Introduction to Time – Varying Electrical Networks

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Lecture No. 67

Time-domain implications of inter-reciprocity and the adjoint network: Example calculation

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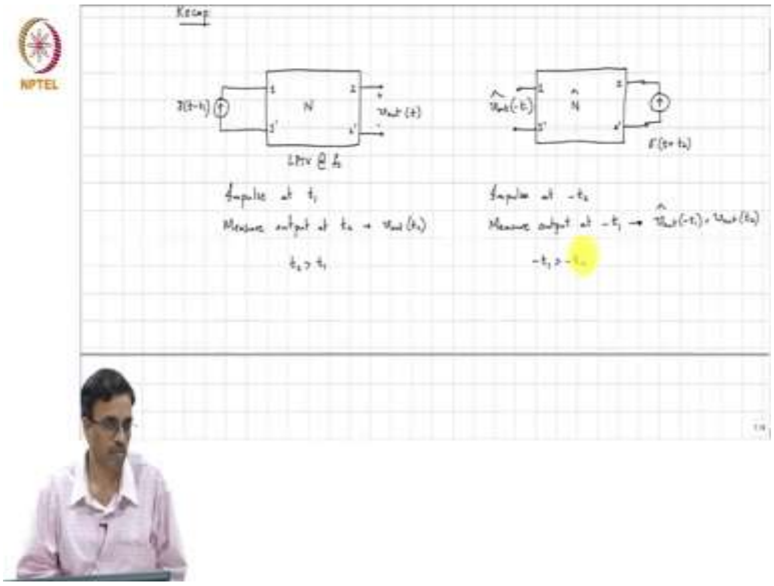

A quick recap of what we did yesterday. So, when we ended the class yesterday, we looked at we were discussing a time domain implication of reciprocity in LPTV networks. So, we were discussing that if we apply an impulse at t1 and measure output at t2, so you will basically get v out of t2. You would get the same response if in the adjoint network are the inter-reciprocal network you excite, so we must put delta of t plus t2 and you call this v out hat and measure it at minus t1.

So, impulse at minus t2 and measure output at minus t1, and so that is basically v out hat of minus t1 and that will be happens to be the same as v out of t2. And we call that t2 of course, must be greater than t1 when you since you applied to the original network and therefore, minus t1 must be greater than minus t1 in the adjoint.

So, there is no, just because the time is negative, it does not really mean that something has become anti-causal. Remember the time being 0 or negative it is all a matter of choosing the origin properly. So, there is nothing wrong with having negative time. The only problem is or unphysical thing is that if the output occurs before the input.

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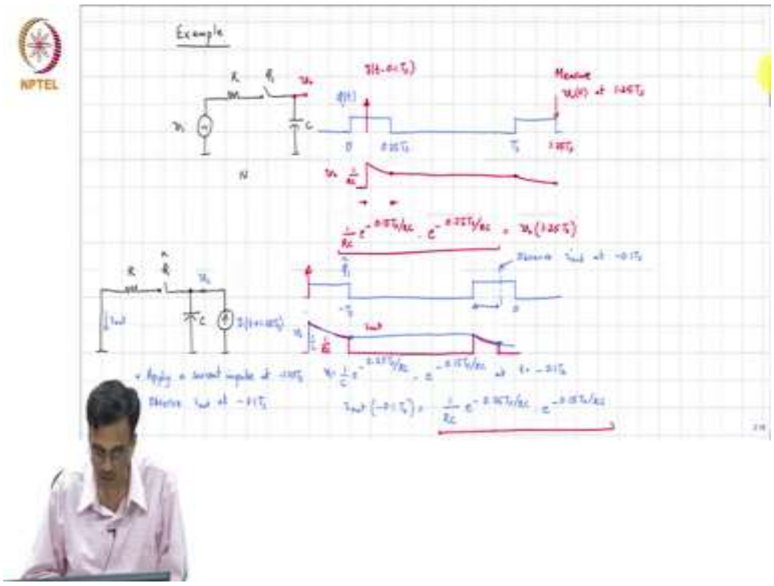

So, let us take a quick example and check this out. And as usual, we will begin with the simplest possible network that we can think of, namely, a resistor, a periodically operated switch and a capacitor. It has one, the smallest amount of memory possible, it has a resistor and of course, even you need to have at least one periodically time varying element and that is the periodically operators.

So, this is vi. This is the network n and phi of t is basically looking like this, this is 0 this is ts and this is 0.25 ts and this is pi of 2 t. Now let us, I do not know let us apply an impulse at say point 1 ts, so this delta of t minus 0.1 ts that is the input we apply, and let us measure the output say at about I do not know here how about 1.25 ts. So, we measure vo of t at 1.25 ts. So, what is, what will be output be?

So when you apply the impulse at point 1 ts, the switch is closed at that point in time. So, the impulse current will be vi, 1 by r, impulse current is basically a delta of t by r, that will go through the capacitor causing a voltage jump, which is 1 by Rc. So, the voltage at so vo basically will jump to 1 by Rc and then will decay exponentially with a time constant Rc. And what will be the voltage at this point? It will be.

Student: 1 minus.

Professor: 1 by Rc times e to the minus point 15 ts by Rc, correct. We are applying the impulse at point 1, up to point 25 it is simply a linear network, I mean linear time invariant network. So, it is the usual Rc decay. At that point what happens the switch opens and whatever voltage is there on the capacitor is trapped and therefore, it remains like that for a long time until the switch is closed again. And what happens to the…

Student: And then it will start decaying.

Professor: Again it will start decaying. And at that point, so that will be e to the minus point 25 ts by Rc. Correct. So, it is decaying for an additional.

Student: Rc.

Professor: Quadricycle.

Student: Quadricycle.

Professor: With the time constant of Rc. So, this is going to be v out of or vo of.

Student: Vo t2.

Professor: Vo of…

Student: 1.25 ts.

Professor: 1.25 ts. Now, let us see what happens when we do the adjoint. So, in the original network, we were looking at voltage transfer functions. In the adjoint we have to look for current. So, this is r, this is going to be replaced by phi 1 hat this is c, this is where to apply an impulse. So, let us first plot phi hat. So, phi hat is going to look like, so this is going to be delta t, phi hat is going to look like this is just the time reversed version of this. So, this is 0. This is ts, and so this phi hat and phi 1 hat. Now, where we are supposed to apply the impulse now?

Students: Minus t2.

Professor: So, we have to, supposed to apply the impulse at minus 1.25 ts. So, instead of so it does not make sense and look for the output at minus?

Student: Minus 0.1 ts.

Professor: Where are we supposed to apply the input at?

Student: Minus 1.25 ts.

Professor: Minus 1.25. So, let us write that down here. So, delta of t plus 1.25 ts, that is because you must apply a current impulse at minus 1.25 ts and observe the output is now a current I out at minus?

Student: Minus 0.1 ts.

Professor: 0.1 ts. So, since we are dealing with only a negative time. I mean, rather than plot 0 and ts, we can, I will shift the whole thing by 1 ts. So, this is 0 and this is minus ts. Correct. I am just drawing the previous period. So, we should apply the input at minus 1.25 ts. So, the input therefore must be applied here. So, what will happen to the output voltage, I mean to the current?

Student: Output we have to see at point 1, 0.125.

Professor: So, this capacitor voltage will basically will be 1 by.

Student: C.

Professor: We will 1 by C. So, that is vc. And then it will decay. So, this is 1 by c. At this point it is c e to the minus 0.25 ts by rc. Then what happens to the capacitor voltage?

Student: It will remain same.

Professor: It remains the same. And?

Student: Again it will decay in two e point.

Professor: And again, and we are supposed to observe at minus 0.1, so that is I out at minus 0.1 ts. Correct. So, this voltage is going to remain the same and then it is going to decay again exponentially. So, what will be the voltage at this point? It will be this multiplied by e to the minus.

Student: Minus 0.1 ts.

Professor: No. What is this time? That is minus 0.1 ts.

Student: Minus 0.1 ts by Rc.

Professor: And so, that time is basically minus 0.15 ts by Rc. And so, the output current, i out at minus 0.1 ts is simply.

Student: By r.

Professor: vc by r, which is 1 by Rc, e to the minus 0.25 ts by Rc, times e to the minus 0.15 ts by Rc. And so i out before basically will look like this. Go to 0 then do that and then.

Student: Rc.

Professor: So, there is basically i out is in red and it is 1 by it starts off with an initial value of 1 by Rc. So, as you can see, this is the same as this. And another thing that I like to point out is that, it does not mean that the waveforms will be flipped or anything. As you can see, the i out waveform looks very different from the vo waveform in the original network? It is just that the values at those points are the same. You understand?

So, this basically with this, we have also seen an example and you can also try it out with other time varying networks yourself and convince yourself that this is indeed true. It turns out that this will be used. I mean, this is a useful observation to make, and we will use this going forward in the lectures to come.