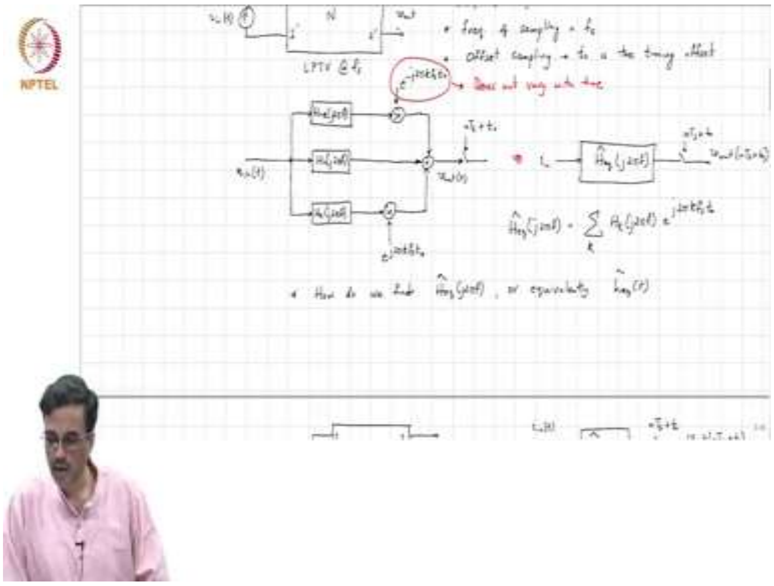

So, this is how you determine the equivalent impulse response of the system, rather the impulse response to the equivalent LTI filter. If the.

Student: Sampling is offset.

Professor: The sampling is offset. That makes sense?