Introduction to Time – Varying Electrical Networks
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Lecture 61
Inter-reciprocity in Signal-flow Graphs

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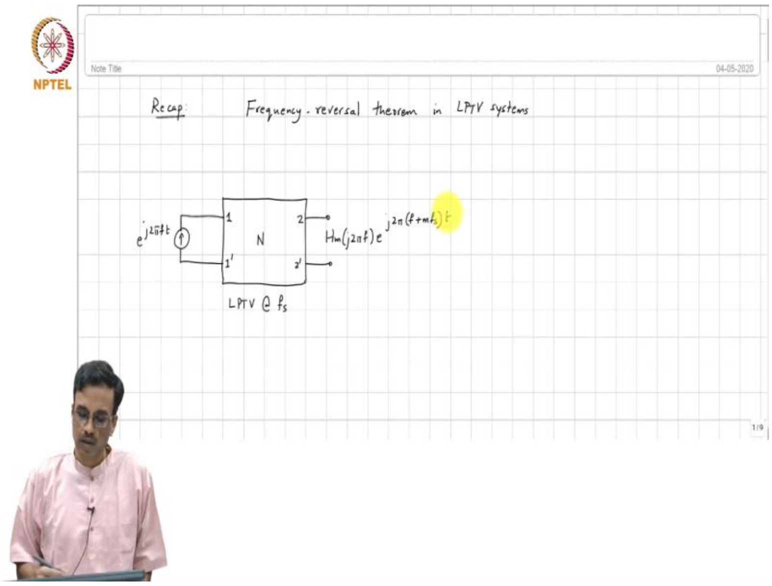

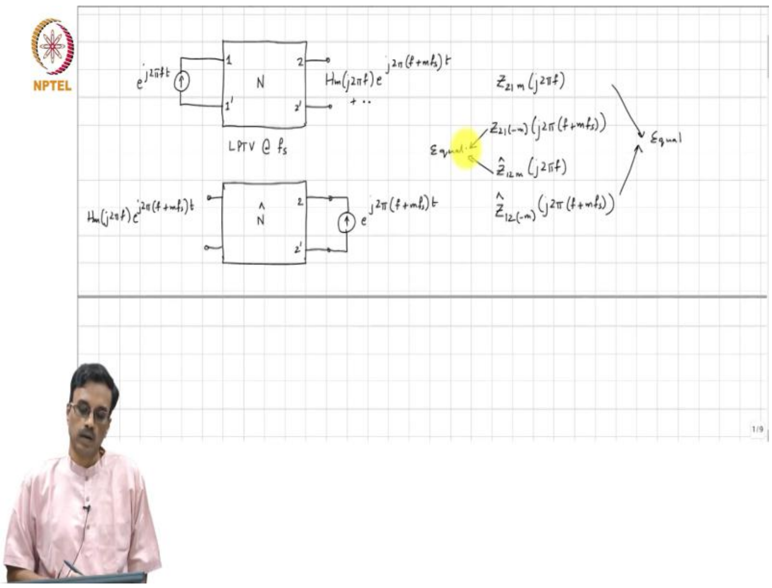

A quick recap of what we did in the last class. We looked at or we derived the frequency reversal theorem in connection with the operation of linear periodically time varying systems. And the result that we derived yesterday was the following. if you have a network N which is LPTV at fs and V excited at port 1 with a current e to the j 2 pi ft and measure the strength of the voltage at the output at frequency, H sub m j 2 pi f times e to j 2 pi f plus m f s times t.

You will get the same transfer function if you formed the adjoint, is N hat and you excite the adjoint with a tone e to the j 2 pi f plus m f s time t and measure the voltage at port 1 and you will get the same H sub m of j 2 pi f times e to the j 2 pi f plus m f s times t. Alright, so equivalently, this is, what this is saying if you, this is the forward transmission, for example, since the input is a current and the output is a voltage, you can think of this is Z 2 1 of m of j 2 pi f. And this is nothing but Z hat of 1 2 of minus m of j 2 pi f plus m f s.

And what these, what if N hat is chosen to the adjoint, then these two quantities are equal, which is also equivalent to saying Z 2 1 minus m of j 2 pi f plus m f s is the same as Z hat 1 2 of m of j 2 pi f. These two are equal or maybe, yeah, these two are equal. And it is often very useful to see this as a picture rather than words.

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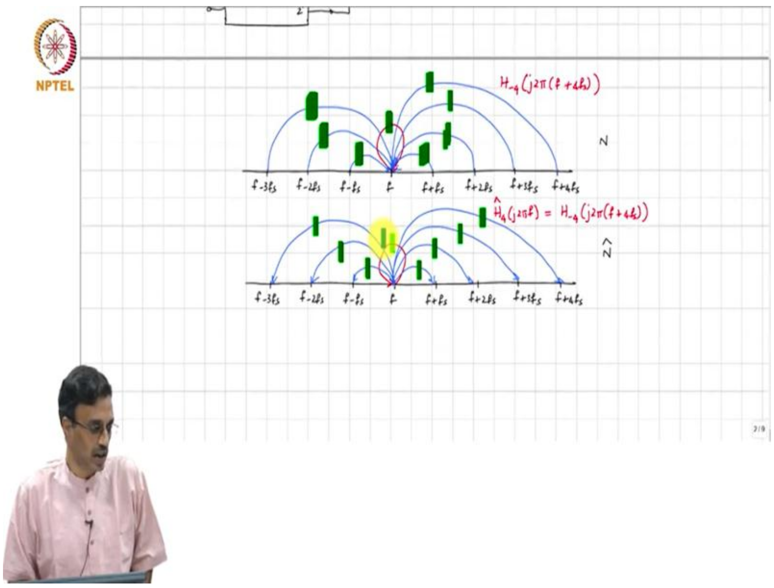

So, let us say, this is f, this is f plus fs, this is f plus 2 f s, f plus 3 f s, f plus 4 f s and so on. This is f minus f s, f minus 2 f s, f minus 3 f s and so on. And let us say, this is the original network N and this is the adjoint network, let me copy and paste this, this is N hat. So, what this is saying is that the transfer function from f to f in the original network is the same as that in the adjoint.

You still remember that the ports are interchanged and the transfer function from f minus 2 f s to f is the same as the transfer function from f to f minus 2 f s, from f minus 3 f s to f is the same as the transfer function from f to f minus 3 f s. So, this is in general, this is H sub, this is H minus 4 of j 2 pi f plus 4 f s. And this is also going to be the same, this actually is H hat 4 of j 2 pi f and because N hat is the adjoint network, it turns out to be exactly equal to H minus 4 j 2 pi of f plus 4 f s.

So, if we wanted to find the contribution from multiple, from a single source to an output frequency at f, you would have to calculate the strength of all these guys, 1, 2, 3, 4, 5, 6, I mean in this case, 8 transfer functions. And that would basically, if you did this by superposition, you would do it one source at a time. Now, what are we doing in adjoint, you just basically apply f and in one shot you get all these transfer functions in one shot, without sweat, or rather with lesser amount on sweat than you would need otherwise. Excellent.

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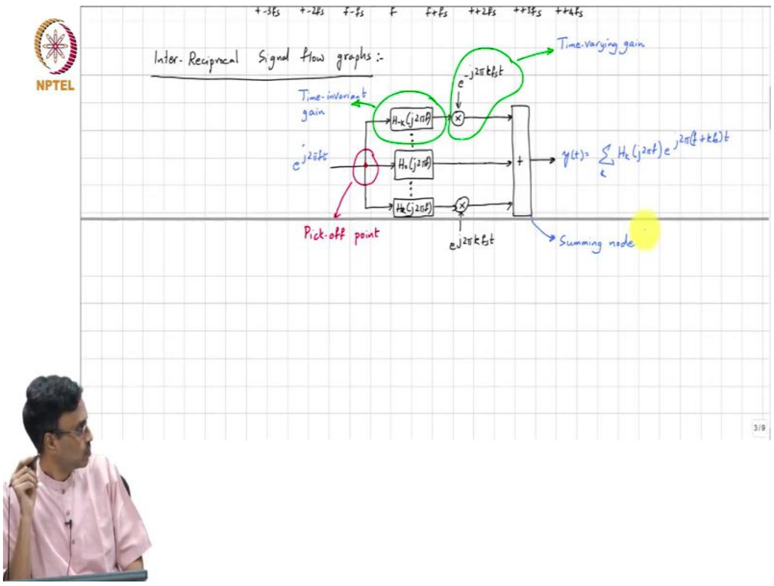

So, the next, I think what I would like to draw your attention to is the, which I find useful, is reciprocity, or let me say inter-reciprocal, signal flow graphs and it turns out, I mean a lot of times, when we are working with these networks, you often encounter signal flow graphs and just like how, in a network it is useful to figure out, I mean it is useful to understand reciprocity because it allows us to find multiple transfer functions at the same time and or transfer functions in a multiple frequencies to the same output frequency in one shot.

Likewise, the same thing happens when signal flow graphs and let me just take a typical signal flow graph that we have already seen so far and that is this other expansion. So, remember that any LPTV system from input to output transfer function can be basically mapped to this signal flow graph. So, this is H of j 2 pi f, this is, there is a whole bunch of stuff there, this is H of k j 2 pi f or we have always been doing H minus k so, and this is multiplied by e to the minus j 2 pi k f s times t, e to the j 2 pi k f s times t.

So, this is x of t and this y of t. This is an example signal flow graph. So, let us see what all elements you find in a signal flow graph and then, we will figure out how to create an inter-reciprocal signal flow graph. So, what all do we see? The first thing that we see is this. So, this is what is called a pick-off point. And what is happening? Well, input signal is coming here and branching out into three. The outputs of each of these branches is exactly the same as the input.

It is also possible that each of these branches has got, can also have some gain. And this is a branch with a, actually let us assume that we are working in the frequency domain. Everything is, just put it as j 2 pi f t and then you get some output y of t, k H sub k of j 2 pi f, e to the j 2 pi f plus k f s times t. So, this is an example of a branch with a time invariant gain, correct?

Because remember that, these H sub k's are all frequency response, corresponding to frequency response of filters that are linear and time invariant. This branch represents a block with a time varying gain or periodically time varying gain. And finally, we have this, which is nothing but a summing node.

So, to find the inter-reciprocal graph, the idea is very similar, you can write a, with our networks we wrote network equations which is our set of linear equations which relate the unknowns to the excitations and then, we found the unknowns. Then we selected which of those unknowns we are interested in and therefore, g inverse times excitation, times, I mean measurement times g inverse times excitation is our output, we transformed it and got the other network.

Well, we could do the same thing actually with a signal flow graph. But since we have already seen what we have done with a network, let us try to see if we can use that knowledge to, without having to write the equations all over again.

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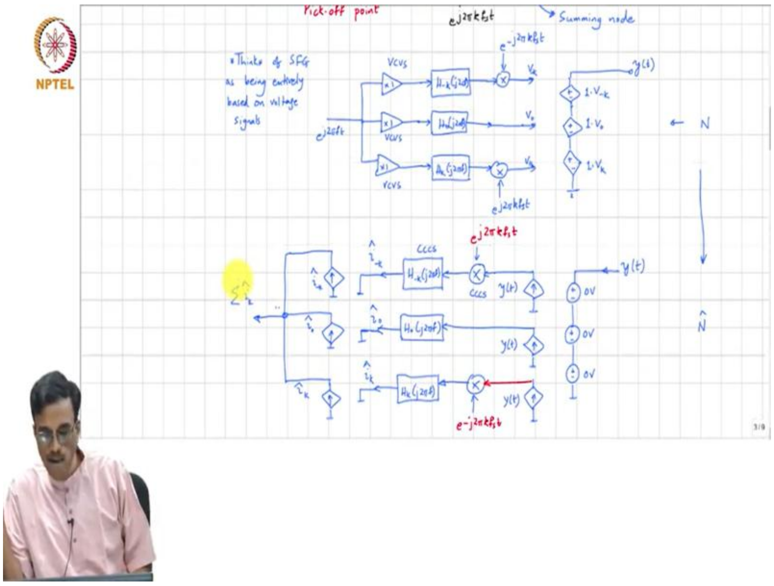

So, to do that, I am going to think of the signal flow graph as being, as everything being voltages. So, the input is a voltage and if you want to generate three copies of the input voltage, what would you do? You have three branches and each one of them is a voltage-controlled voltage source with, so, based on voltage signals, so this is e to the j 2 pi f t and this is a VCVS with a gain 1. And so, this branch will be H sub minus k of j 2 pi f, this will be H sub 0, this is H sub k of j 2 pi f, the output of these is also voltage e to the j 2 pi k f s times t.

And how do we realize summation in voltage? Well, we have a voltage-controlled voltage source like this. Then, so this voltage source is basically, let us call this Vk, V0 sorry V minus k, V0, Vk. So, this is 1 into V minus k. Then we have voltage control voltage source which is basically 1 times V0 and finally, we have, this is 1 times Vk. And this is basically, y of t. So, everything we have is now basically a, I mean this, for instance, represents a voltage-controlled voltage source whose gain is varying periodically with time. This is a voltage controlled voltage source on the other hand whose gain is fixed with frequency, it is some H sub k of whatever.

So, now this is a network with whole bunch of control sources. Well, to find the adjoint, all that we need to do is apply our standard rules. So, what does this become? We start from the right now. So, this is our N. So, what is N hat? We will, so this is the voltage control, each one of them is a voltage controlled voltage source and now they become a current controlled current source, except that the controlling and control ports have to be flipped.

So, what happens, so therefore, the current controlled current source basically means that we have three 0 voltage sources, this is now a current y of t. So, this is 0, this is 0 volts, this is 0 volts and we basically have, so the current is y of t so this must be y of t. This must also be y of t, this must also be y of t. And what happens to the gain? Well, hold on. Before we do that, let us say, let me, so let us go further, so we have a gain here. This gain is a time varying gain, so this is a voltage controlled voltage source in N. It must now become a current controlled current source. And what must its gain be?

Student: Gain has to be like (()) (20:46).

Professor: Very good. So, you have to, this current y of t, basically must be, now a current controlled current source, so I am going to just draw the same multiplier, except say that, this is the CCCS, current input, current output. And what must be the, the key point is to note that the gain must be time reverse. So, this must be e to j to pi plus k times f s times t. We will lower this a little bit.

So, this is going to be e to the j plus 2 pi k f s times t and so on. And here, you will have e to minus j 2 pi k s f times t. Then, we had, in N we have voltage control voltage source going this way, from left to right. So, now what must we have? We must have a current controlled current source, so this is a CCCS with gain H minus k of j 2 pi f. This is H 0 of j 2 pi f. This is H 0 of j 2 pi f, sorry, H sub k of j 2 pi f.

And so, the output is a current. So, what should you do as far as the, what is the next step? Well, earlier we had VCVS's in this direction, they must now become, they must become CCCS. So, let us call this V minus k hat, sorry I minus k hat, this is I 0 hat and this is I plus k hat. And we have, so this is basically I 0 hat plus I minus k hat plus I k hat plus I k hat plus or I just basically, I suppose it is easier to simply write sigma I k hat. So, now therefore, what has happened here? The same input y, now this is N hat, so how do we interpret this in the signal flow graph?

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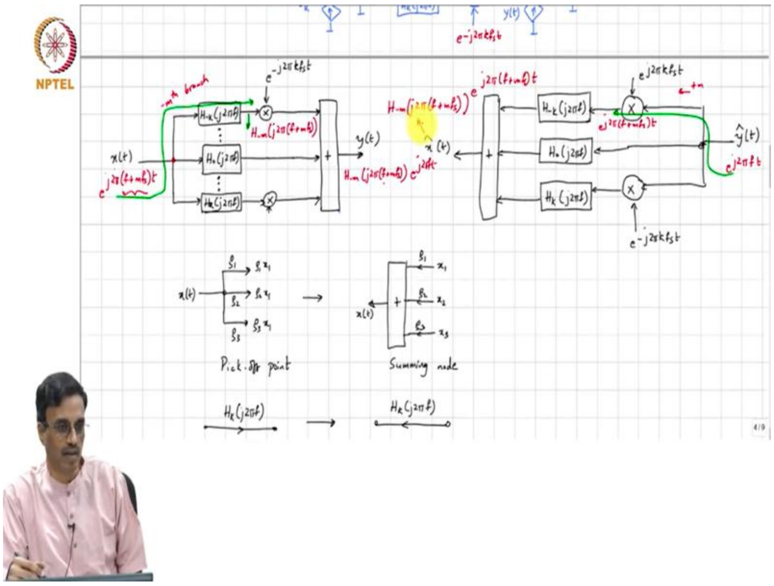

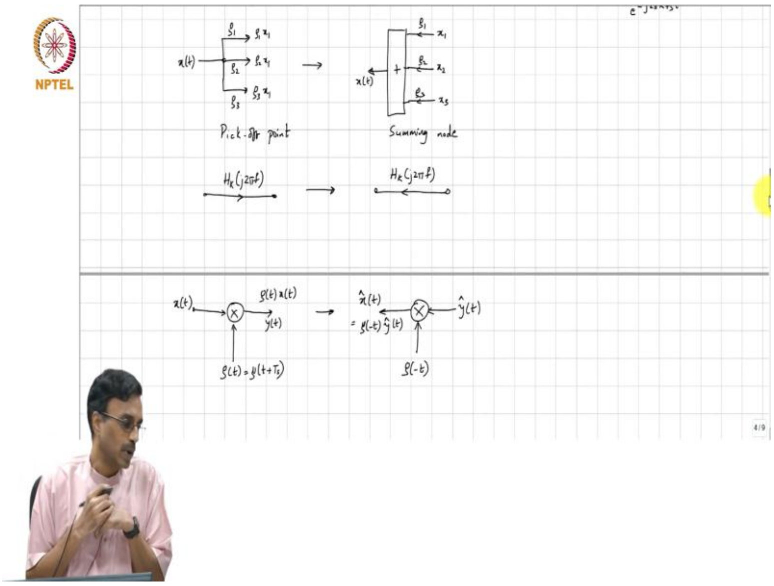

Well, this is a signal y of t, what is happening? y of t, the same y of t is getting multiplied by, so basically the same y of t is going to, so this is e to the plus j 2 pi, k f s time t and this goes through a gain which is H minus k of j 2 pi f. This signal goes through H 0 of j 2 pi f, this is e to the minus j 2 pi k f s time t, this goes through H sub k of j 2 pi f

And what are we doing to the output of these three guys? We are simply adding all of them up to get x hat of t. So, this was the original signal flow graph, this is the original signal flow graph and this is the new one. So, this is, say this is x of t and this is y of t. This is the adjoint signal flow graph on the right. So, what all conclusions can we draw from this?

The pick-off point in the original network, in the original signal flow graph. And we had gain here, let us call this g 1, g 2, g 3. This is g1 times x 1, g 2 times x 1 and g 3 times x 1. And what will happen in the adjoint? It simply becomes a weighted summing node, so this is x 1, this is x 2, this is x 3, they are multiplied by g 1, g 2, g 3 and they become a summing node. The pick-off point therefore becomes a summing node.

So, what comment can we make about a time invariant gain? So, if in the original flow graph, it was say, some H sub k of j 2 pi f, in the adjoint signal flow graph, what should you do? The gain remains the same but direction must be reversed. And if we have a time varying or let me say, I will make this, g of t, a time varying gain, a periodically time varying gain rather, what will happen in the adjoint? And what? The directions are all reversed.

So, x of t, this is g of t times x of t which is y of t, so what should you do? Well, we should reverse, time reverse the gain control waveform. We plop in y of t on the right and what we get here is, y hat of t, I mean whatever you get the x hat of t which is nothing but g of minus t times y hat of t.

The next thing is to see if we can relate these two signal flow graphs to something that we already know. So, for instance if, we excite the signal flow graph with e to j 2 pi f t or let us say, let us excite this with e to the j 2 pi f plus m f s times t. And let us say I am looking at the output here, the output at, I am interested in the output of the signal flow graph at a frequency f. So, what will that be?

Well, if I want this frequency to be f, that can, it must follow that it can only come from a branch where, you must, exactly, it will only come from a branch where, it will come from the mth, minus mth branch so that this H minus m, so the mth branch is how the signal will travel. So, for example, so this will go like this, get processed by H minus m of j 2 pi f. So, what will the output be there? The input frequency is f plus m f s. It goes through a filter whose transfer function is H minus m. So, what will be the output, strength of the output tone here? What will be the signal there?

Student: Signal is, there f plus m f s only.

Professor: Yes, exactly. So, basically, we are talking about a time invariant one, so the signal here will be H minus m of j 2 pi f plus m f s times e to the, it is no minus j here, it will be e to the j 2 pi f plus m f s. Then it is getting multiplied by e to the minus j 2 pi m f s t. So, what will come out here will be H minus m of j 2 pi f plus m f s times e to the j 2 pi f t.

Well, this is something that we know, if you take an LPTV network, the transfer function from an input at f plus m f s to an output at f is simply H minus m of j 2 pi f plus m f s. Now, the equation is, what will happen in the adjoint, what would you expect if we excite the output port of the adjoint with e to the j 2 pi f t, from the signal flow graph, what do we see? If we are interested in looking at what the strength of this waveform, x hat is at f plus m f s, what do we get? How do we, through which branch will it go through?

Student: (()) (35:30) multiplied with e to the j 2 pi m f s.

Professor: So, it will go through the plus mth branch here. So, what you will see is the signal go like this, it is first getting frequency translated, so that will give you e to the j 2 pi f plus m f s times t and is then processed by a filter whose transfer function is H minus of m of j 2 pi f. So, what you get here will simply be H minus m of j 2 pi f plus m f s times e to the j 2 pi f plus m f s times t.

So, this is something that we knew already. So, if you take the adjoint, the transfer function from this H minus m of f plus m f s in the original network is the same as the transfer function from f to f plus m f s in the adjoint signal flow graph. So, this is something that we knew already and then this is consistent. So, that, and this is a useful trick to know, a lot of times people are dealing with signal flow graphs and if you are interested in finding multiple transfer functions, this is an easy trick to know.