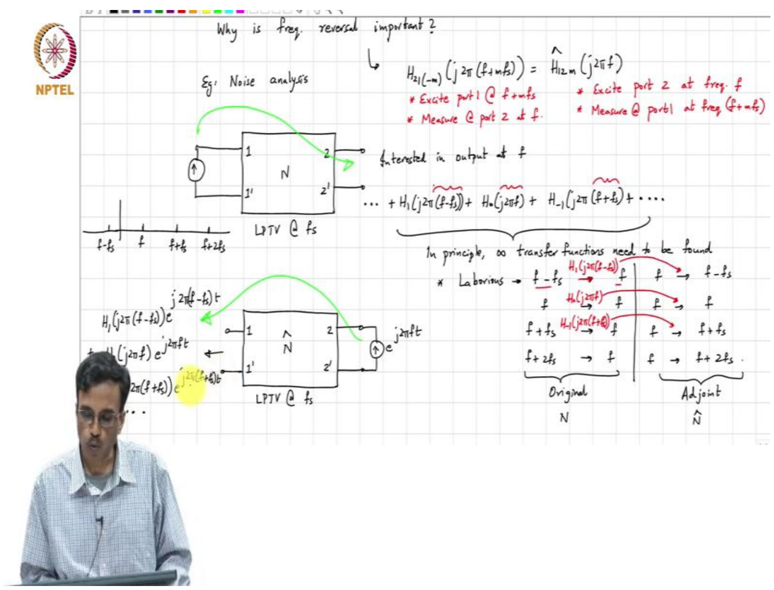
Introduction to Time - Varying Electrical Networks
Professor Shanthi Pavan
Department of Electrical Engineering
Indian Institute of Technology Madras
Lecture 60
Why is the frequency-reversal theorem important?

(Refer Slide Time: 0:24)



Is and like we saw in the time invariant case inter reciprocity and reciprocity were particularly useful when we were calculating noise at the output because in the linear time invariant network you need to find transfer functions from many sources to the output, to a single output. Fortunately, there all the sources are at the same frequency and the output is at the same frequency because you are dealing with a time invariant network.

In a time, in a periodically time varying network not only do we have multiple sources but even with a single source you not only have to worry about transfer function from, if you are interested in finding what the noise is at an output frequency f, you not only have to worry about transfer functions from a, transfer function from the source to the output at frequency f, you also have to worry about transfer functions from other frequencies of the form f plus k fs.

So, for example, in noise analysis, so suppose that we have a network N, this is LPTV at fs and there was a noise current with some specified frequency content. So, there is some strength at f, there is some strength at f plus fs, there is some content at f plus 2 fs, this is f minus fs and so on.

If you do not like to think about it as noise, you can think of an excitation which has got components not only at f but at multiples of at f plus integral multiples of fs.

And let us say we are interested in the output and we are interested in output frequency f. So, what would we have to do? We would have, I mean so we are interested in output at f. So, what would you have to do? We would have to find say H minus, we would have to do, we need to find H1 of j 2 pi f minus fs plus H0 of j 2 pi f plus H minus 1 of j 2 pi f plus fs and so on and so forth.

For how many frequencies, we do this for all the, for k equal to minus infinity to infinity. So, in principle, infinite number of transfer functions need to be formed. Now, this is very laborious, because, I mean what would you do this, how would you do this? You would put in a tone at f minus fs measure the output at f, put in a tone at f measure the output at f and so on and so on, so this would be terrible to compute or determine.

So, the smart way of doing this is to use reciprocity or inter reciprocity. So, you form the adjoint network. So, this is also LPTV at fs, we already know what, we already know the what do you call, we already know how to form the adjoint given the original network. And so what do we do, how do we exploit reciprocity? We will apply? We will apply what tone at the output, what tones at the output? We are interested on in an output only at f. So, what will we do? So, if you apply a tone e to the j 2 pi ft at the output. What will you get here?

Student: Tone at f1, the other tones at the f plus kfs, like H naught into j 2 pi…

Professor: So, basically I mean, what you will get, I mean here, remember the transfer function from a frequency, from a frequency f minus fs to f in the original network is the same as the transfer function from the frequency f to f minus fs in the adjoint network. Do you understand? And similarly, what is the transfer function from an input frequency of f minus fs to f in the original network? That is nothing but H1 of j 2 pi f minus fs, here something is confusing you. What is the problem?

Student: Transfer function from the, this one is like H1 to H2 are not related.

Professor: Correct, so, I mean that is clear because the input port and the output port have been interchanged. Then this, remember what is the frequency reversal theorem saying? The frequency reversal theorem is saying that H21 of minus m of j 2 pi f plus mfs is equal to H hat 12 of m of j 2 pi f. So, what does this mean in English?

It means that if I apply a tone at port 2 in the adjoint network. So, basically let me write this down, excite port, which port? Port 2, at frequency? At frequency f and measure at which port? Port 1 at frequency f plus mfs, I will get the ratio will be the same as when I excite the original network at f plus excite port 1 at f plus mfs and measure at port 2 at f.

And what are we interested in doing here? We are carrying out multiple measurements, we are exciting at f minus fs and measuring only at f, exciting at f, measuring at f, exciting at f plus fs, measuring it f. So, the input frequencies are changing here but the output frequency is fixed. So, in the adjoint or the inter reciprocal network the input becomes the output, input port becomes the output port, the input frequency becomes the output frequency.

So, write that down for you. So, this is you are exciting at f minus fs. Then measure at f excite at f minus, I mean f, measure at, sorry excite at f, measure at f, you excite f plus fs, measure at f, f plus 2 fs you measure at f1 not fs. So, this is the original network, network that is N. So, now in the adjoint N hat what we have to do?

Remember what is frequency reversal saying that the input and the output ports are reversed but the input and output frequencies are also reversed. So, what should we do, at the output port you should excite the adjoint with f, you measure at f minus fs, you excite at, f you measure at f, you excite at f, you measure it f plus fs, you excite at f, f plus 2 fs and so on.

And what is the transfer function which relates this input to this output in the original network? H1 of j 2 pi f plus f, sorry f minus, f minus fs, what this inter reciprocity was telling you is that the transfer function here is going to be the same as the transfer function there, the transfer function here which is H0 of j 2 pi f is going to be the same in the adjoint.

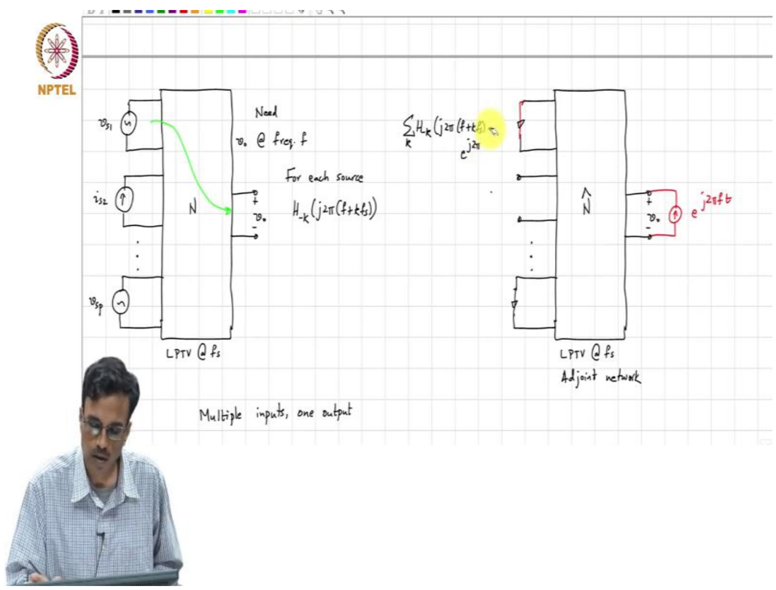
Similarly, this is going to be H minus 1 of j 2 pi f plus fs. And that is going to be the same there and so on and so forth. So, what do we do in the, so now what do we need to do in the adjoint network we just need to excite the output port with a current only at one frequency, namely the output frequency of interest which is f and automatically the voltage that is developed at port 1.

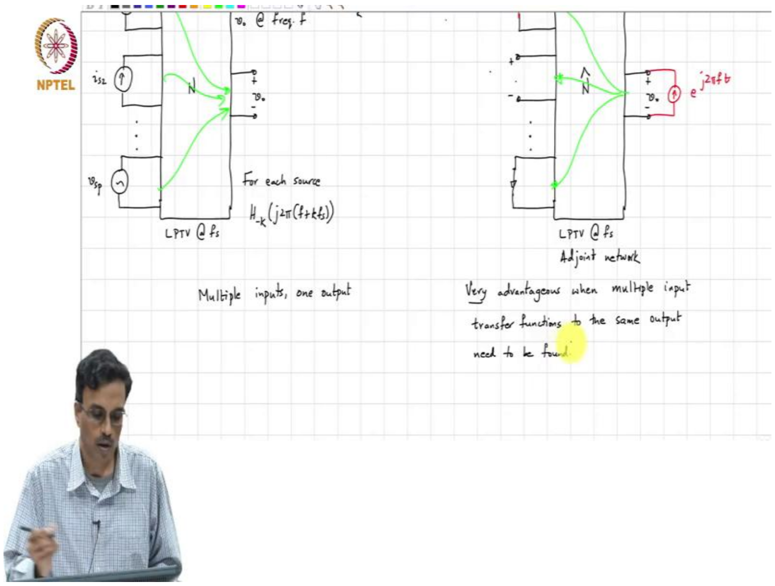
Remember what is that if you, it is nothing but by reciprocity it should be H hold on we will start with H1 of j 2 pi into f minus fs times e to the j minus 2, sorry there should be e to the j 2 pi f minus fs times t plus H0 of j 2 pi f times e to the j 2 pi ft plus H minus 1 of j 2 pi f plus fs e to the j 2 pi ft and so on. So, what is the advantage of using the adjoint network therefore?

Student: It is like not only multiple ports, multiple frequencies we can also apply at single frequency and…

Professor: Yeah exactly. So, basically the advantage of this is that only one experiment or one evaluation of N hat exciting it with, exciting it with a single tone and the voltage developed across port 1 automatically gives you all the harmonic transfer functions in one shot. And this is the advantage with respect to, when you have only, even when you have only one input port and one output, when you have multiple input ports.

(Refer Slide Time: 15:01)





So, this is N, this is LPTV at fs, so multiple inputs, same output port or I would say one output which is what is common most of the time. So, what do you do? Well, let us say you had a voltage source here is2, blah, blah, blah, some other voltage source vsp. And if we want to find the strength of the output let us call this vo, let us say we need vo at frequency f. What do you do?

I mean if you did not know about inter reciprocity and reciprocity what would you do? You will follow supervision principle, you not only have to do, so basically you need to calculate for example in this situation, you will need to calculate the H sub k for corresponding to, I mean we need to find for instance H sub 0 corresponding to source 1 of j 2 pi f or in other words in general for each source you would need to compute H sub minus k of j 2 pi f plus k fs.

Now, this is now doubly painful, because you not only have to find, for each source you have to find these multiple transfer functions. And if you have multiple sources, you do this all over again for each source. Fortunately, if you did, if you understand reciprocity you can be much smarter about it.

And therefore what do you do? Use the adjoint network and so when you do that, you will excite the output port with, what do we do? So, let us say is the adjoint network, how do we deal with it, how do we be able to exploit the adjoint to find, we apply a current source at our output frequency of interest which is e to the j 2 pi ft. And as the case may be if the input was a voltage then you replace it with a short circuit, if the input is a current you replace it with an open circuit.

And what would you do? You would have to measure this. So, this would give you all the harmonic transfer functions in one shot. So, this will give you H sub minus k of j 2 pi f plus k fs e to the j 2 pi f plus, let me write this clearly, this will give us sigma k, H sub minus k of j 2 pi f plus k fs e to the j 2 pi into f plus kfs times t.

And so this is for source one, similarly at the same time by looking at the voltage across this port you will get the corresponding transfer functions here and so in one shot you are able to get all these transfer functions. So, you just need to excite the output port to the e to the j 2 pi ft and at the same and in one shot you not only get the transfer functions to multiple ports but you also get transfer functions to multiple frequencies.

So, like in the time invariant case the use of the adjoint greatly simplifies noise calculations. And finding the adjoint network is trivial because you already have the MNA matrix of the original network, you do not, I mean transposition basically, all that it means is that as far as a computer is concerned for instance it is just simply changing the address with which it has to pick out that the row column address, that is all.

And if you are working things out it is easy enough, you simply flip the orientation of the sources and you will get all these, you do not have to work very hard, in one shot you can basically get all these transfer functions. So, the I think this is a good breakpoint to stop, so let me summarize, so this is very advantageous when multiple input transfer functions to the same output need to be found.