Introduction to Time – Varying Electrical Networks
Professor. Shanthi Pavan
Department of Electrical Engineering
Indian Institute of Technology, Madras
Lecture No. 72
Equivalent LTI filter for a switched-RC network

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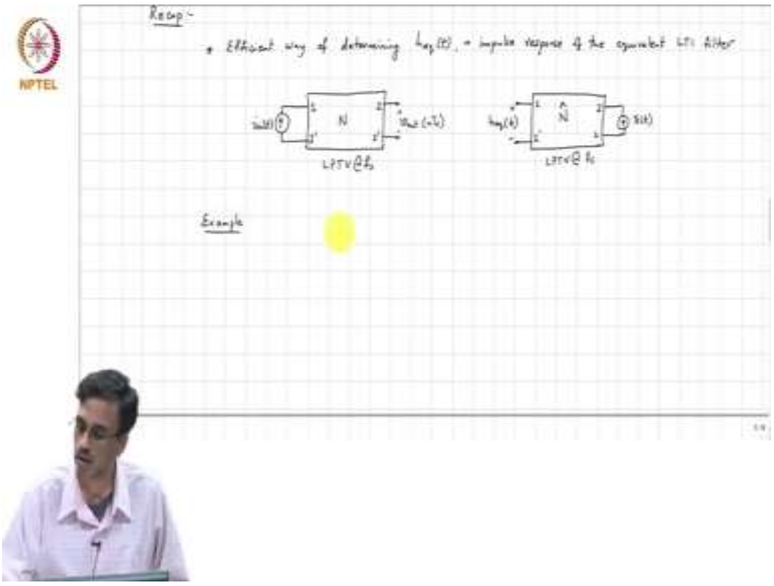

A quick recap of what we were doing in the last class, we were trying to we, we found an efficient way of determining h equivalent of t which is the impulse response of the equivalent LTI filter. And the context in the statement is that we have an LPTV network, and let us say we are interested in in a current input, voltage output transfer function and we are interested in v out of n times Ts.

And the trick that we found was that you will get the, we recognize we know now, that if you are only interested in the samples of the LPTV network output, one need not worry about all the harmonic transfer functions only the sum is relevant and the impulse response corresponding to that can be found by forming that adjoint network, we already know how to do that. You inject an impulse current in the adjoint and measure the voltage across the input port, and what you see here will give you h equivalent of t.

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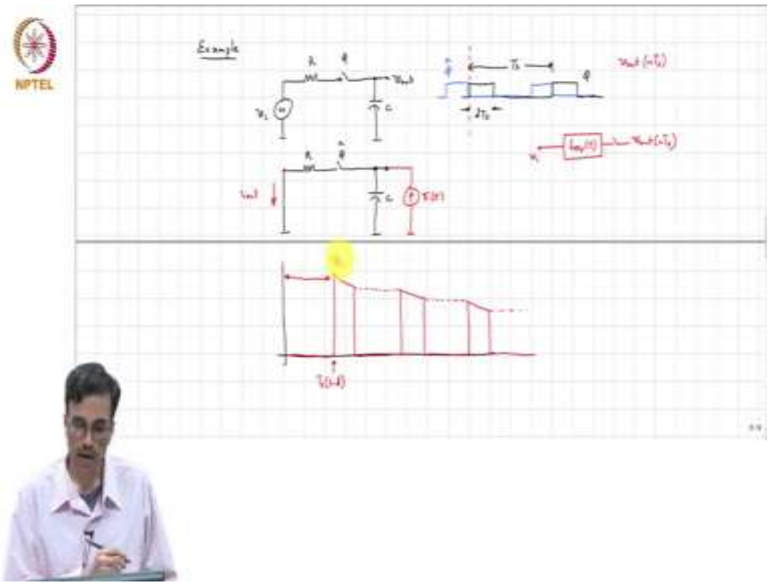

And we were considering an example, we will continue with that this afternoon. So, this is vi, this is R, this is c, this is v out, and this is phi. This is Ts, and this is d times Ts. And to find and let us say we are interested in v out of n times Ts. And what to do that, we basically understand that we can think of this as vi, pass through h equivalent of t and the output is sampled at nTs, so we will get n times Ts.

So, to do that, we have phi hat, this is c, this is r, this is delta t, this is i. And as we saw yesterday if you plot the waveform at iout, and by the way, phi hat is going to be a waveform which looks like this is going to be phi hat. The black waveform is phi, the blue waveform is phi hat. So, if we inject, we are interested in vo out of n times t.

So, if you inject an impulse current and plot iout, basically, you will see that the current is 0 until the time the Ts into 1 minus d, at this point it is going to jump up, then decay, come down then so it is going to start off here again. Do this, and so on. So, if you kind of neglect this delay this peak is 1 by Rc.

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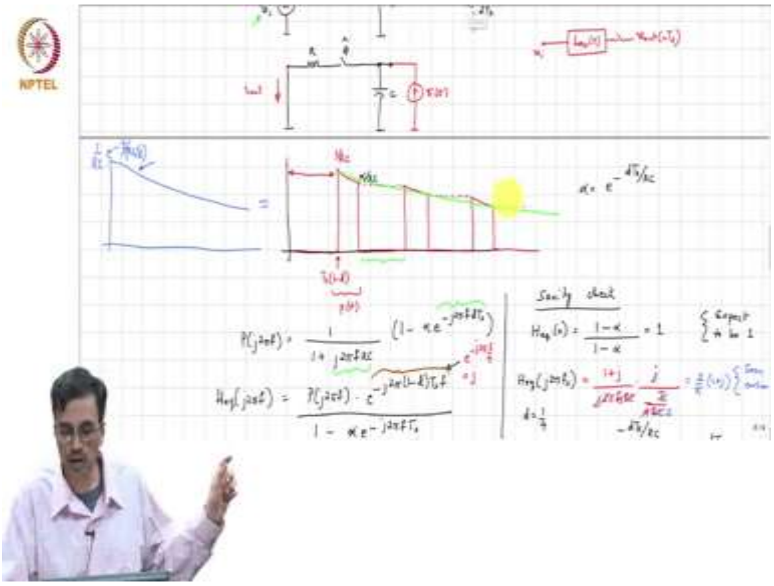

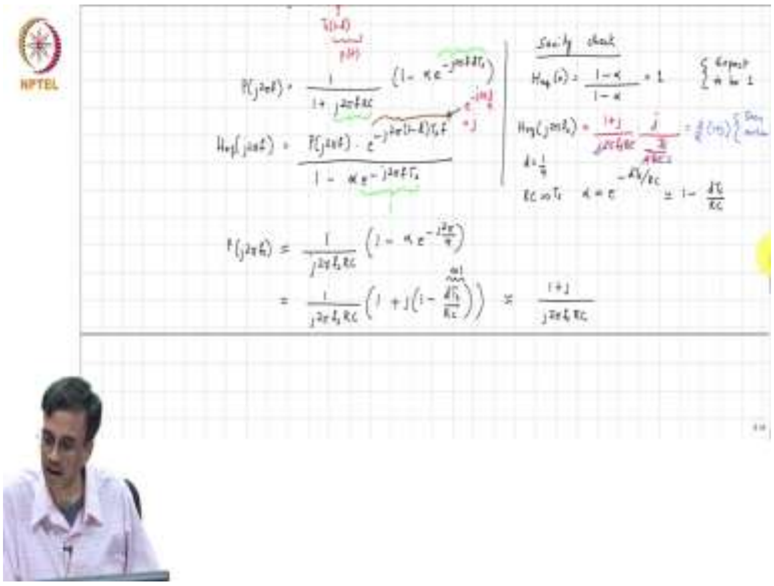

And if you call this p of t p of f, as we saw yesterday or p of j 2 pi f, if you will, is over plus j 2 pi fRc times 1 minus t, 1 minus if you call this ratio alpha, alpha is nothing but e to the minus dTs over Rc. So, this is alpha our Rc, so this is 1 minus alpha times e to the minus j 2 pi f, into dTs. And so, h equivalent of f of j 2 pi f is simply p of j 2 pi f times e to the minus j 2 pi into 1 minus dTs times f, that is simply the delay of the whole waveform, that does not change the magnitude. And this divided by 1 minus, 1 minus alpha e to the minus j 2 pi fTs.

Let us now do a sanity check. So, h equivalent of 0, we expect this to be 1, simply because if you have a DC voltage source here, the voltage across the capacitor simply going to be the same DC voltage. So, this must be 1. And let us see that that works out. So, as f tends to at f equal to 0, this goes to 0. Similarly, this becomes 1 and likewise, this also becomes 1 and therefore, p of f actually becomes or reduces to 1 minus alpha and the rest of it also, this also, this term in the denominator also becomes 1. So, that becomes 1 minus alpha and therefore this becomes equal to 1.

Now, let us take another case where we know the answer already. So, e to the minus j 2h equivalent of s, h equivalent of j 2pi fs. And so we have done this before and let us assume that d is equal to one-forth because this is the case that we have looked at before. Now, so this term, let us go term by term and if Rc is much, much larger than ts, then t of j 2pi fs basically will approximately be equal to 1 over j 2pi fs Rc, because you can neglect the 1 in comparison with j 2pi fsRc.

And the numerator will become 1 minus alpha times e to the minus j 2pi times, j 2pi by 4. And so, this and further recognize that alpha is e to the minus dTs by Rc, which is approximately 1 minus dTs by Rc. Correct. So, this is 1 over j 2pi fsRc times 1 minus dTs by Rc, times e to the minus j pi by 2, which is minus j. Now, since dTs by Rc is much smaller than 1 therefore, this is approximately going to be one plus j divided by j 2pi fsRc. This factor on the other hand will be e to the minus j 2 pi, e to the minus j 2 pi three-fourth 3 by 4 since fs times Ts is 1 and therefore, this is nothing.

Student: J.

Professor: This is nothing but j. So, h equivalent of f therefore, is nothing but p of j 2 pi fs, which is 1 plus j divided by 1 j 2 pi fsRc times this factor here, which we just saw was j and the denominator you have one minus alpha, which we know is nothing but dTs by Rc and d is nothing but one-forth.

So, the Rc, Rc goes away the fs, Ts goes away 2 and 4 go away and left the 2, j and j go away and this is nothing but 2 by pi times 1 plus j which we have seen. Remember, when we saw this, the result earlier, this was the average value of the voltage across the capacitor. Whereas what we are seeing now, is for the value sampled on the capacitor at the end of, yeah, at the at the end of the clock phase right are that just at the beginning of the clock.

The two of them give us the same results in the only in the limit that this rc is much, much larger than d times Ts. And the reason for that is that if RC is much larger than ts, then during when the switch is on, the voltage across the capacitor is changing very, very little. So, it does not matter whether you sample at the end of the phase or you look at the average or you sampled at the beginning of the phase you will always get the same answer. And you can verify for yourself that all this, the results that we have derived at other frequencies are also valid.

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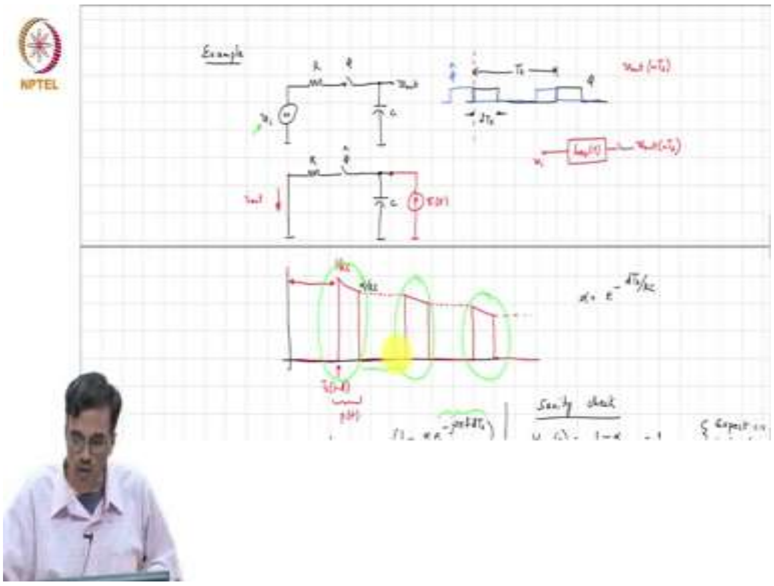

One thing that I would like to draw your attention to is the following. And this is again when we discuss n-path filters in a little more detail. This is something that will come in handy. Can you guess, if I break up all these pieces and put them together what will I, what is the shape that I will see? In other words, if I break up, if I remove this portion there and join all those pieces.

Student: It is like exponential.

Professor: You will simply see the, that is like basically not turning off the switch at all and therefore, you will see.

Student: The exponential curve.

Professor: The exponential, the standard exponential curve. Now, because the switch is periodically opening, this discharge through the capacitor is I mean, through the resistor is actually getting interrupted, so it is basically stretching out in time.

So, what comment can you make about this envoy up here?

Student: It is exponential.

Professor: It is also exponential, but is it decaying slower than e to the minus Tr by Rc or faster than e to the minus t by Rc?

Student: Slower than the equivalent.

Professor: It is, so it is decaying slower than the than e to the minus t by Rc. By what factor?

Student: By 4 times.

Professor: By I mean in general it will be if you have if.

Student: One-fourth of the side.

Professor: If d is one-fourth it will be decaying four times. So, rough so this is roughly equal to you can think of this, as taking an exponential which is decaying very slowly and decaying slowly by a factor of this is you can think of this, as you take a waveform which is 1 by Rc, e to the minus t by Rc by d. Correct? And you multiply, and but that is not the waveform that you are seeing here.

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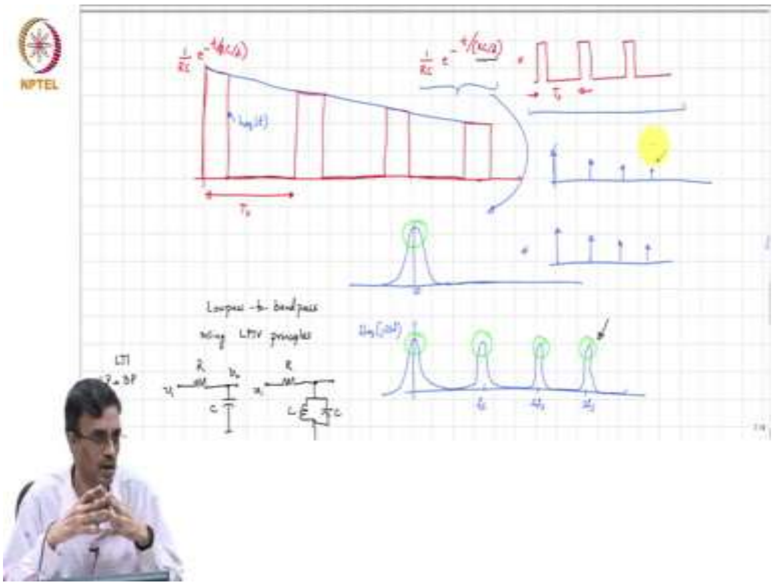

To get to that waveform, what would you do with this blue waveform?

Student: Shifted and scaling.

Professor: No, no hold on. So, we have let us say you have a waveform like this? What you are interested in is a waveform which does this. So, given the blue waveform and you know this period how will you get the red waveform?

Student: It is like multiplying the scale.

Professor: It is multiplying. It is like so, you take this 1 by Rc, e to the minus t by Rc by d and multiply it with the clock waveform. The clock waveform, so basically you take 1 by Rc e to the minus t by Rc by d and multiply it with a waveform like a periodic waveform like this. This is Ts. And so, if you look at the Fourier transform of this waveform, what do they expect to see? You know how you got this waveform now? That is simply you take this waveform and multiply this by that waveform. So, the question I am now trying to ask you is how would you expect the Fourier transform of this? This is nothing but h equivalent of t.

Student: This waveform will be at the ms.

Professor: Why I mean, so and why do you think that will be the case?

Student: In frequency it is the current.

Professor: Very good. So, h equivalent of t is this waveform, which can be thought of as the, you can take you can think of it as a waveform which is related to the time constant. I mean the decay time constant of the original network if you did not switch it off, if you did not turn off the resistor and you take that waveform and then you multiply that by a periodic waveform with the duty cycle d and period Ts. So, that is roughly how h equivalent of t will look like.

Now, how, so in the frequency domain therefore multiplication in the time domain is convolution in the frequency domain. So, how will this look like? So, how will this chap look like? The frequency response corresponding to an exponential decay is simply a low pass transfer function, and it keeps going off to 0. And this will, what is the frequency domain? In the frequency domain this will look like a train of impulses. A periodic is simply the Fourier series components. So, basically you will see this, you will see second harmonic, you will see third harmonic and so on.

So, when you convolve this with something which has impulses in the frequency domain, what do you expect to see? What do you expect to see as the frequency response of the equal of h equivalent of t?

Student: We have to consider.

Professor: So, h equivalent of j 2 pi f is simply the convolution of, this is 0, and this is the convolution of this response with.

Student: That impulse train.

Professor: That impulse train. And so, when you convolve this with the impulse train, you will basically get a response like this, you will get a response around fs 2fs, 3fs, and so on.

The magnitudes of all these of course, depend on the details of the pulse. So, intuitively therefore, you should expect to see, see multiple pass bands. Each of those pass bands. What comment can you make about the shape of this pass band? What comment can we make about the shape of that guy and that guy, and all these? All of them are have the same shape as the Rc low-pass filter that just shifted around.

Student: Fs.

Professor: Multiples of fs. So, basically by taking a low-pass filter I mean, we, this is what we should expect. Of course, that is also brought out by the math as we saw yesterday.

The denominator of this h equivalent of f will keep peeking at multiples of fs. And then there is some p of f and therefore, we expect peaking.

Intuitively what is happening is you can think of it as this this way where the low-pass transfer function is simply translated to around all multiples of fs. So, this is an example of what do you call a low-pass to band-pass transformation using LPTV principles. So, this uses low-pass band-pass, this is what implements what is basically a band-pass filter using the property of time variance.

To do the same thing with time invariant networks what would you do? Well, this was our low-pass filter, so LTI low-pass to band-pass or this will be a low pass filter. So, what is the job of the capacitor? The capacitor is an open at DC and becomes shot at high frequency. So, if you replace, if you want the behavior at DC to happen at fs, what would you have to do?

Student: That is f.

Professor: It is a time invariant network. So, you have an Rc network if.

Student: We have to aid the system with f.

Professor: No, the question I am asking is. So, this looks like this, frequency response of the Rc networks look like, looks like this because the capacitor is open at DC and falls off at high frequency. Now, what you want is this kind of behavior, but around some fs. So, what comment can you make about z.

Student: z has to be open at fs.

Professor: z has to be open at fs and short at a very high frequency and very low frequency. So, what do you think that z can be? Can you think of an impedance, which is open at fs and shot at other frequencies? So, what have we done? The, I mean, if you want to convert a low-pass filter into a band-pass filter.

Student: Like LTI case that is what time.

Professor: So, that is not what we are discussing. So, in the LTI case to convert from low-pass to band-pass you have to put an inductor in parallel with the capacitor so that it becomes.

Student: Both.

Professor: Now, the problem is as we have discussed earlier it is very difficult to make a lossless inductor correct, and it is very difficult to tune the. Once you have made the LC network to tune it is very difficult. In contrast basically, the, if you look at the frequency response around at the output of the capacitor, you can see that. This basically, also shows similar kind of behavior. Except that there are multiple pass-bands, but the center frequency of the band-pass response is governed by.

Student: Clock frequency.

Professor: By the clock frequency. Does it make sense? And if we change the clock frequency you can basically get, you can change the center frequency. What comment can we make out the bandwidth?

Student: Bandwidth depend on that Rc.

Professor: Bandwidth only depends on the Rc by d, correct. So, the center frequency and the bandwidth are independent. If you change fs the bandwidth is not changing. The same shape is getting moved two fs or less than fs.

So, we will come back to this later when we discuss n-path filters, but this is the intuition behind why the frequency response looks periodic. With the magnitudes of course, depend on the details of the pulse, but the intuition why it looks like that is because the equivalent impulse response that relates the, what do you call the sample capacitor voltage to the input the impulse response looks like to take the low-pass response and then multiply it by a periodic wave, the Fourier expansion of the periodic wave will contain Dirac impulses.

And therefore, when multiplication in the time domain corresponds to convolution in the frequency domain, and therefore, when you multiply these 2 you should expect to see in the frequency domain, you should expect to see multiple pass-bands. Does it make sense? So, let us take the next example.