Introduction to Time – Varying Electrical Networks
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Lecture No. 71
Finding the equivalent LTI filter of a sampled LPTV system: example

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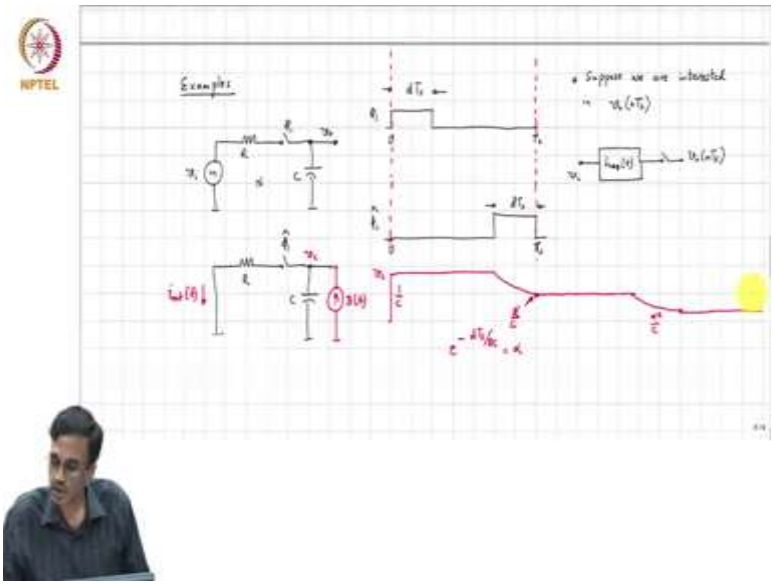

So, let us now do some examples. And as usual, we will begin with the simplest possible network that we can think of. We have a voltage source, a resistor, a periodically operated switch and a capacitor c, and this is driven periodically. So, let us assume that this is 0 this ds is phi 1. And let us say this is a duty cycle d times ts because of switches being periodically operated. And suppose we are interested in only the samples of the capacitor waveform. So, this is vo, vo of n times ts. So, the sampling instants are 0 ts and so on.

So, what do we do? So, we need to find. So, the, in other words what we are trying to do is we have vi, there is some h equivalentof t, the output of which is sampled and this output is v out of nts.

So, what comment can we make with how do we get a h equivalent of t? Well, you form the adjoint network. So, this is n, this is a voltage input, voltage output. So, what should we do?

Student: Get the current input.

Professor: We first draw the adjoint network, the voltage source remains vi. And what happens to?

Student: Phi 1 of minus t.

Professor: So, this becomes phi 1 of minus t and therefore, phi 1 hat simply becomes something like this. So, this is the adjoint network and we are interested in we are interested in the what do you call the equivalent transfer function that is at the end of, I mean at 0 ts and so on. So, what should we do? We would basically like to inject.

Student: Impulse current.

Professor: An impulse current delta t. And what are we supposed to do? Measure the output current i out of t. So, what happens? When you inject a current what will happen?

Student: The voltage that is developed across the capacitor.

Professor: The voltage that is developed across the capacitor is?

Student: 1 by c.

Professor: And remain the same throughout up to d times ts. At this point what will happen? This is the voltage correct. This is okay.

Student: But dt is periodically discharged like cap is instantly charges and then.

Professor: Correct. So, basically the voltage so if I call this vc, vc is like this, then what happens?

Student: The exponential decay.

Professor: It will decay. Then what happens?

Student: The authorities will be 1 by sorry current.

Professor: This will be 1 by C times.

Student: D.

Professor: Let us call this. Let us call e to the power minus d times ts by rc let us call this alpha. So, this will be, this voltage will therefore be 1 by c times alpha. Correct? Then what will happen? It will remain the same till the next period, and it will decay, alpha square by c. Decays remains becomes alpha square, alpha cube by c and so on.

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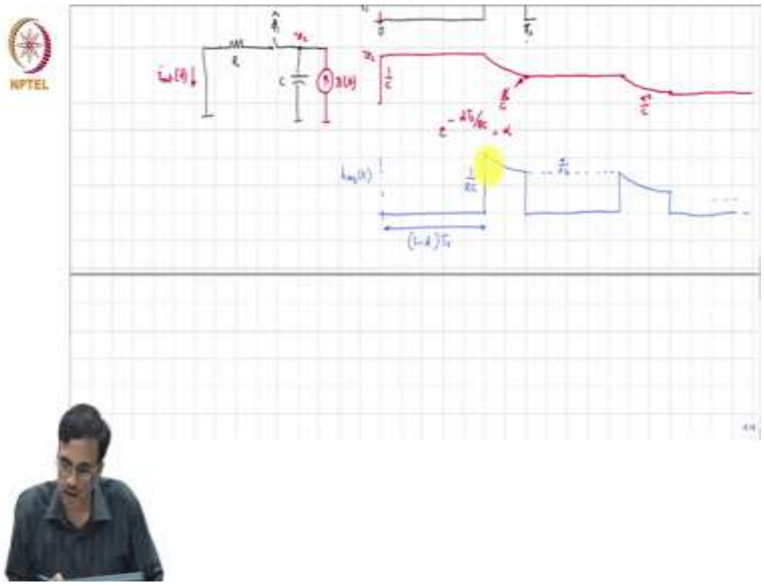

So, but that is not exactly the output waveform we are after, we are interested in the current through the resistors. So, what would be the current in the resistor?

Student: Like alpha by Rc by r.

Professor: No, think carefully.

Student: Waveform will be?

Professor: What will it be before when the switch is open?

Student: 0, sir.

Professor: Then when the switch is closed what will it be?

Student: Alpha by Rc scaling factor.

Professor: By Rc so 1 by Rc then it will become 0. Then it will start off.

Student: Alpha by Rc.

Professor: Alpha by Rc. Like this. And so on.

So, this is h equivalent of is iout and h equivalent of t is basically divided this divide by ampere second so that is. In fact that 1 already is got Coulomb in it. So, when you divide by ampere second you basically you will get just 1 by Rc and all this stuff. So, what is the Fourier transform? How does this look like in the frequency domain? Well, this is delayed by what is the delay? It is 1 minus d times.

Student: Ts.

Professor: Ts. So, that so basically we find the Fourier transform of the rest of this, we can just simply multiply by e to the minus j 2 pi f times one minus d times ts to get the Fourier transform of the of h equivalent.

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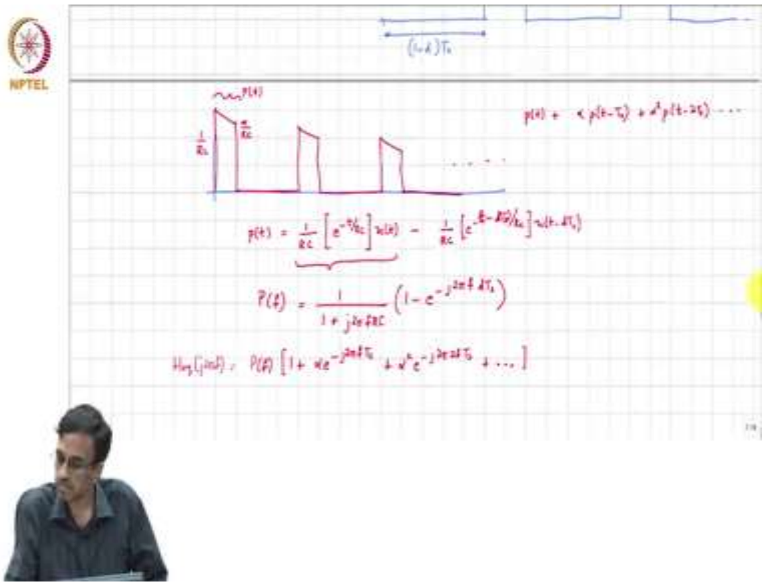

So, what the question is, what is the Fourier transform of let me call this of this pulse, which is basically. So, let us call this p of t and see what the Fourier transform this is. Let me make this. Let us call this so this is 1 by Rc, and this is alpha by Rc. If we call this pulse p of t, what is this waveform?

Student: It is alpha.

Professor: It is p of t plus alpha times t minus ts, plus alpha square p of t minus 2 ts and so on. Correct? So, what is the Fourier transform? So, if you, if we know the Fourier transform of p of t, we will be able to.

Student: Shifting.

Professor: Simply as a matter of scaling and shifting and then adding them all up and then that is. So, the Fourier transform of p of t is actually pretty straightforward. It is p of t therefore you can see is nothing but 1 by Rc times e to the minus t by Rc minus this times u of t minus 1 by Rc, e to the minus t minus d ts by Rc, u of t minus.

Student: D ts.

Professor: dTs. So, in other words for p of t itself, if you find the Fourier transform of this we will be able to just shift and delay, and then you will be able to get the Fourier transformer the pulse. This is nothing but the exponentially decaying pulse waveform, and this is nothing but 1 by 1 plus j 2 pi fRc. That is the Fourier transform of just a decaying exponential. This times 1 minus e to the minus j 2 pi f into is delayed by dTs. That is the.

Student: P of f.

Professor: P of f. And so, what is the Fourier transform of this periodic waveform is p of f times 1 plus alpha e to the minus j 2 pi fTs plus alpha square.

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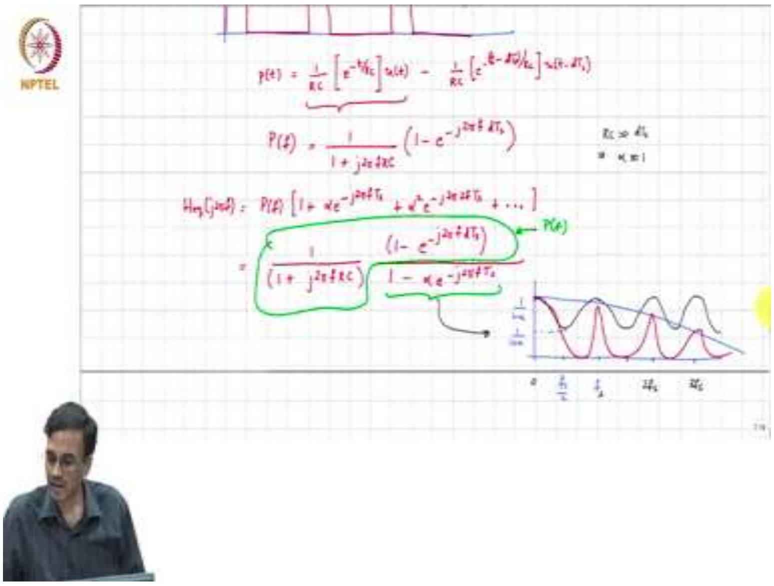

So, h equivalent of j 2 pi f is this plus alpha square e to the minus j 2 pi into 2 fTs and so on, which therefore, is given by 1 over 1 plus j 2 pi fRc into 1 minus e to the minus j 2 pi f times dTs divided by this is nothing but a geometric progression. So, that is nothing, but one minus alpha e to the minus j 2 pi fTs.

So, if you plot this, what do you think you will see? Well, this is the p of f. But I am most interested in in this function the denominator function. What do you think it will be at t equal to 0? I mean at def equal to 0?

Student: So, sir it will gain up 1 byone kind of something.

Professor: This is going to be, dc is going to be 1 by 1 minus alpha. That is just the denominator then at ts by 2 what will it be? At f equal to ts I mean 2 by ts what will it be? So, you will have 1 byone plus alpha. So, at fs what will happen again?

Student: 1 by 1.

Professor: So, basically this function alpha is going to be. I mean if Rc is much, much greater than dTs what comment can you make about alpha?

Student: Alpha is almost 1.

Professor: Alpha will be approximately 1. So, 1 by 1 minus alpha will be a number which is very large compared to 1, and 1 by 1 plus alpha will be some number close to 2. So, you can see that this, the denominator, which is this quantity 1 by that quantity is periodic with so it will peak at 0, it will peak at fs, it will peak at 2fs. It will peak at 3fs and so on. And this is multiplied by his Fourier transform of that pulse. And that basically, it turns out to be some we will take a look at in the next class, but it turns out to be something like this.

So, when you multiply these two things, what common can you make about the sampled value on the capacitor there will still be peaks at. So, you will have peaks at at, fs 2s, 3fs and four and this we knew already.

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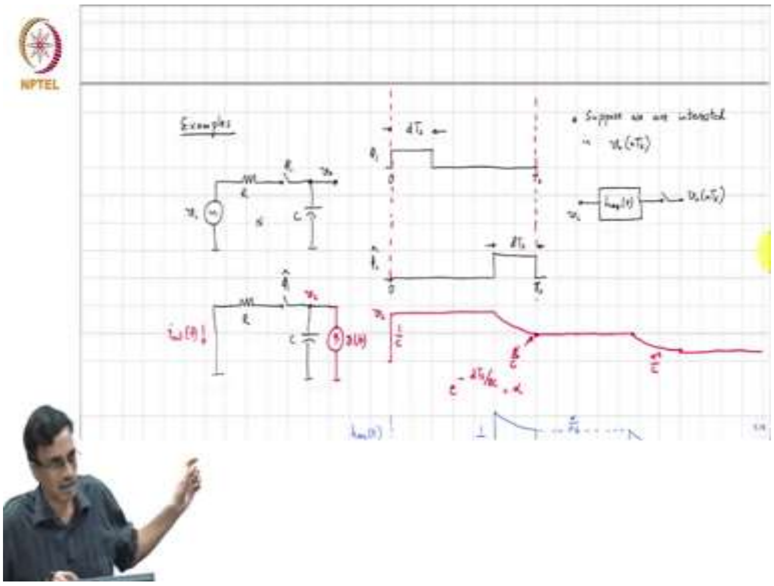

Remember, when we discussed this intuitively, if Rc is much, much larger than c i mean then t or d times ts then the capacitor will see the same, if the input is a multiple of the the switching frequency then the capacitor will see the same section of the input voltage every cycle and therefore, you will expect to see a maximum a local maximum at.

Student: Fs.

Professor: At fs. And that of 2fs and at 3fs and at 4fs. I mean this is basically just telling you that that that is indeed correct. We will continue with this next lecture.