Introduction to Time – Varying Electrical Networks
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Lecture No. 73
Finding the equivalent LTI filter of a sampled LPTV system: example of a continuous-time delta-sigma modulator

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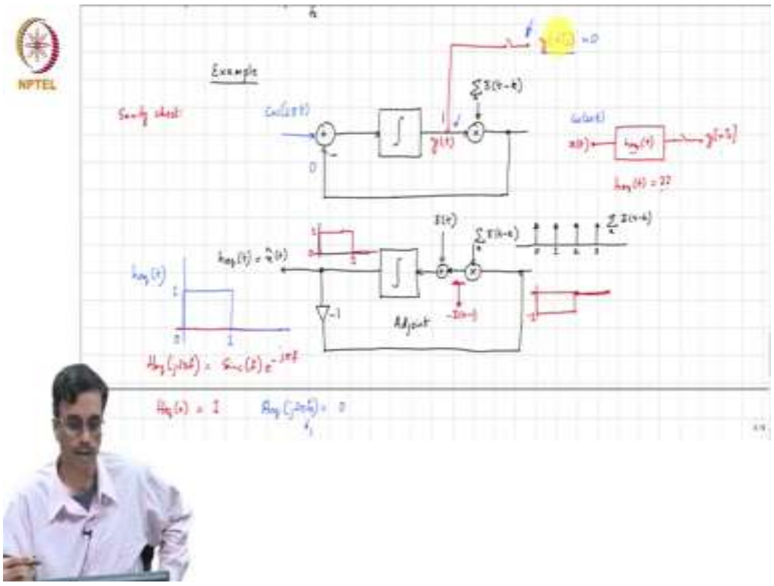

So, I will take an example of in a different domain and this is a continuous time delta sigma converter. But if you do not, have not seen this before do not worry about it, just think of this as a linear time varying system whose output is a sample. So, minus k. So, I am going to assume that the sampling rate is 1 hertz. So, I multiply the output of the integrator with a periodic impulse strain. And the, so therefore, this will be an impulse strain. And let us say I feed this back like this.

This is my LPTV system it is clearly LPTV because I am multiplying. I mean, there is basically some time varying gain somewhere. And so my output is, my desired output is simply the samples of let me call this y of t. My desired output is basically y of nTs. So, I am interested in only the samples of y of t at the instant at instance of 0, 1, 2 and so on.

So, what should I do? We know that equivalently, therefore, we have, I mean, let me call this x of t. And we know that there is some h equivalent of t, which when sampled will give you y of nts. But, and we are trying to find what is this. And so, what do we do?

Student: Adjoint network.

Professor: We have to first make the adjoint network. So, how do you do the adjoint? Well, there is a signal flow graph. So, this is a time invariant gain so it just remains like that. All the directions of the arrows are removed. This is summing node so that becomes a pickoff point with signs. This becomes x hat of t and this becomes minus one. Then what do you do? Well, this is a pickoff point.

So, we need to this becomes a summing node, and this is a multiplier we have to time reverse it, but the impulses are coming of 1, 2 and so on. So, time reversing makes no makes no difference. 1, 2, 3, and so on this is sigma over k of delta of t minus k. So, if you make t equal to minus t what will happen? So, therefore, there is a reversal of time has no effect, and this becomes this. So, this is the adjoint. So, what should we do now? As you said, I formed a giant network.

Student: We have to apply the.

Professor: We have to apply the?

Student: We have to detect the output.

Professor: Which is the output port here?

Student: The output port is by nft.

Professor: So, where should I put the. What input should I put? And where should I put it?

Student: We have to sum.

Professor: We have to add. What should we? What should we have to put a Dirac impulse delta of t. Alright? So, what do we expect to see at x hat of t?

Student: X hat 0.

Professor: No, we will do them, we will solve it and get some waveform there, but what does that waveform indicate?

Student: That is like h of h equivalent of t.

Professor: So, what we get here is nothing but h equivalent of t. So, you inject an impulse here. So, what happens and as soon you take an integrator and inject an impulse what happens?

Student: We will get a step.

Professor: You will get a step here. And so, what happens to the step? What happens if you take this step and if it goes in gets inverted? This becomes an inverted step. So, this becomes minus one. And so, this will keep going. So, what will happen to. So, this is a negative step is coming here. And when does the sample again?

Student: Ht.

Professor: At what time?

Student: Time 1.

Professor: Time 1. So, once these samples at time one, what will be the input? What will be given what will be the output of the multiply when you sample?

Student: Delta of t minus 1.

Professor: Look at the sign properly.

Student: Minus delta of t minus 1.

Professor: Minus delta of t minus 1. So, if minus delta of t minus 1 goes into an integrator, what will happen to the output of the integrator?

Student: It will become d.

Professor: It will. Very good. It goes it goes to a t equal to 1 therefore, the output will go to, at t equal to 1 the output will go to. And so, this minus so, then what happens what is fed back before? What comment can you make about the feedback waveform?

Student: 0.

Professor: You get 0 there. So, the next time the sampler samples, it will be sampling?

Student: 0.

Professor: 0? Correct. And so, what is fed back?

Student: 0.

Professor: And everything is dead. So, what is h equivalent of t therefore? It is h equivalent of t is basically 1. This is h equivalent of t. What comment can you make. Sanity check, can we run any sanity check? What is the frequency response corresponding to this? A rectangular pulse what is h equivalent of f? Basically it is sync f, e to the minus j 2 pi f, t is half. So, basically j pi.

So, what is the h equivalent of 0 that is the DC gain? What is it? Sync 0 as 1e. Let us see if that makes intuitive sense. So, if you put in one volt DC here what comment can you make about that voltage?

Student: We should now add 1e volt.

Professor: You should have 1 volt there because otherwise.

Student: Integrator will saturate.

Professor: Integrator will saturate. If we assume that the circuit is working then what must be fed back must be also be 1 volt. That way the DC going into the integrator will be 0. So, if this is 1 volt, what comment can you make about this?

Student: That has to be equivalent.

Professor: At the sampling instant it has to be 1. So, if this is one then what comment can you make about the sampled output is also going to be 1. So, the DC gain therefore, is 1. Make sense from the intuitive. What comment can you make about h equivalent of j 2 pi fs? Well I mean, we can fs which is one so h equivalent of one is basically sync 1 which is is 0. So, what comment can we make if you put in say put in cost 2 pi t, what comment can you make in make about the average value of the signal here?

Student: That will get 0.

Professor: 0. So, again, this is an integrator. If we have an integrator inside a circuit and it is, the circuit is working properly the average value of the quantity going into the integrator must be 0. So, this must be 0 on average.

If this is 0, then what is sample here must be at the sampling instance this must be 0. So, y of nTs must be equal to 0. That make sense? So, because if you put in cos 2 pi t, here h equivalent of j 2 pi 1 is 0, so what you get here is 0. So, if you sample 0 you get 0. You understand? So, as you can see you can I mean, the adjoint makes finding these h equivalents of Ts.